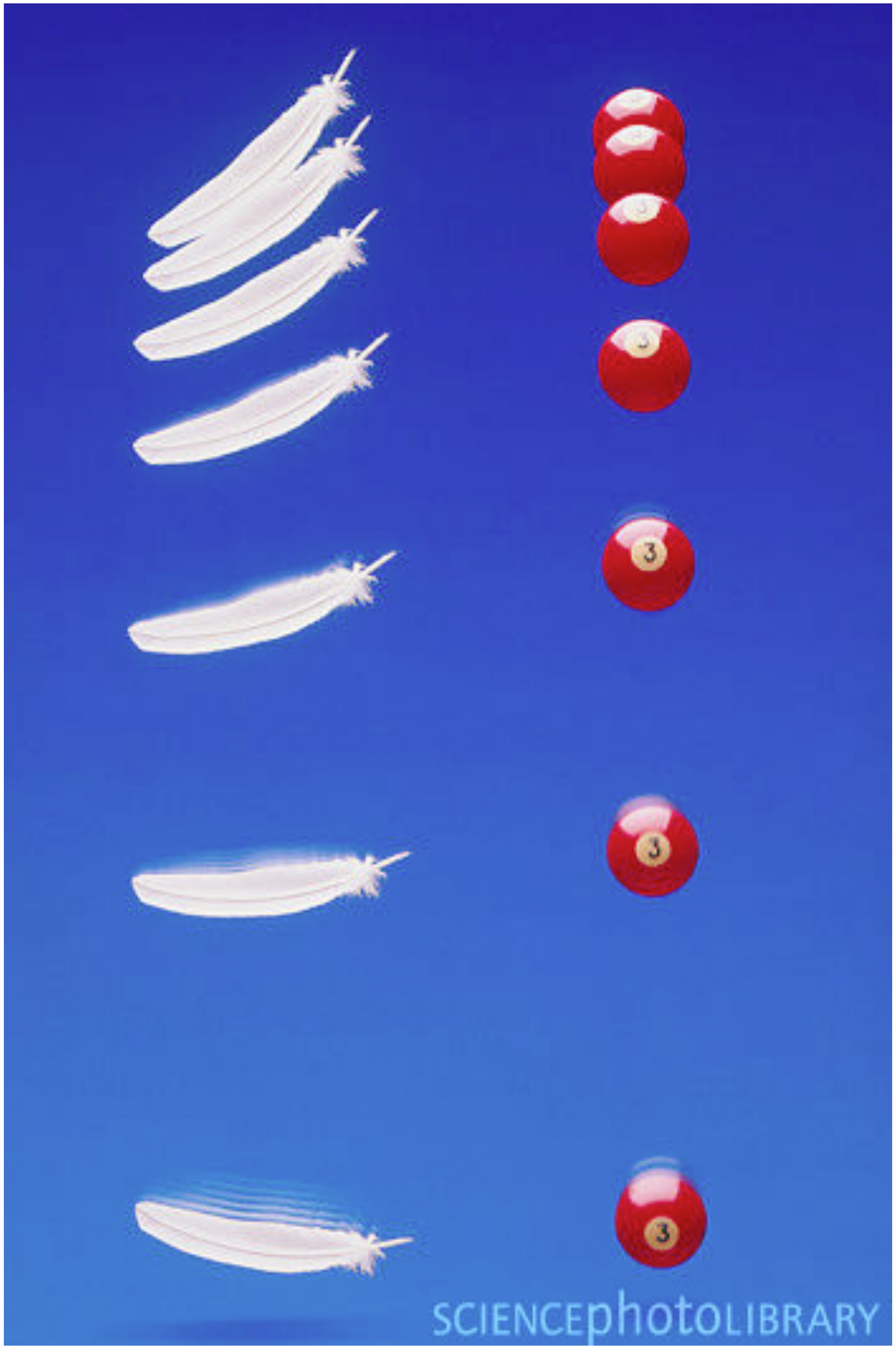
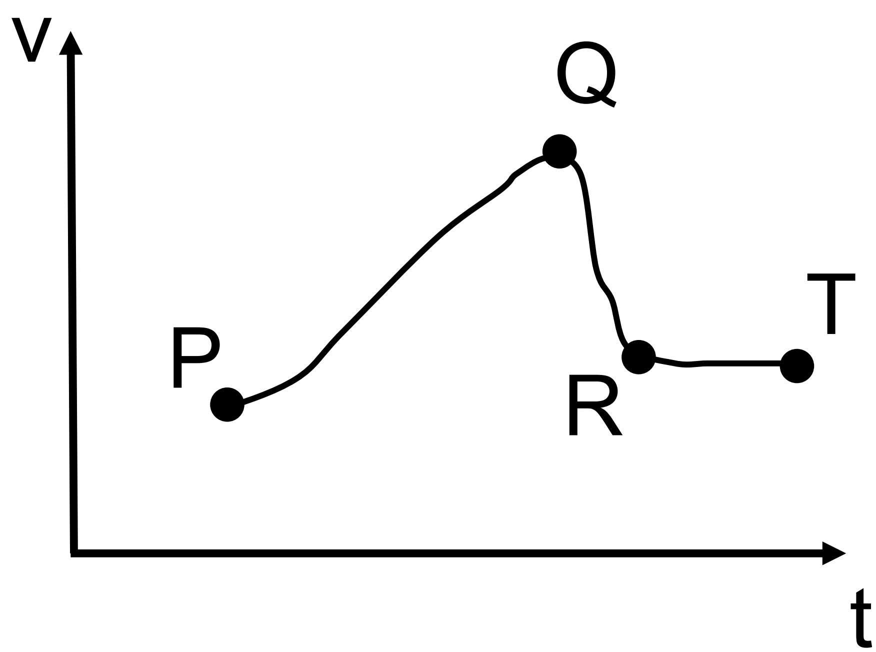


Welcome back to Physics 211

Lecture 2-1

- Last time:
  - Displacement, velocity, graphs
- Today:
  - Using graphs to solve problems
  - Constant acceleration, free fall

1-2.6-8: Acceleration from graph of v(t)



What is a_{av} for

6. PQ ?
7. QR ?
8. RT ?

1. $a_{avg} > 0$
2. $a_{avg} < 0$
3. $a_{avg} = 0$

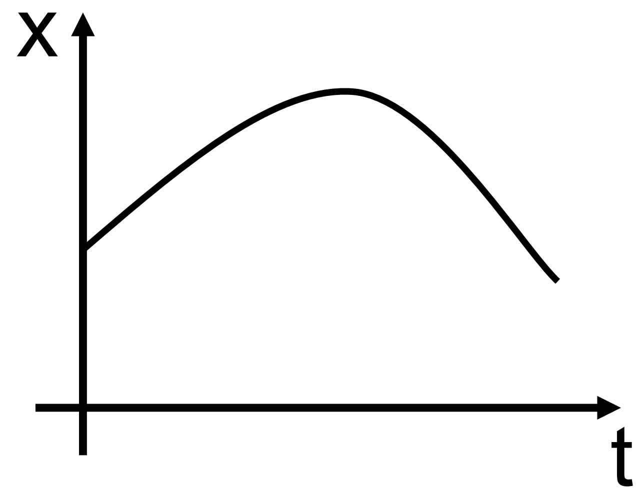
- Slope measures acceleration
  - –Positive a means v is increasing
  - –Negative a means v decreasing

Cart demo

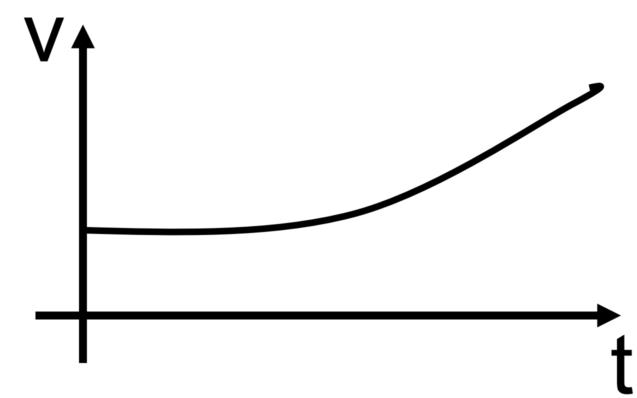
- Sketch graphs of position, velocity, and acceleration for cart

Interpreting x(t) and v(t) graphs

- Slope at any instant in **x(t)** graph gives instantaneous **velocity**



- Slope at any instant in **v(t)** graph gives instantaneous **acceleration**

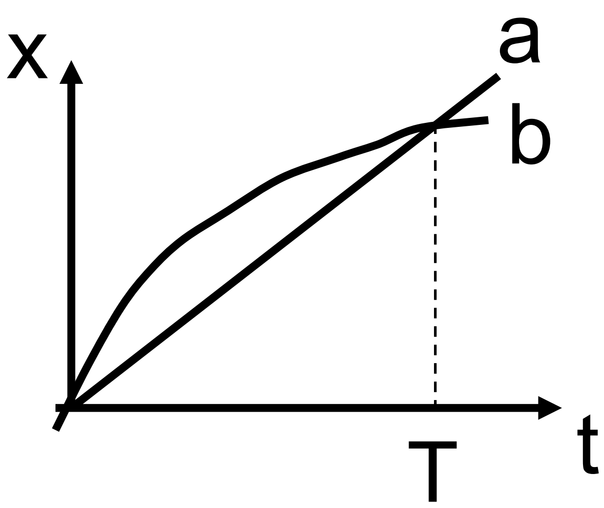


- What else can we learn from an x(t) graph?

Clicker question 2-1.1: You are throwing a ball up in the air. At its highest point, the ball's

1. Velocity v and acceleration a are zero
2. v is non-zero but a is zero
3. v and a are both non-zero
4. a is non-zero but v is zero

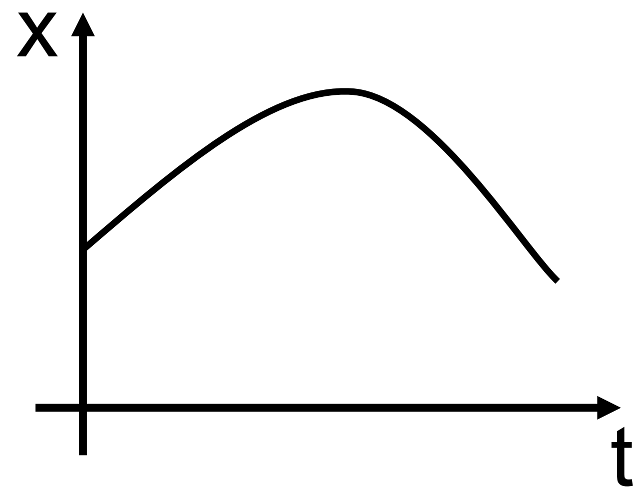
2-1.2 The graph shows 2 trains running on parallel tracks. Which is true:



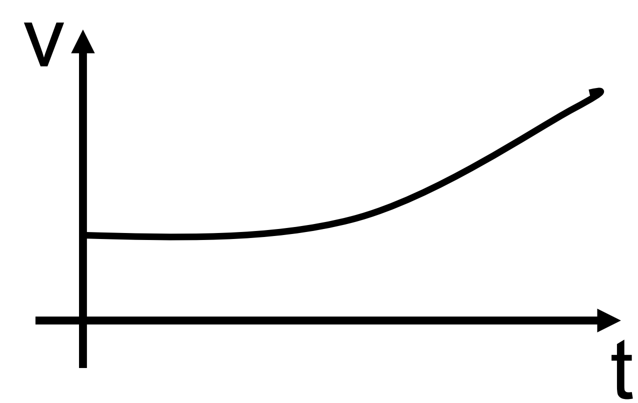
1. At time T both trains have same v
2. Both trains speed up the whole time
3. Both trains have the same v for some t<T
4. Somewhere, both trains have the same acceleration

Interpreting x(t) and v(t) graphs

- Slope at any instant in **x(t)** graph gives instantaneous **velocity**



- Slope at any instant in **v(t)** graph gives instantaneous **acceleration**



- What else can we learn from an x(t) graph?

Acceleration from x(t)

- Rate of change of slope in x(t) plot equivalent to *curvature* of x(t) plot

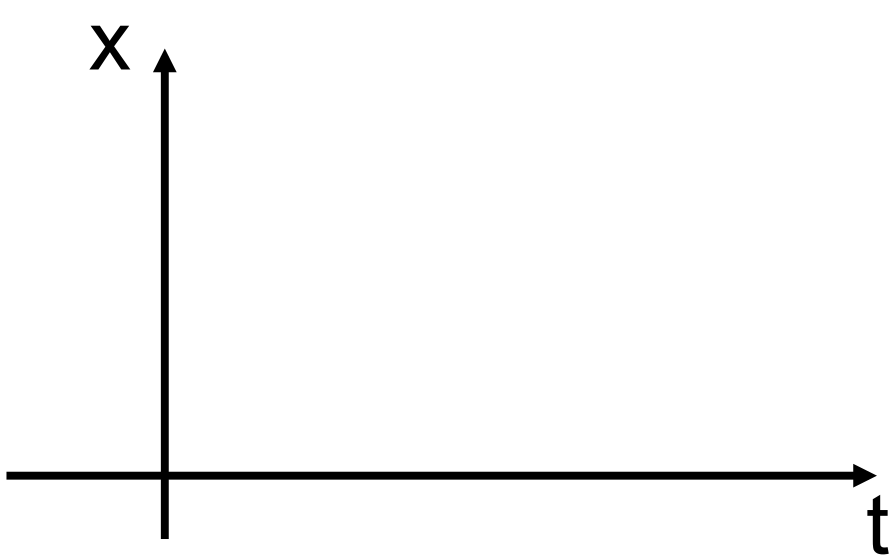
- Mathematically, we can write this as

  a =

  – Negative curvature
    - a < 0

  – Positive curvature
    - a > 0

  – No curvature
    - a = 0



Sample problem

- An object's position as a function of time is given by $x(t) = (3.00\ m) - (2.00\ m/s)\ t + (3.00\ m/s^2)\ t^2$.
  - Clicker 2-1.3:Calculate the avg. accel. between t = 2.00s and t = 3.00 s.

1. $3\ m/s^2$
2. $2\ m/s^2$
3. $4\ m/s^2$
4. $12\ m/s^2$
5. None of the above

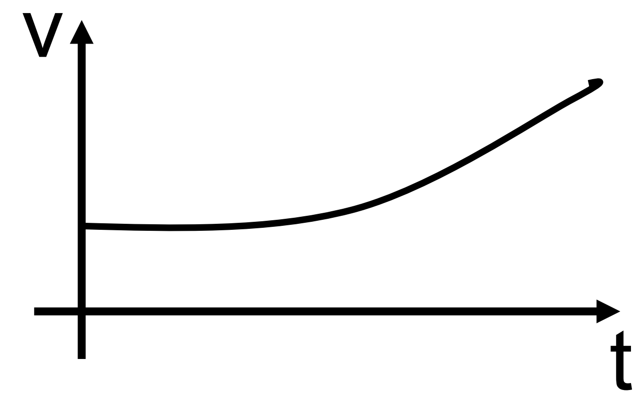
Sample problem

- An object's position as a function of time is given by $x(t) = (3.00\ m) - (2.00\ m/s)\ t + (3.00\ m/s^2)\ t^2$.
  - Calculate the avg. accel. between t = 2.00s and t = 3.00 s.
  - Clicker 2-1.4: Calculate the inst. accel. at (i) t = 2.00 s; (ii) t = 3.00 s.

1. 6 m/s^2, 6 m/s^2
2. 3 m/s^2, 3 m/s^2
3. 4 m/s^2, 6 m/s^2
4. 6 m/s^2, 4 m/s^2
5. None of the above

Displacement from velocity curve?

- Suppose we know v(t) (say as graph), can we learn anything about x(t) ?



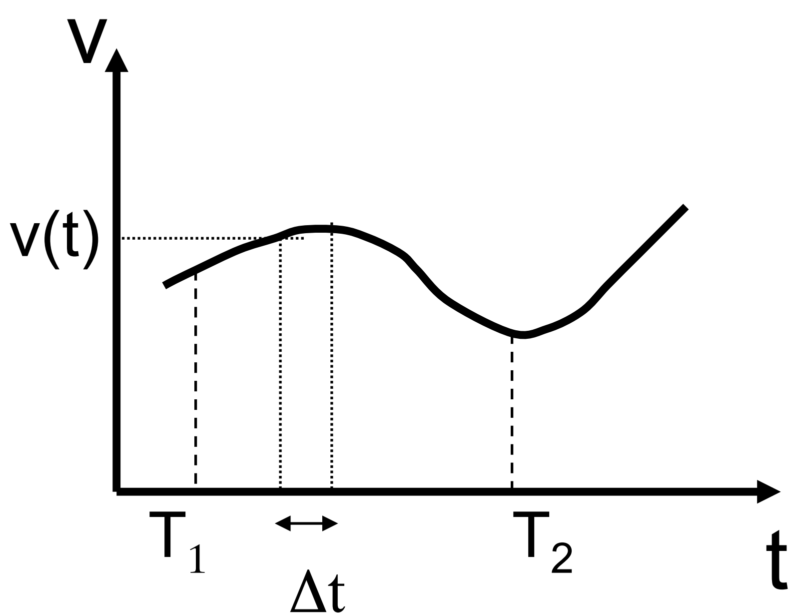
- Consider a small time interval Δt

$$v = \Delta x/\Delta t \rightarrow \Delta x = v\Delta t$$

- So, total displacement is the sum of all these small displacements Δx

$$x = \Sigma\, \Delta x = \lim_{\Delta t \rightarrow 0} \Sigma\, v(t)\Delta t =$$

Graphical interpretation



Displacement between T_1 and T_2 is area under v(t) curve

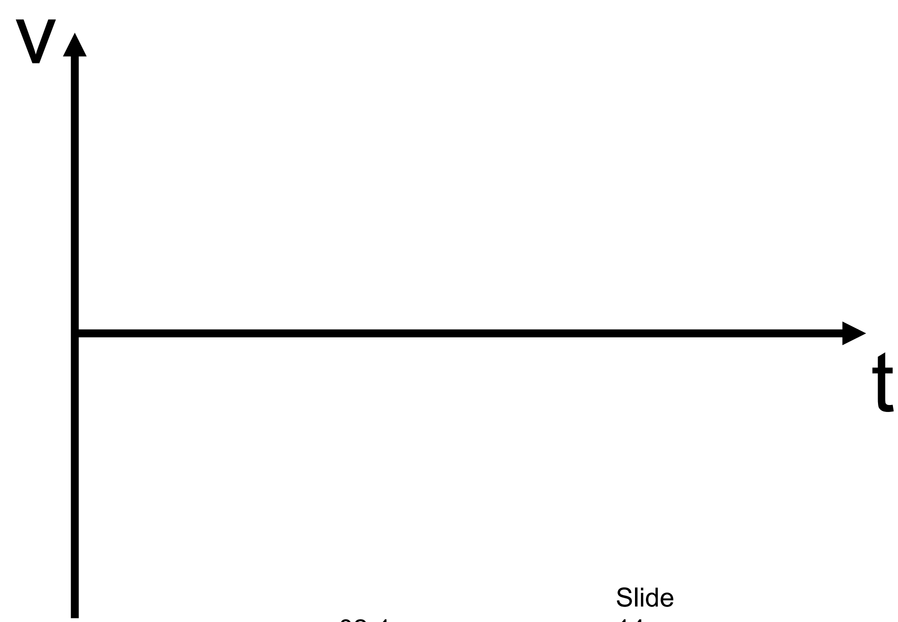
Displacement – integral of velocity

$\lim_{\Delta t \to 0} \Sigma\, \Delta t\, v(t)$ = area under v(t) curve

note: `area' can be positive or negative

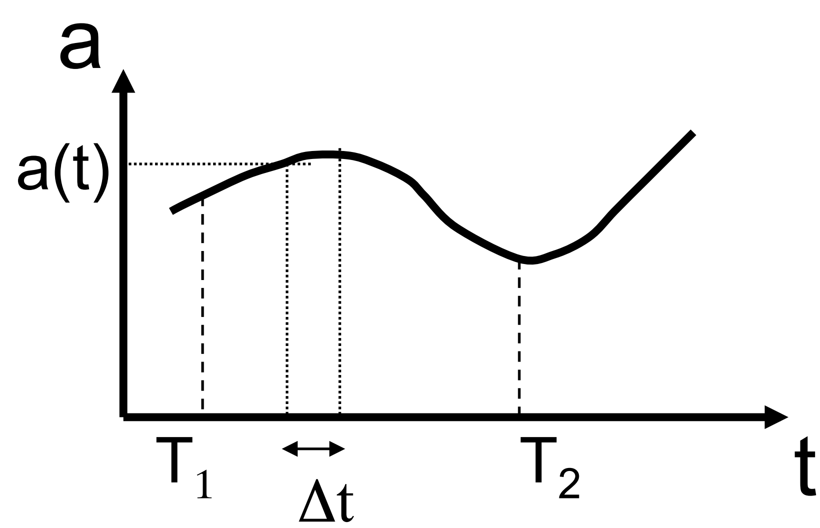
*Consider v(t) curve for cart in different situations…

*Net displacement?



Velocity from acceleration curve

- Similarly, change in *velocity* in some time interval is just area enclosed between curve a(t) and t-axis in that interval.



Summary

- **velocity** $v = dx/dt$
  = *slope* of x(t) curve
  – NOT x/t !!
- **displacement** Δx is
  $$\int v(t)dt$$
  = *area* under v(t) curve
  – NOT vt !!

- **accel.** $a = dv/dt$
  = *slope* of v(t) curve
  – NOT v/t !!
- **change in vel.** Δv is
  $$\int a(t)dt$$
  = *area* under a(t) curve
  – NOT at !!

Simplest case with non-zero acceleration

- Constant acceleration: $a = a_{av}$
- Can find simple equations for x(t), v(t) in this case

1st constant acceleration equation

- From definition of a_{av}: $a_{av} = \Delta v/\Delta t$

$$\text{Let } a_{av} = a,\ \Delta t = t,\ \Delta v = v - v_0$$

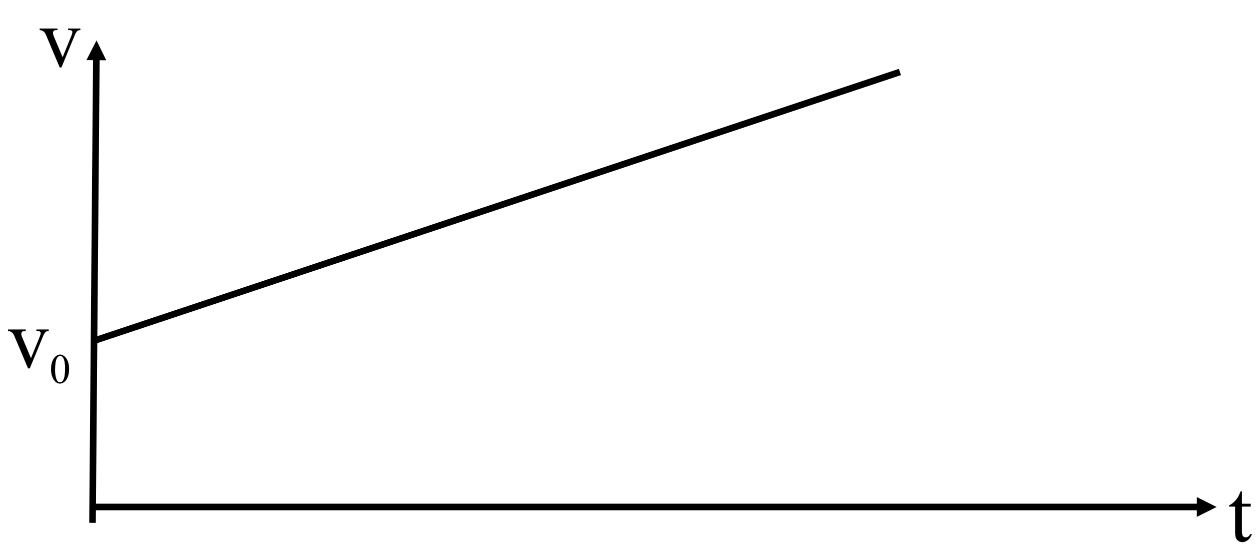
1^{st} constant acceleration equation

- From definition of a_{av}: $a_{av} = \Delta v/\Delta t$

$$\text{Let } a_{av} = a,\ \Delta t = t,\ \Delta v = v - v_0$$

$$\text{Find: } v = v_0 + at$$

*equation of straight line in v(t) plot

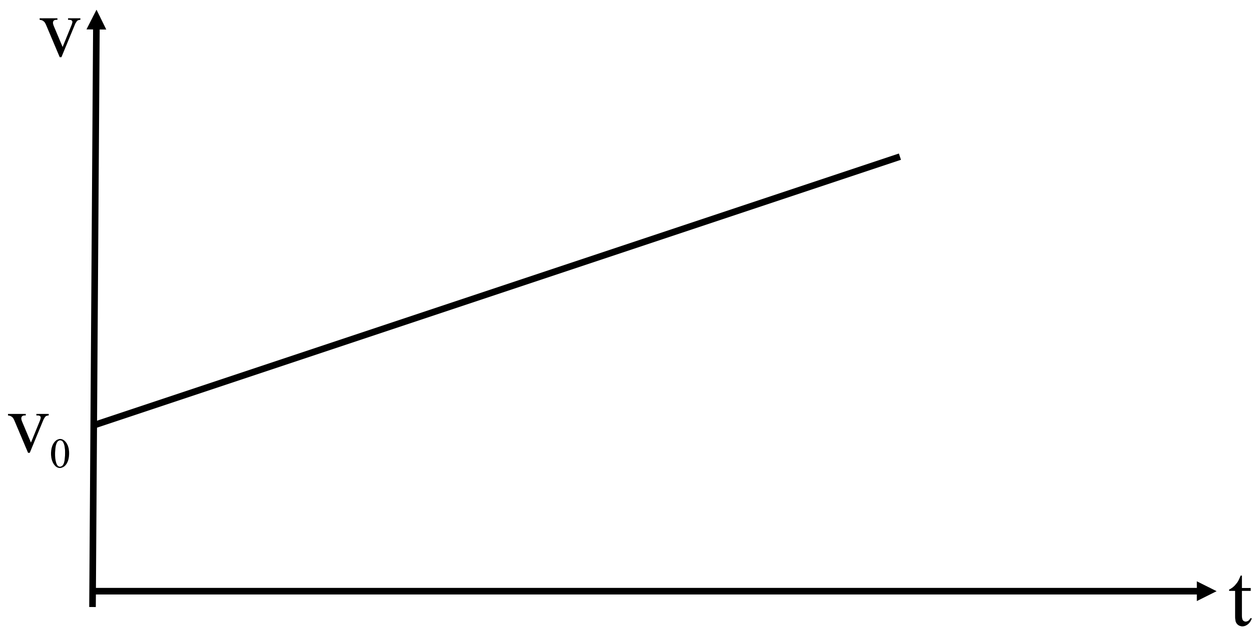


2^{nd} const. acceleration equation

- Notice: graph makes it clear that

$$v_{av}=(1/2)(v + v_0)$$

$$x - x_0 = (1/2)(v + v_0)t$$



3rd constant acceleration equation

- Using 1st constant acceleration equation

$$v = v_0 + at$$

insert into relation between x, t, and v_{av} :

$$x - x_0 = v_{av}t = (1/2)(v + v_0)t$$

$$(\text{set } \Delta t = t, \text{ i.e. take } t_0 = 0)$$

3^{rd} constant acceleration equation

- Using 1^{st} constant acceleration equation

$$v = v_0 + at$$

insert into relation between x, t, and v_{av} :

$$x - x_0 = v_{av}t = (1/2)(v + v_0)t$$

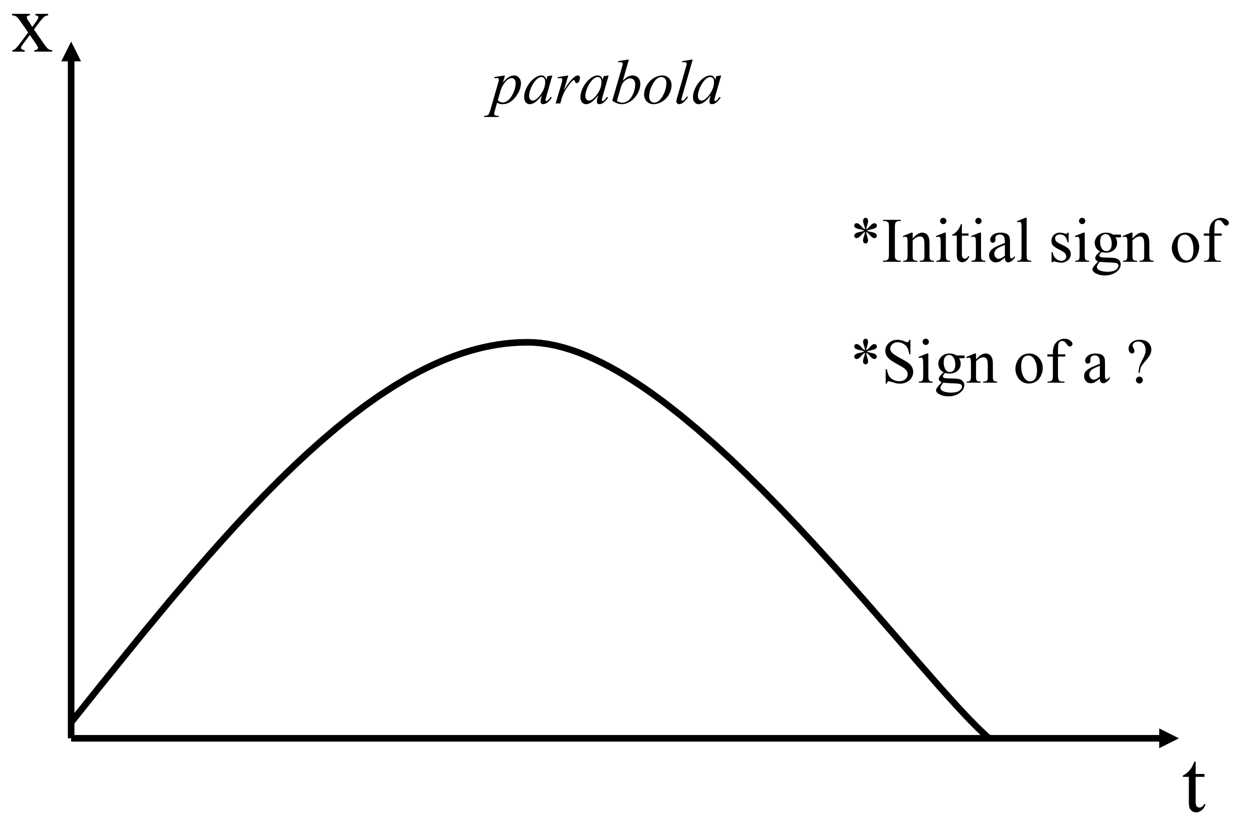
(set $\Delta t = t$, i.e. take $t_0 = 0$)

yields: $$x - x_0 = (1/2)(2v_0 + at)t$$

or: $$x = x_0 + v_0 t + (1/2)at^2$$

x(t) graph- constant acceleration

$$x = x_0 + v_0 t + (1/2)at^2$$



parabola

*Initial sign of v ?

*Sign of a ?

4^{th} constant acceleration equation

- Can also get an equation independent of t
- Substitute $t = (v - v_0)/a$ into

$$x - x_0 = (1/2)(v + v_0)t$$

4^{th} constant acceleration equation

- Can also get an equation independent of t
- Substitute $t = (v - v_0)/a$ into

$$x - x_0 = (1/2)(v + v_0)t$$

we get: $$2a(x - x_0) = v^2 - v_0^2$$

or: $$v^2 = v_0^2 + 2a(x - x_0)$$

Clicker 2-1.5: An object moves with constant acceleration, starting from rest at t = 0 s. In the first four seconds, it travels 10 cm.

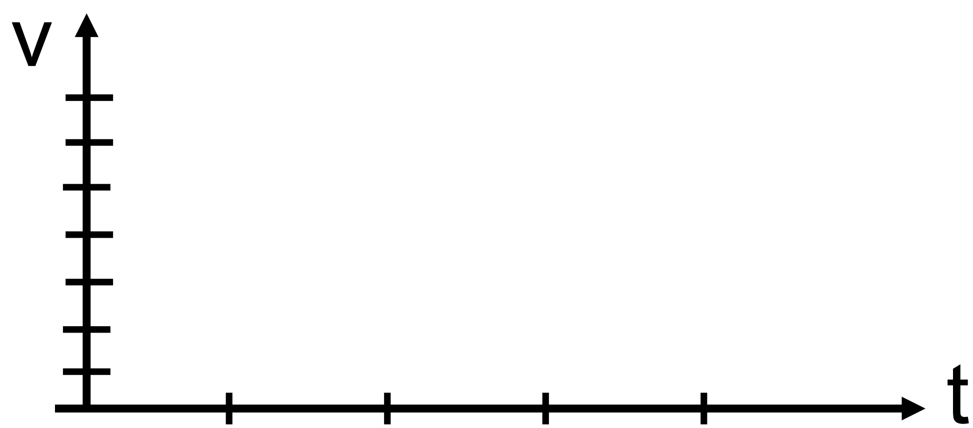
What will be the displacement of the object in the following four seconds (*i.e.* between t = 4 s and t = 8 s)?

1. 10 cm
2. 20 cm
3. 30 cm
4. 40 cm

Rolling disk demo

- Compute average velocity for each section of motion (between marks)
- Measure time taken (metronome)
- Compare v at different times

(i) 10 cm (ii) 30 cm (iii) 50 cm (iv) 70 cm



Motion with **constant** acceleration:

$$v = v_0 + at$$

$$v_{av} = (^1/_2)\,(v_0 + v)$$

$$x = x_0 + v_0 t + (^1/_2)\,a\,t^2$$

$$v^2 = v_0^2 + 2a\,(x - x_0)$$

*where x_0, v_0 refer to time = 0 s ;

x, v to time *t*

Freely Falling Bodies:

- Near the surface of the earth, neglecting air resistance and the earth's rotation, <u>all</u> objects experience the same downward acceleration due to their gravitational attraction with the earth:

$$g = 9.8\ \text{m/s}^2$$

  – Near = height small relative to radius of earth

Example of constant acceleration:

- ***Free fall demo***

- Compare time taken by feather and billiard ball to fall to the ground from the same height

- Influence of air in room?

Sample problem

- A brick is dropped (zero initial speed) from the roof of a building. The brick strikes the ground in 2.50 s. Ignore air resistance, so the brick is in free fall.
  – How tall, in meters, is the building?
  – What is the magnitude of the brick's velocity just before it reaches the ground?
  – Sketch a(t), v(t), and y(t) for the motion of the brick.

Motion in more than 1 dimension

- Have seen that 1D kinematics is written in terms of quantities with a magnitude and a sign
- Examples of 1D **vectors**
- To extend to $d > 1$, we need a more general definition of vector

Reading assignment

- Vectors, 2D motion
- 3.1-3.4, and reread 1.3 in textbook (review of vectors)
- 4.1-4.2, motion in 2D

Vectors: basic properties

- are used to denote quantities that have magnitude *and* direction
- can be added and subtracted
- can be multiplied or divided by a number
- can be manipulated **graphically** (*i.e.,* by drawing them out) or algebraically (by considering components)

Vectors: examples and properties

- Some <u>vectors</u> we will encounter:
  position, velocity, force

- Vectors commonly denoted by boldface letters, *or* sometimes arrow on top

- Magnitude of **A** is written |**A**|, *or* no boldface and no absolute value signs

- Some quantities which are <u>not</u> vectors:
  temperature, pressure, volume ….

Drawing a vector

- A vector is represented graphically by a line with an arrow on one end.
- Length of line gives the **magnitude** of the vector.
- Orientation of line and sense of arrow give the **direction** of the vector.
- Location of vector in space does not matter -- two vectors with the same magnitude and direction are equivalent, independent of their location