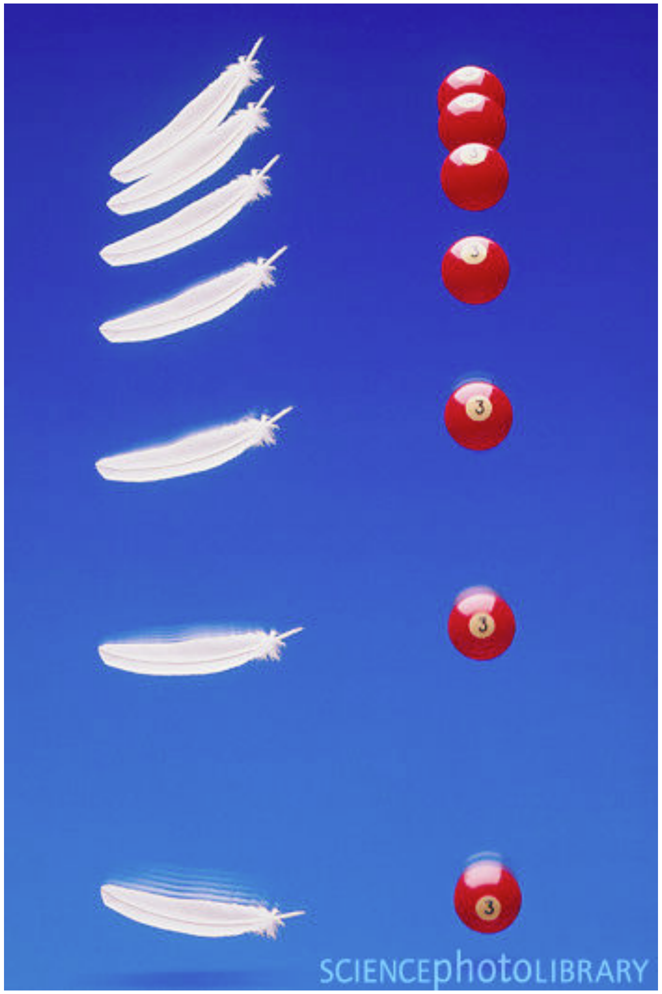
Welcome back to Physics 211



Today's agenda:

- *Vectors in Mechanics*
- *Motion in two dimensions*

Motion with **constant** acceleration:

$$v = v_0 + at$$

$$v_{av} = (^1/_2)\,(v_0 + v)$$

$$x = x_0 + v_0 t + (^1/_2)\,a\,t^2$$

$$v^2 = v_0^2 + 2a\,(x - x_0)$$

*where x_0, v_0 refer to time = 0 s;

x, v to time *t*

Freely Falling Bodies:

- Near the surface of the earth, neglecting air resistance and the earth's rotation, all objects experience the same downward acceleration due to their gravitational attraction with the earth:

$$\mathbf{g = 9.8\ m/s^2}$$

  – Near = height small relative to radius of earth

Sample problem

- A brick is dropped (zero initial speed) from the roof of a building. The brick strikes the ground in 2.50 s. Ignore air resistance, so the brick is in free fall.
  - How tall, in meters, is the building?

Sample problem

- A rock is tossed straight up with a speed of 20 m/s. When it returns, it falls into a hole 10 m deep.

  How long is the rock in the air, from the instant it is released until it hits the bottom of the hole?

Sample problem

- When jumping, a flea accelerates at an astounding 1000 m/s^2, but over only the very short distance of 0.50 mm. If a flea jumps straight up, and if air resistance is neglected (a rather poor approximation in this case), how high does the flea go?

Motion in more than 1 dimension

- Have seen that 1D kinematics is written in terms of quantities with a magnitude and a sign
- Examples of 1D **vectors**
- To extend to d > 1, we need a more general definition of vector

Vectors: basic properties

- are used to denote quantities that have magnitude *and* direction
- can be added and subtracted
- can be multiplied or divided by a number
- can be manipulated **graphically** (*i.e.,* by drawing them out) or algebraically (by considering components)

Vectors: examples and properties

- Some vectors we will encounter: *position, velocity, force*

- Vectors commonly denoted by boldface letters, *or* sometimes arrow on top

- Magnitude of **A** is written |**A**|, *or* no boldface and no absolute value signs

- Some quantities which are not vectors: *temperature, pressure, volume ….*

Drawing a vector

- A vector is represented graphically by a line with an arrow on one end.
- Length of line gives the **magnitude** of the vector.
- Orientation of line and sense of arrow give the **direction** of the vector.
- Location of vector in space does not matter -- two vectors with the same magnitude and direction are equivalent, independent of their location

Adding vectors

<u>To add vector **B** to vector **A**</u>

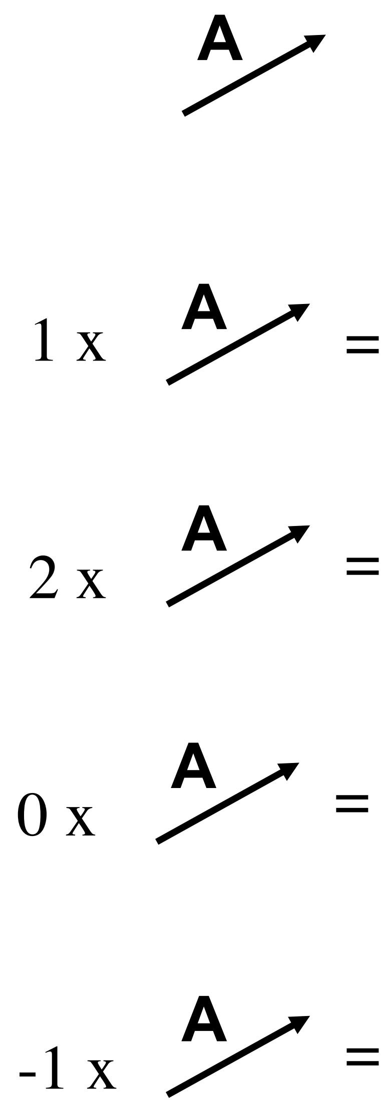
- Draw vector **A**

- Draw vector **B** with its tail starting from the tip of **A**

- The sum vector **A**+**B** is the vector drawn from the tail of vector **A** to the tip of vector **B**.

Multiplying vectors by a number

- Direction of vector not affected (care with negative numbers – see below)
- Magnitude (length) scaled, e.g.
  - 1***A**=**A**
  - 2***A** is given by arrow of twice length, but same direction
  - 0***A** = **0** null vector
  - **-A** = **-1*A** is arrow of same length, but reversed in direction

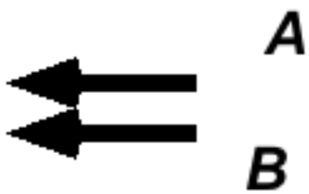


$A + B =$

$B - A =$

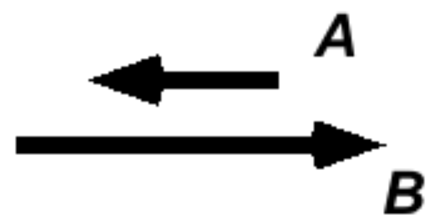
$A + B =$

$A - B =$



$A + B =$

$B - A =$

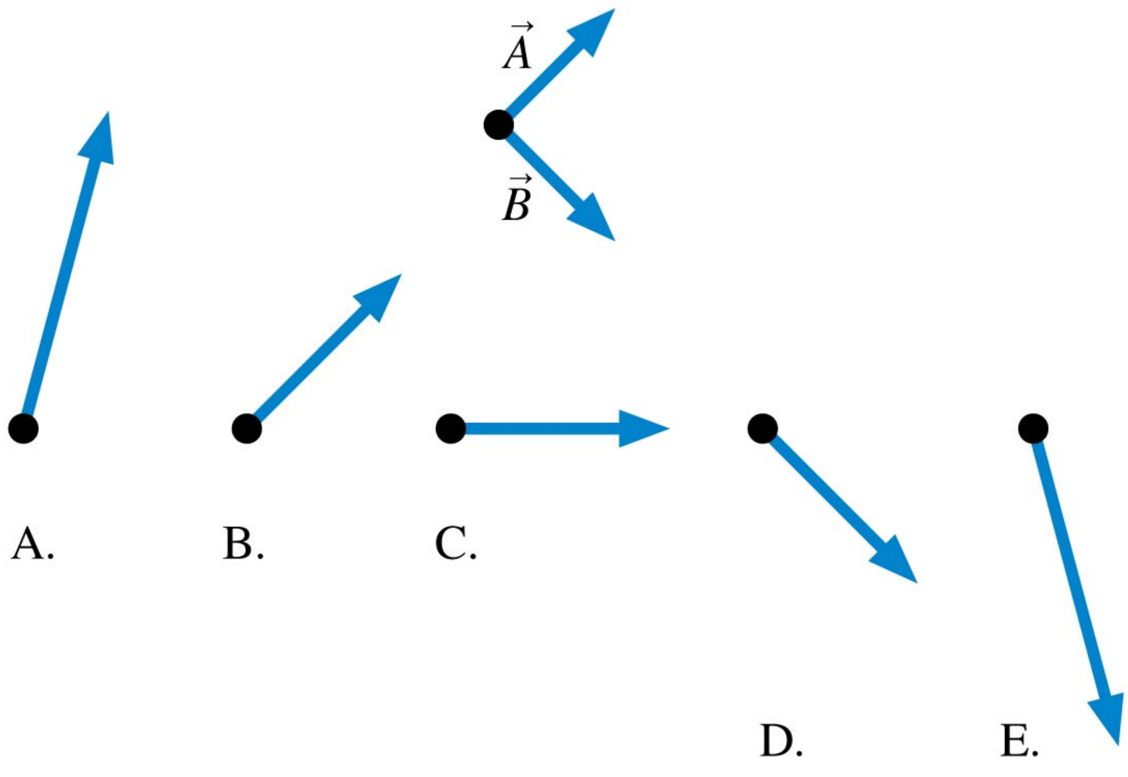


$A + B =$

$A - B =$

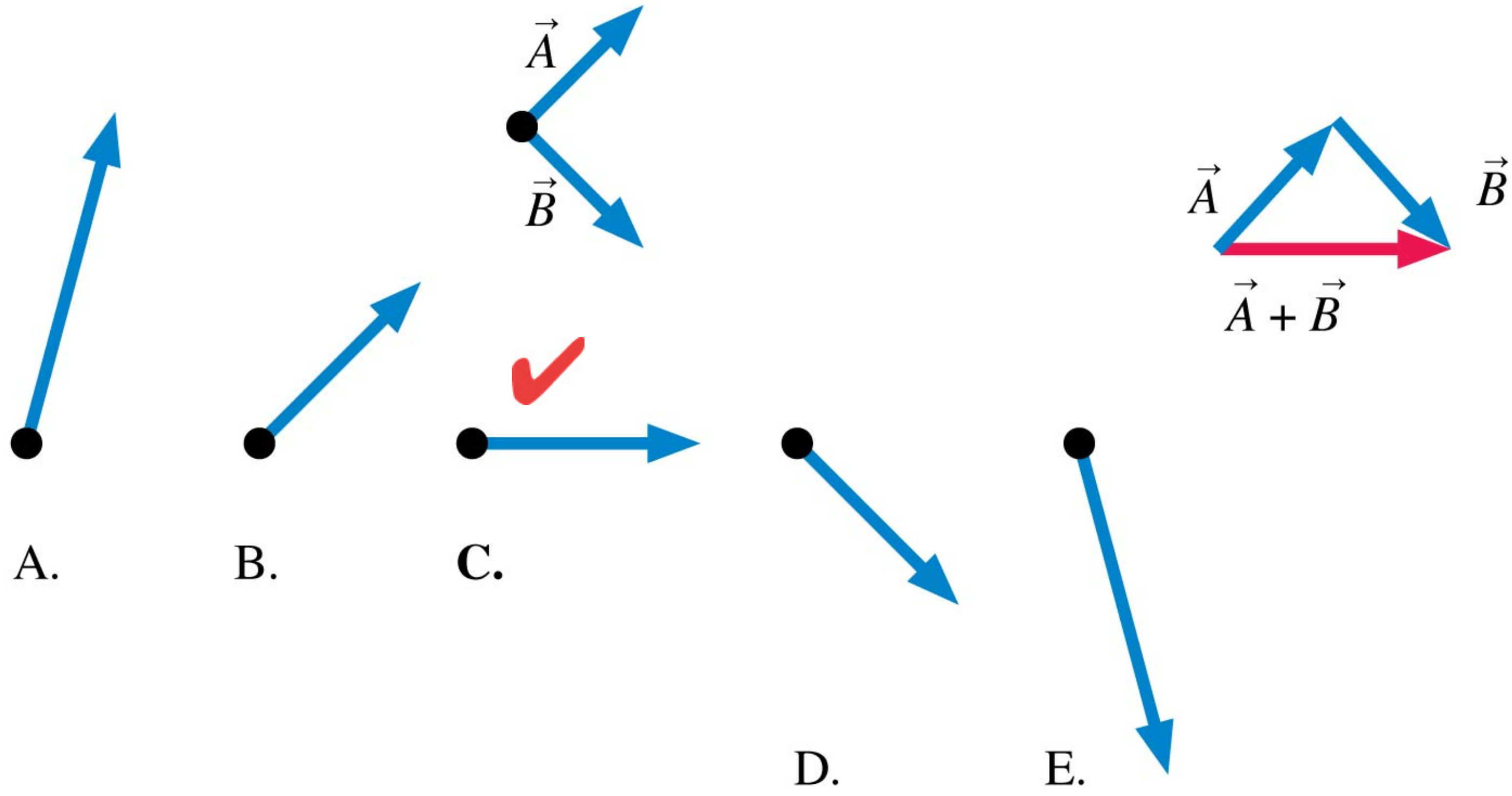
Clicker 3-1.1:

Which of the vectors in the second row shows $\vec{A} + \vec{B}$?



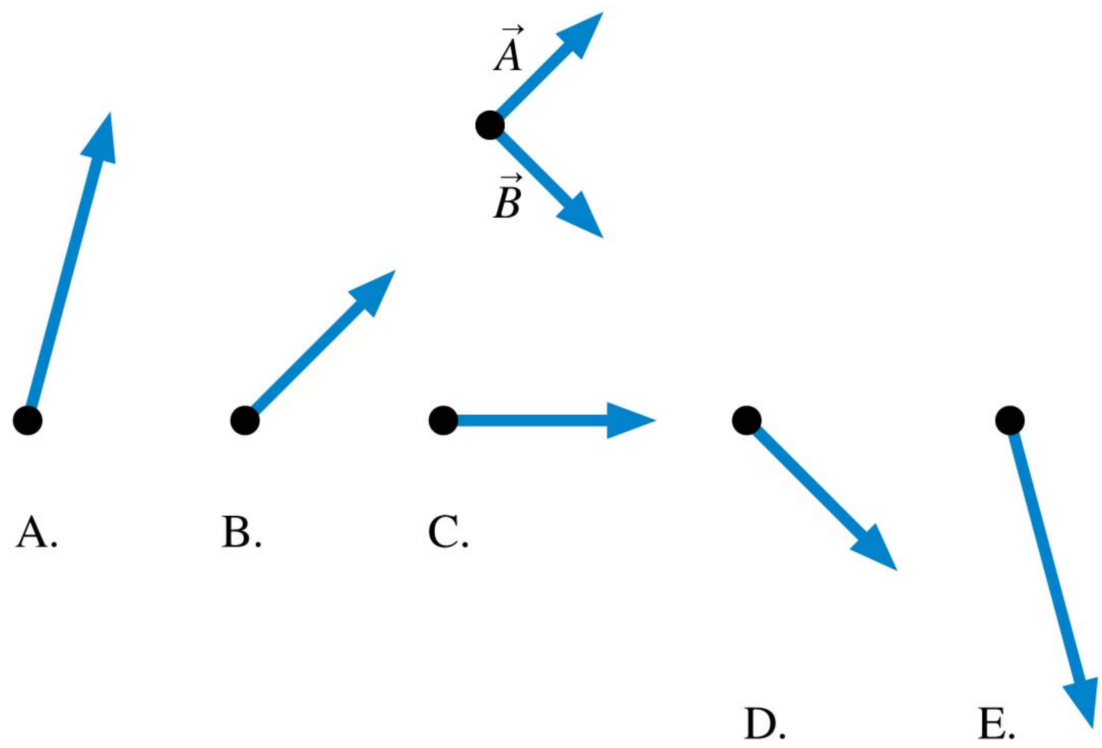
Clicker 3-1.1:

Which of the vectors in the second row shows $\vec{A} + \vec{B}$?



Clicker 3-1.2:

Which of the vectors in the second row shows $2\vec{A} - \vec{B}$?



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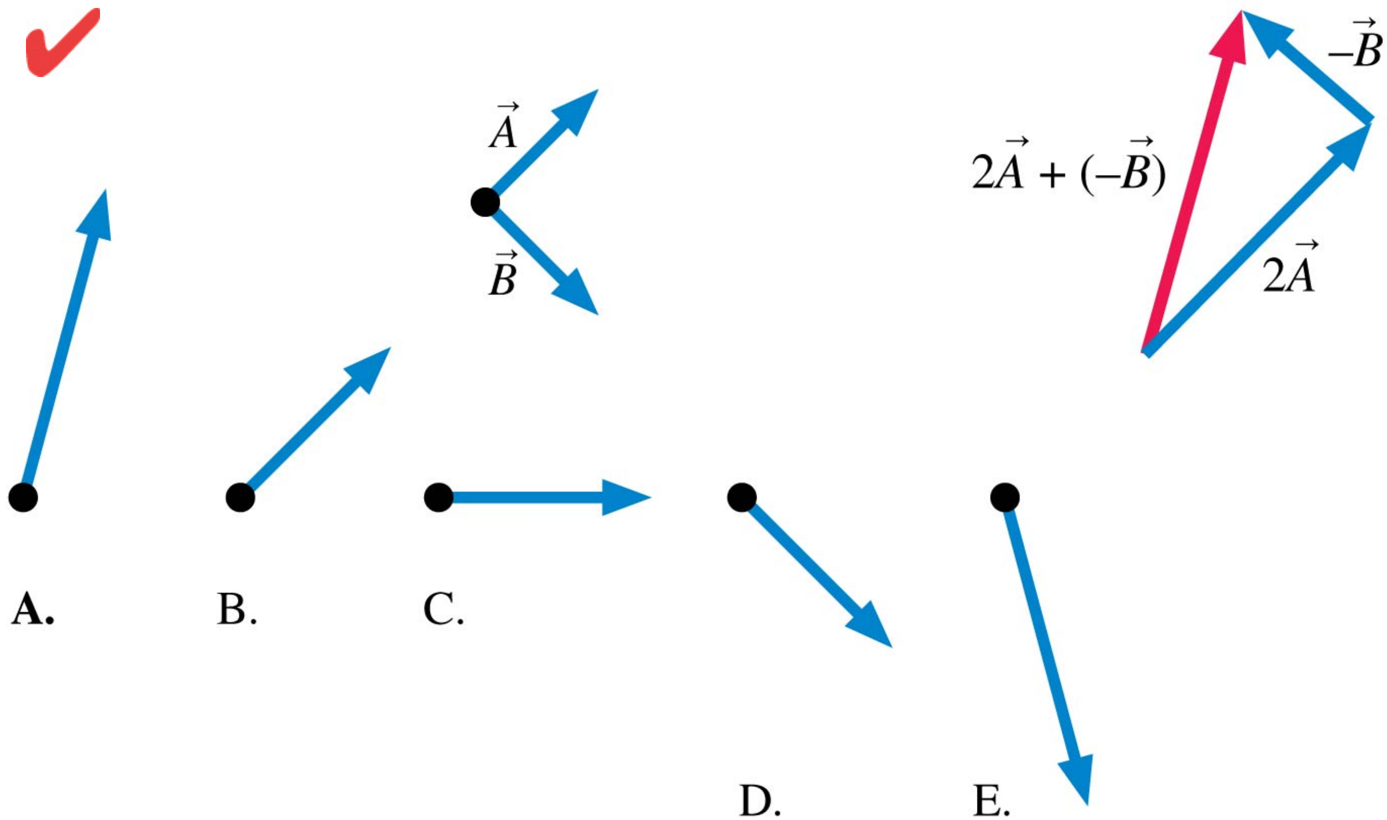


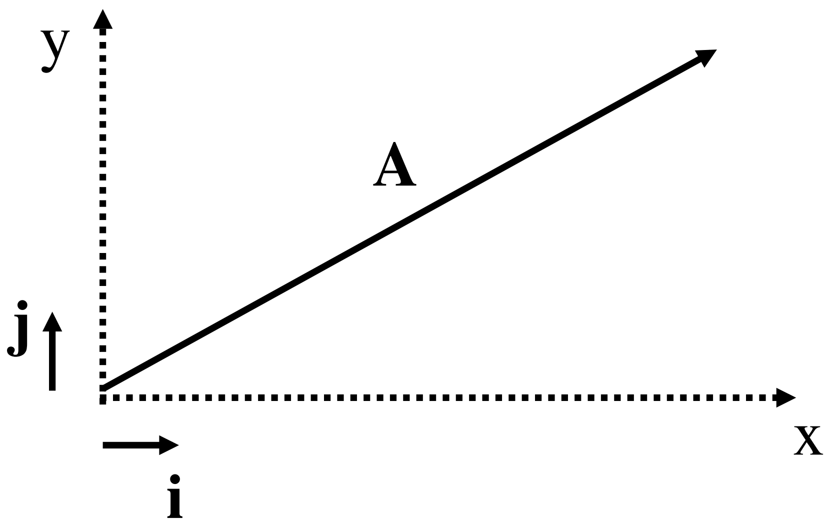
Projection of a vector

"How much a vector acts along some arbitrary direction"

Component of a vector

Projection onto one of the coordinate axes *(x, y, z)*

Components



$\mathbf{A} = \mathbf{A_x} + \mathbf{A_y}$

$\mathbf{A} = a_x\mathbf{i} + a_y\mathbf{j}$

i = *unit vector* in x direction

j = *unit vector* in y direction

<u>Projection</u> of **A** along coordinate axes

a_x, a_y = *components* of vector **A**

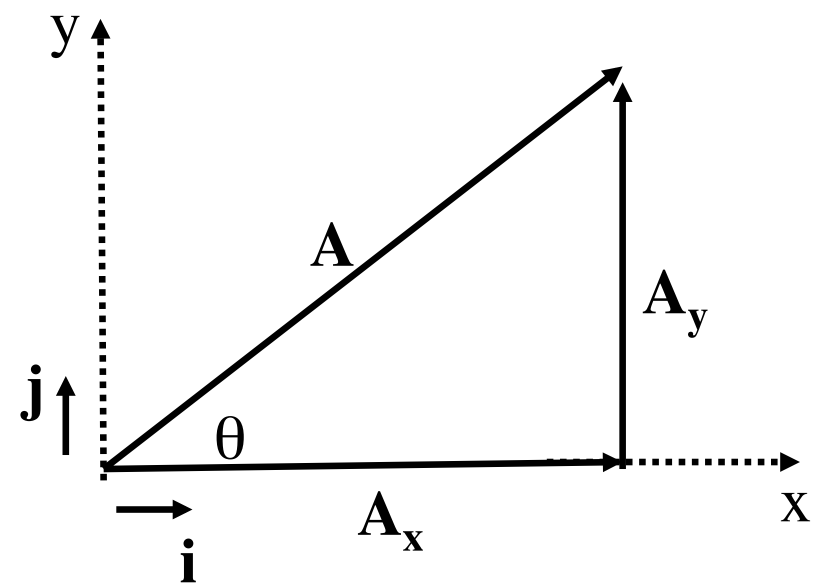
Vector demo

More on vector components

- Relate components to direction (2D):

$$a_x = |\mathbf{A}|\cos\theta,$$

$$a_y = |\mathbf{A}|\sin\theta$$



or

- *Direction:* $\tan\theta = a_y/a_x$
- *Magnitude:* $|A|^2 = a_x^2 + a_y^2$

Clicker 3-1.3: A bird is flying along a straight line in a direction somewhere East of North. After the bird has flown a distance of 2.5 miles, it is 2 miles North of where it started.

How far to the East is it from its starting point?

1. 0 miles
2. 0.5 miles
3. 1.0 mile
4. 1.5 miles

Why are components useful?

- *Addition:* just add components

  e.g. if **C** = **A** + **B**

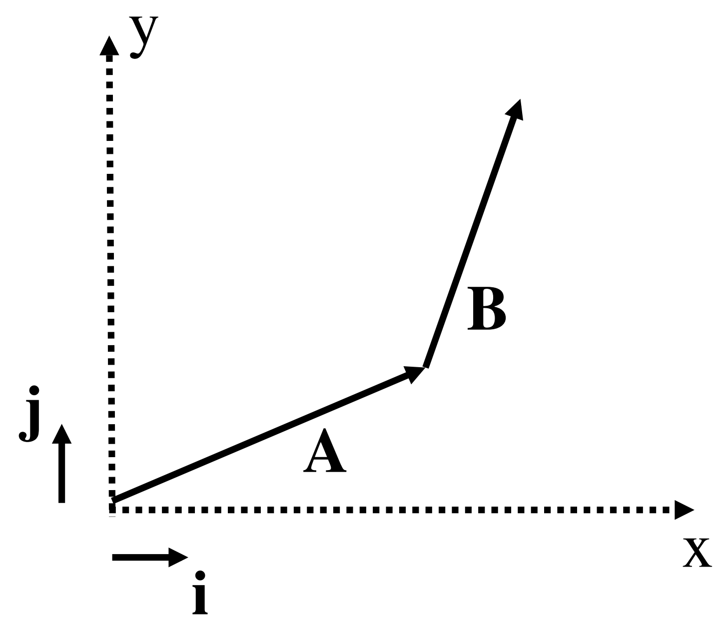
  $c_x = a_x + b_x$; $c_y = a_y + b_y$

- *Subtraction* similar

- *Multiplying* a vector by a number – just multiply components: if **D** = n***A**

  $d_x = n*a_x$; $d_y = n*a_y$

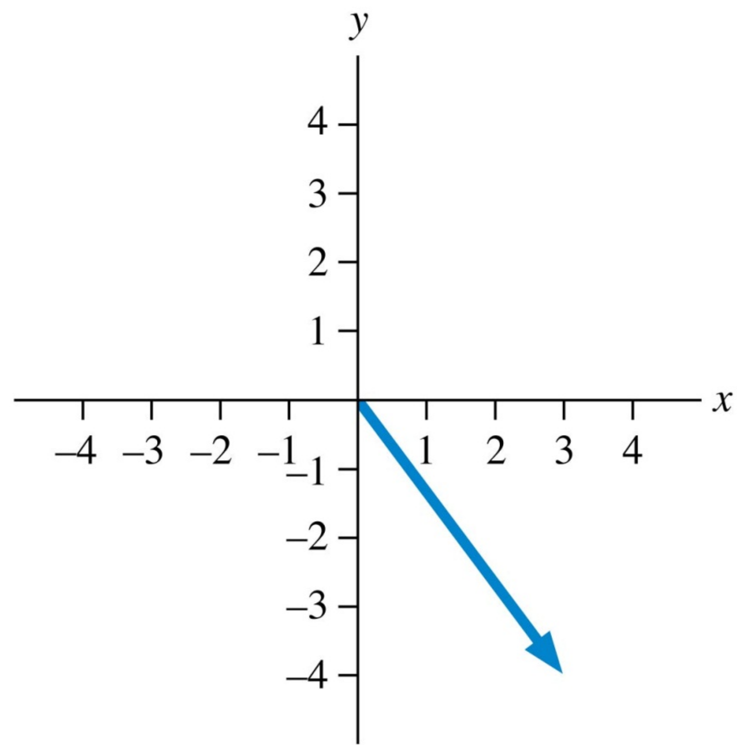
- Even more useful in 3 (or higher) dimensions



Clicker 3-1.4

What are the x- and y-components of this vector?

A. 3, 4
B. 4, 3
C. –3, 4
D. 4, –3
E. 3, –4



Clicker 3-1.4

What are the x- and y-components of this vector?

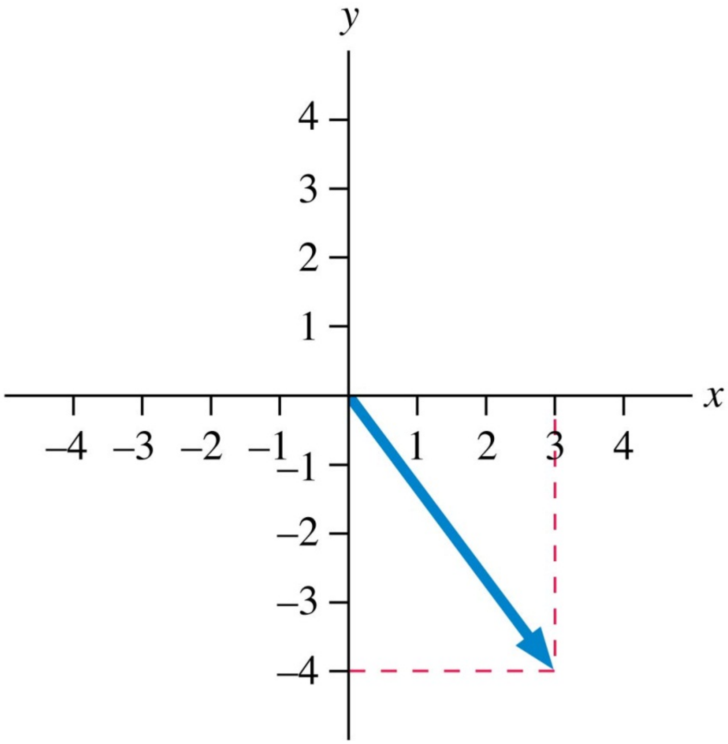
A. 3, 4

B. 4, 3

C. –3, 4

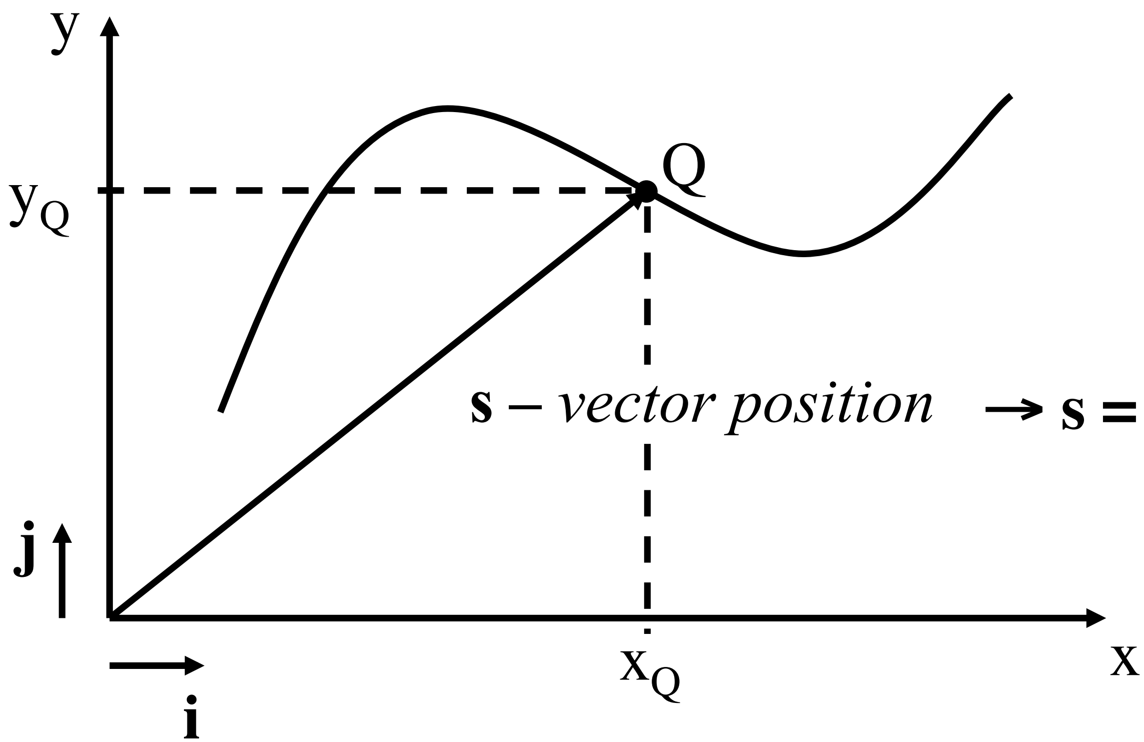
D. 4, –3

✔ **E. 3, –4**



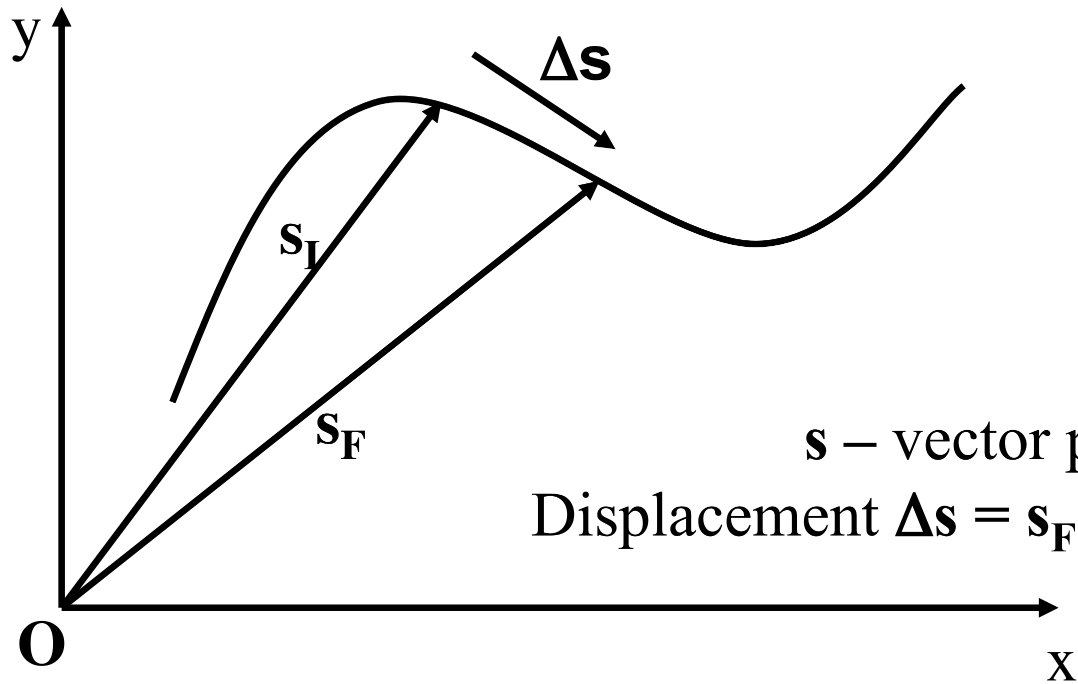
2D Motion in components

Note: component of position vector along x-direction is the x-coordinate!



s – *vector position* $\rightarrow \mathbf{s} = x_Q\mathbf{i} + y_Q\mathbf{j}$

Displacement in 2D Motion



s – vector position

Displacement $\mathbf{\Delta s} = \mathbf{s_F} - \mathbf{s_I}$, also a vector!

Describing motion with vectors

- Positions and displacements

$$\boldsymbol{s}, \qquad\qquad \Delta\boldsymbol{s} = \boldsymbol{s}_{\mathbf{F}} - \boldsymbol{s}_{\mathbf{I}}$$

- Velocities and changes in velocity:

$$\boldsymbol{v}_{\mathbf{av}} = \underline{\hspace{2cm}}, \qquad \boldsymbol{v}_{\mathbf{inst}} = \lim_{\Delta t \to 0} \underline{\hspace{1.5cm}}$$

$$\Delta\boldsymbol{v} = \boldsymbol{v}_{\mathbf{F}} - \boldsymbol{v}_{\mathbf{I}}$$

- Acceleration: $\boldsymbol{a}_{\mathbf{av}} = \underline{\hspace{2cm}}, \quad \boldsymbol{a}_{\mathbf{inst}} = \lim_{\Delta t \to 0} \underline{\hspace{2cm}}$

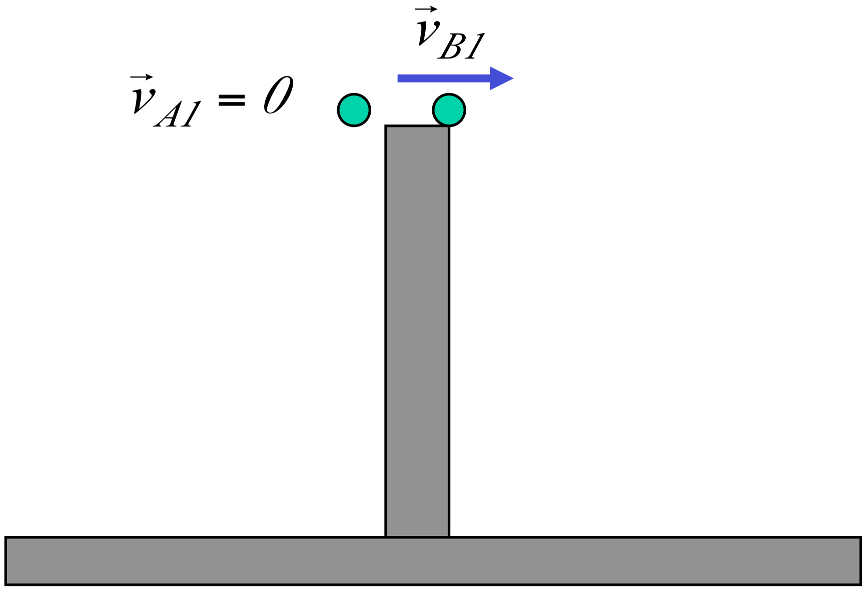
2D Motion in components

- x and y motions *decouple*

- $v_x = \Delta x/\Delta t$; $v_y = \Delta y/\Delta t$
- $a_x = \Delta v_x/\Delta t$; $a_y = \Delta v_y/\Delta t$

- If acceleration is only non-zero in 1 direction, can choose coordinates so that 1 component of acceleration is zero
  - e.g., motion under gravity

Clicker 3-1.5 Ball A is released from rest. Another identical ball, ball B is thrown horizontally at the same time and from the same height. Which ball will reach the ground first?



A. Ball A

B. Ball B

C. Both balls reach the ground at the same time.

D. The answer depends on the initial speed of ball B.

Motion under gravity

$a_y = -g$

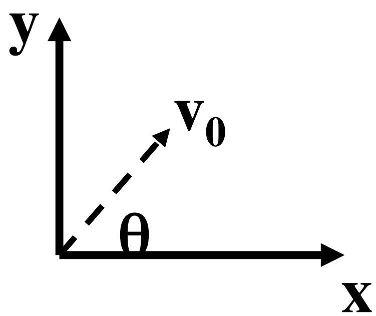
$v_y = v_{0y} - gt$

$y = y_0 + v_{0y}t - (1/2)gt^2$

$a_x = 0$

$v_x = v_{0x}$

$x = x_0 + v_{0x}t$



$v_{0y} = v_0\sin(\theta)$

$v_{0x} = v_0\cos(\theta)$

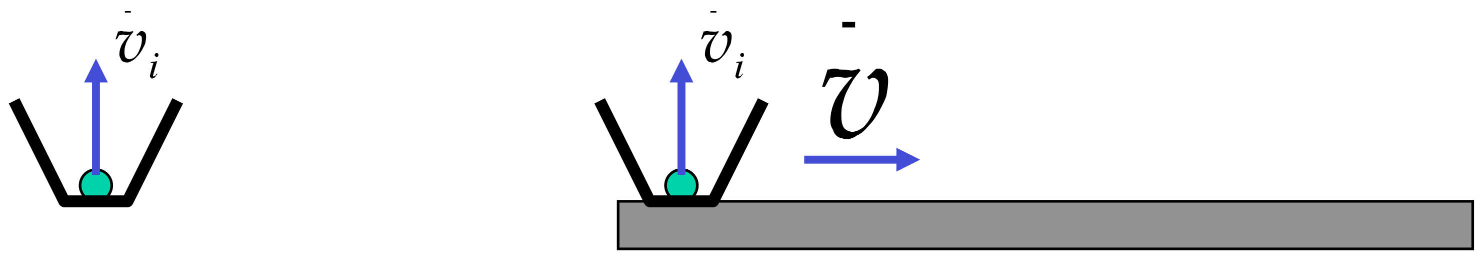
Projectile motion…

Reading assignment

- Kinematics in 2D
- Review 4.1-4.3 and read 4.5-4.7 in textbook

Clicker 3-1.6: A ball is ejected vertically upward from a cart at rest. The ball goes up, reaches its highest point and returns to the cart.

In a second experiment, the cart is moving at constant velocity and the ball is ejected in the same way, where will the ball land?



1. In front of the cart.
2. Behind the cart.
3. Inside the cart.
4. The outcome depends on the speed of the cart.