Welcome back to Physics 211

Today's agenda:

- *Velocity and acceleration in two-dimensional motion*
- *Motion under gravity -- projectile motion*
- *Acceleration on curved path*

Current assignments

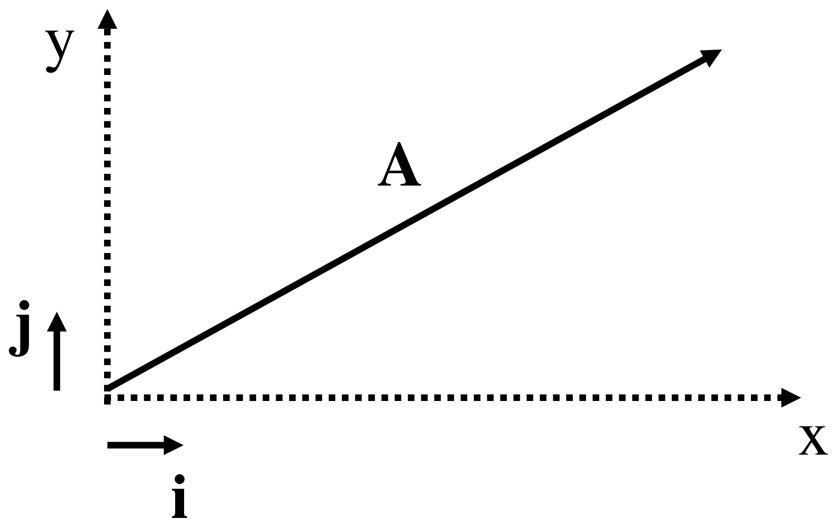
Due Fri 9/13 5pm:

- *HW#3 on Mastering Physics and on paper*

Reminder about course website:

https://jwlaiho.expressions.syr.edu/

Components



$\mathbf{A} = \mathbf{A_x} + \mathbf{A_y}$

$\mathbf{A} = a_x\mathbf{i} + a_y\mathbf{j}$

i = *unit vector* in x direction

j = *unit vector* in y direction

Projection of **A** along coordinate axes

a_x, a_y = *components* of vector **A**

Clicker 3-1.3: A bird is flying along a straight line in a direction somewhere East of North. After the bird has flown a distance of 2.5 miles, it is 2 miles North of where it started.

How far to the East is it from its starting point?

1. 0 miles
2. 0.5 miles
3. 1.0 mile
4. 1.5 miles

Why are components useful?

- *Addition:* just add components

  e.g. if **C** = **A** + **B**

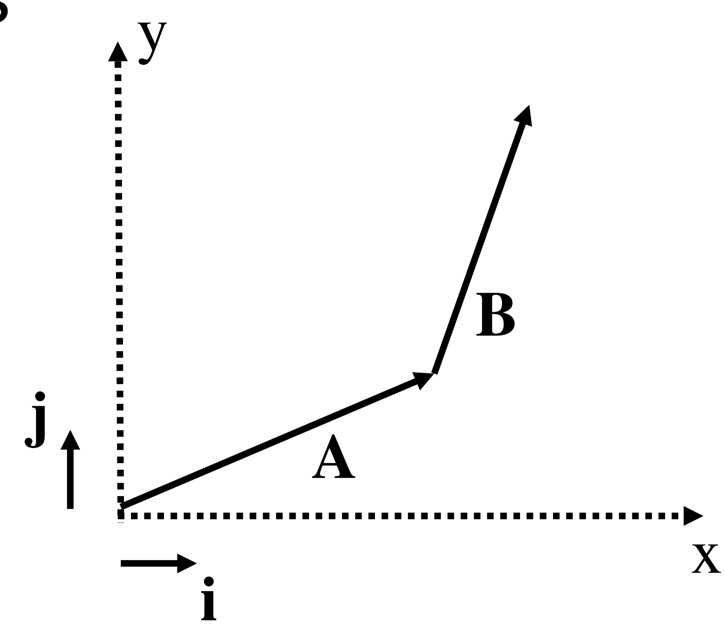
  $c_x = a_x + b_x$; $c_y = a_y + b_y$

- *Subtraction* similar

- *Multiplying* a vector by a number – just multiply components: if **D** = n***A**

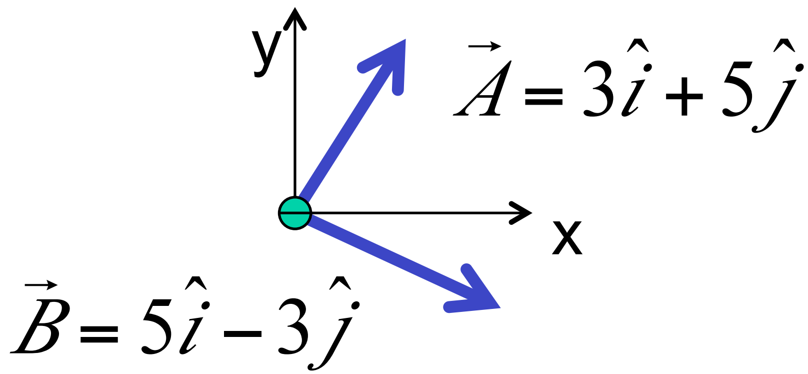
  $d_x = n*a_x$; $d_y = n*a_y$

- Even more useful in 3 (or higher) dimensions



Clicker 3-2.1:

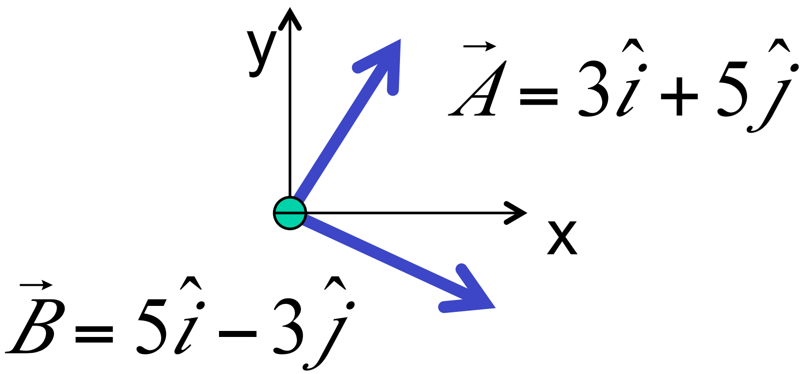
What is the sum of $\vec{A} + \vec{B}$?



1. 8 **i** + 8 **j**
2. 15 **i** – 15 **j**
3. 8 **i** - 2 **j**
4. 8 **i** + 2 **j**
5. **None of the above**

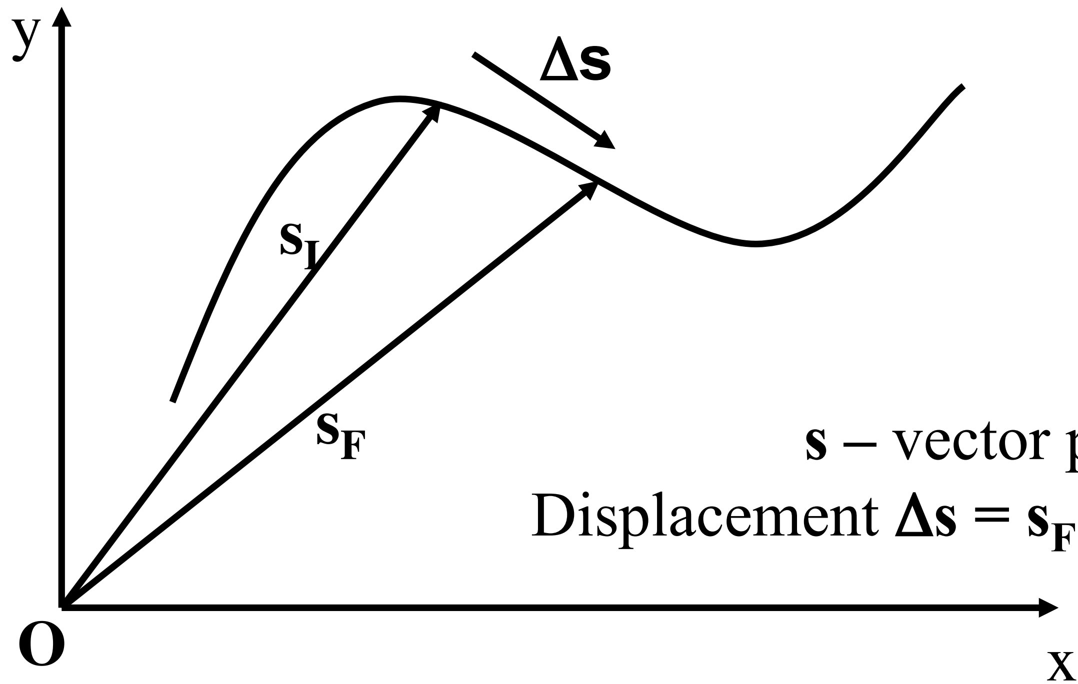
Clicker 3-2.2:

What is the length of $\vec{A} + \vec{B}$?



1. 6
2. 10
3. sqrt(64)
4. sqrt(68)
5. None of the above

Displacement in 2D Motion



s – vector position

Displacement $\mathbf{\Delta s} = \mathbf{s_F} - \mathbf{s_I}$, also a vector!

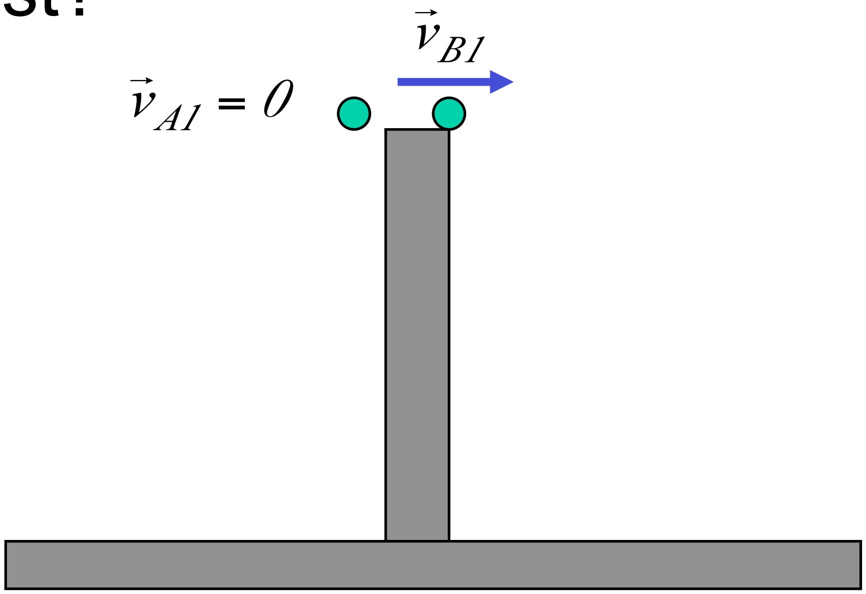
2D Motion in components

- x and y motions *decouple*

- $v_x = dx/dt$ $v_y = dy/dt$
- $a_x = dv_x/dt$ $a_y = dv_y/dt$

- If acceleration is only non-zero in 1 direction, can choose coordinates so that 1 component of acceleration is zero
  – e.g., motion under gravity

Clicker 3-2.3 Ball A is released from rest. Another identical ball, ball B is thrown horizontally at the same time and from the same height. Which ball will reach the ground first?



1. Ball A
2. Ball B
3. C. Both balls reach the ground at the same time.
4. D. The answer depends on the initial speed of ball B.

Horizontal Projectile Demo

Motion under gravity

y- motion

$a_y = -g$

$v_y = v_{0y} - gt$

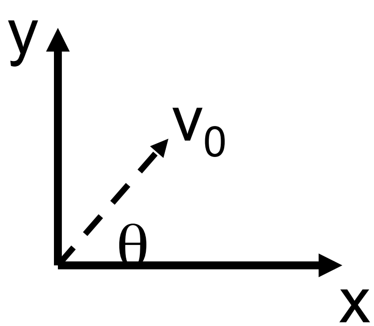
$y = y_0 + v_{0y}t - (1/2)gt^2$

x- motion

$a_x = 0$

$v_x = v_{0x}$

$x = x_0 + v_{0x}t$



$v_{0y} = v_0\sin(\theta)$

$v_{0x} = v_0\cos(\theta)$

Projectile motion...

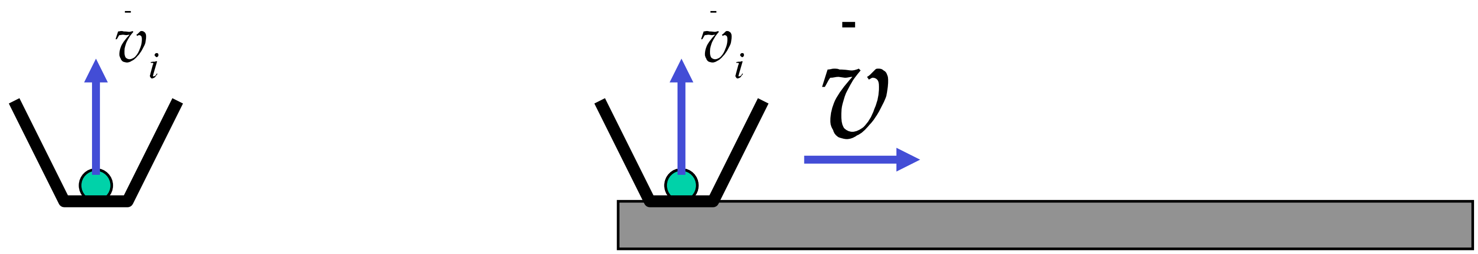
Clicker 3-2.4

A 100 g ball rolls off a table and lands 2.0 m from the base of the table. A 200 g ball rolls off the same table with the same speed. It lands at distance

1. 1.0 m.
2. Between 1 m and 2 m.
3. 2.0 m.
4. Between 2 m and 4 m.
5. 4.0 m.

3-2.5 A ball is ejected vertically upward from a cart at rest. The ball goes up, reaches its highest point and returns to the cart.

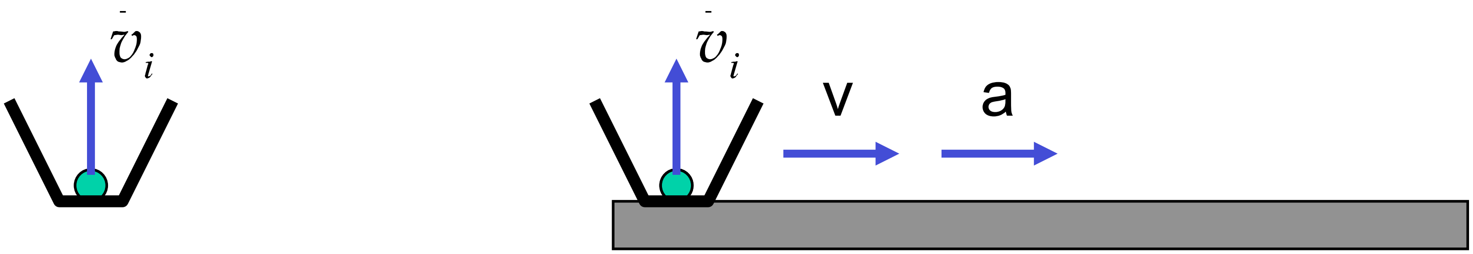
In a second experiment, the cart is moving at constant velocity and the ball is ejected in the same way, where will the ball land?



1. In front of the cart.
2. Behind the cart.
3. Inside the cart.
4. The outcome depends on the speed of the cart.

3-2.6 A ball is ejected vertically upward from a cart at rest. The ball goes up, reaches its highest point and returns to the cart.

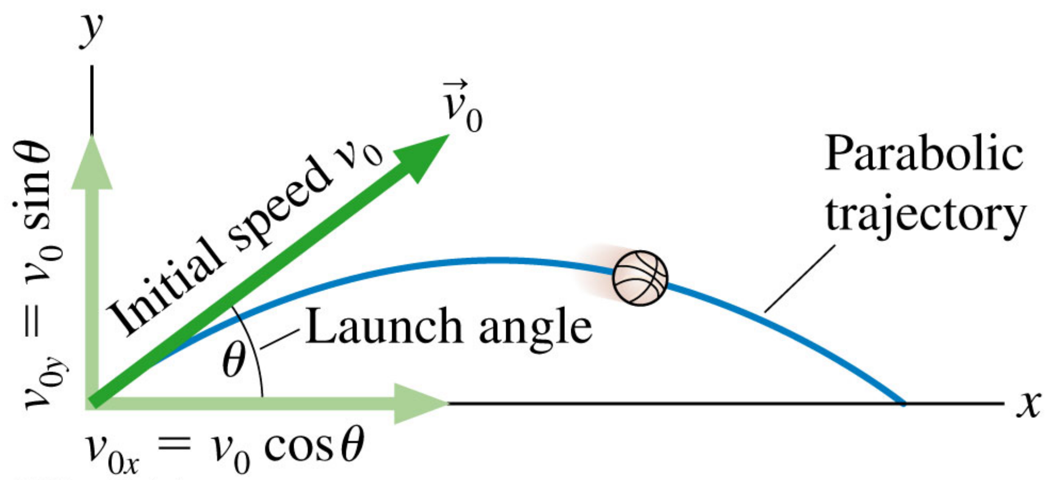
In a second experiment, the cart undergoes an acceleration in the direction of its velocity. If the ball is ejected before the acceleration ends, where will the ball land?



1. In front of the cart.
2. Behind the cart.
3. Inside the cart.
4. The outcome depends on the speed of the cart.

Cart Demo

- The start of a projectile's motion is called the *launch*.
- The angle θ of the initial velocity v_0 above the x-axis is called the **launch angle**.



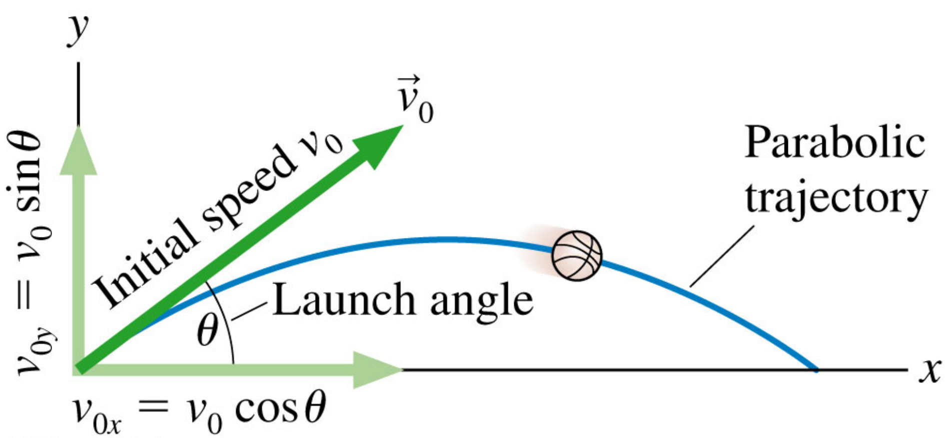
- The initial velocity vector can be broken into components.

$$v_{0x} = v_0 \cos\theta$$

$$v_{0y} = v_0 \sin\theta$$

where v_0 is the initial speed.

- Gravity acts downward.
- Therefore, a projectile has no horizontal acceleration.
- Thus:



$$a_x = 0$$
$$\text{(projectile motion)}$$
$$a_y = -g$$

- The vertical component of acceleration a_y is $-g$ of free fall.
- The horizontal component of a_x is zero.
- Projectiles are in free fall.

Motion under gravity

y- motion

$a_y = -g$

$v_y = v_{0y} - gt$

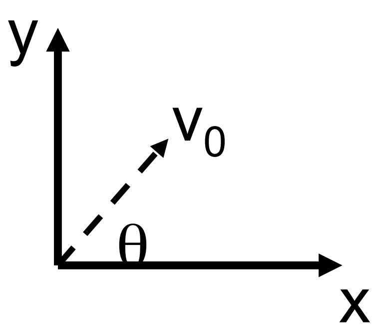
$y = y_0 + v_{0y}t - (1/2)gt^2$

x- motion

$a_x = 0$

$v_x = v_{0x}$

$x = x_0 + v_{0x}t$



$v_{0y} = v_0\sin(\theta)$

$v_{0x} = v_0\cos(\theta)$

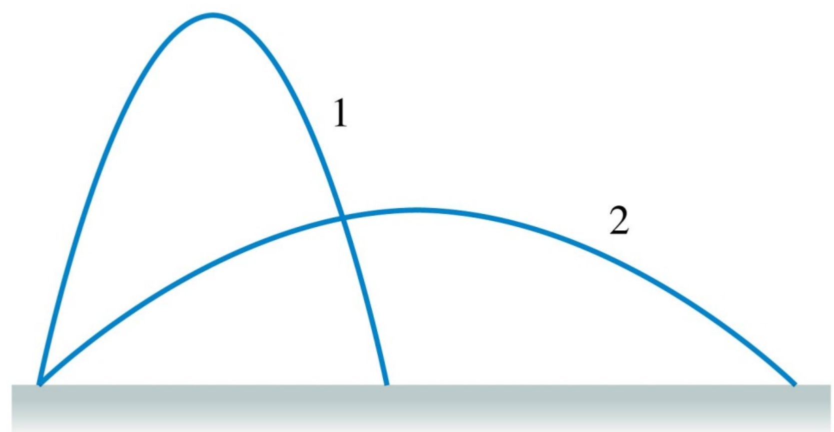
Projectile motion...

Projectile question

- A ball is thrown at 45^o to vertical with a speed of 7 m/s. Assuming $g=10\ m/s^2$, how far away does the ball land?

Clicker 3-2.6

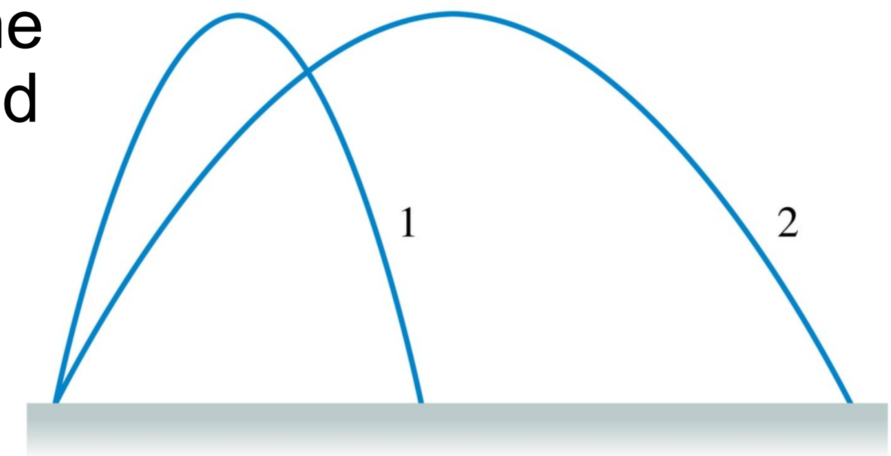
Projectiles 1 and 2 are launched over level ground with the same speed but at different angles. Which hits the ground first? Ignore air resistance.



1. Projectile 1 hits first.
2. Projectile 2 hits first.
3. They hit at the same time.
4. There's not enough information to tell.

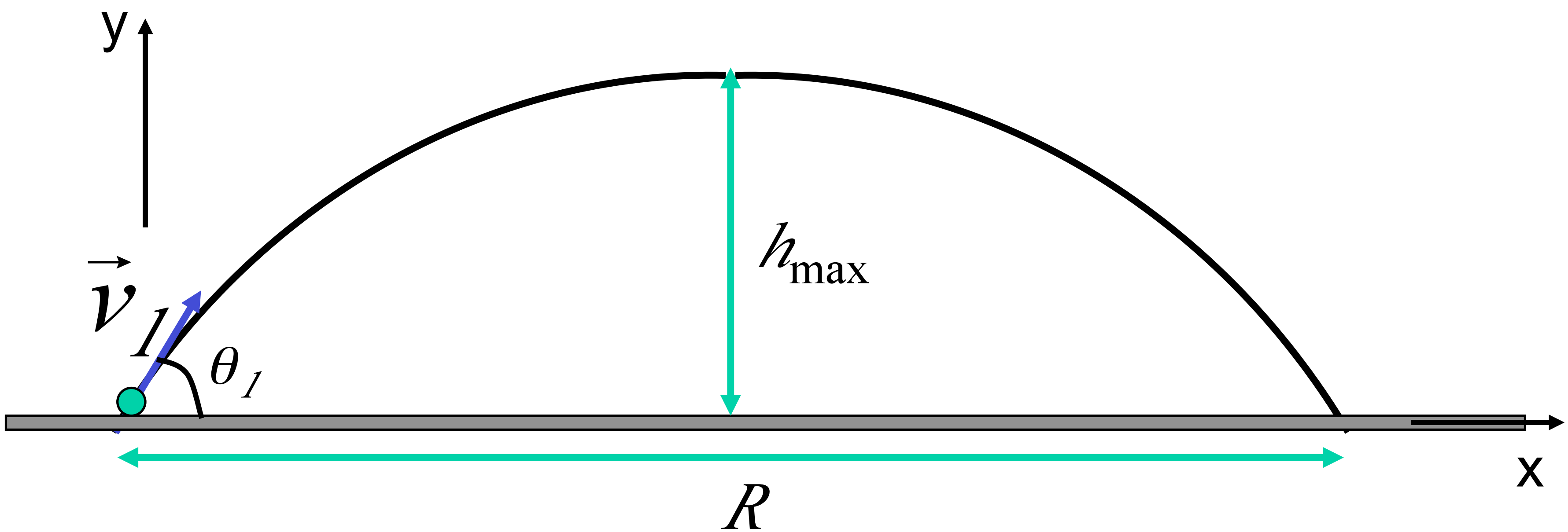
Clicker 3-2.7

Projectiles 1 and 2 are launched over level ground with different speeds. Both reach the same height. Which hits the ground first? Ignore air resistance.



1. Projectile 1 hits first.
2. Projectile 2 hits first.
3. They hit at the same time.
4. There's not enough information to tell.

Projectile motion



R : when is y=0 ?

$t[v_{y1}-(1/2)gt] = 0$

i.e., $T = (2v)\sin\theta/g$ → R (x-eqn.) → h_{max} (y-eqn.)

Maximum height and range (only works on FLAT ground)

$$2\cos\theta\sin\theta = \sin(2\theta)$$

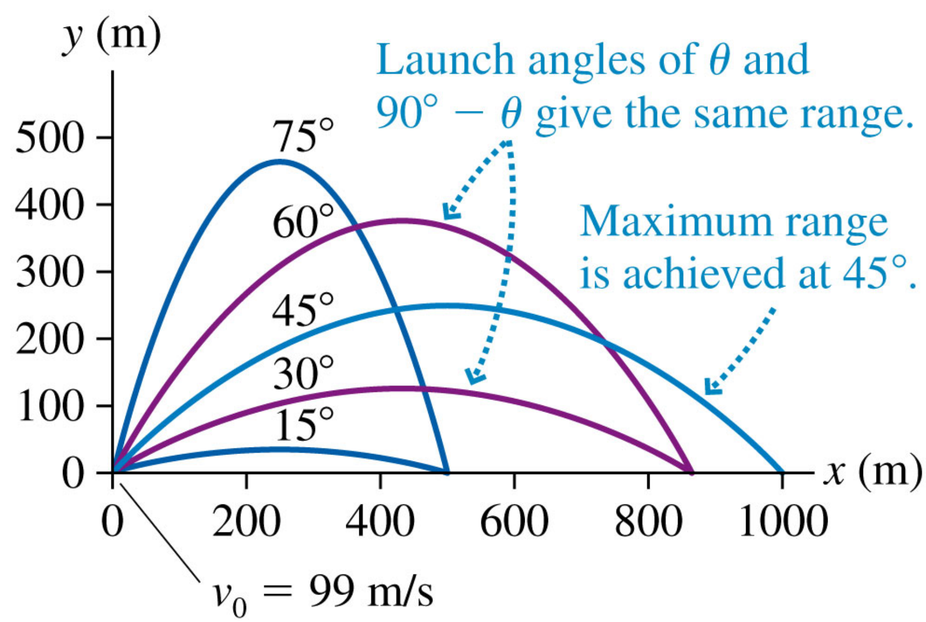
$$R = \frac{v_1^2 \sin\left(2\theta_1\right)}{g}$$

Challenge: Derive the formula for the maximum height:

$$h_{\max} = \frac{v_0^2 \sin^2\theta}{2g}$$

A projectile with initial speed v_0 has a launch angle of θ above the horizontal. How far does it travel over level ground before it returns to the same elevation from which it was launched?

Trajectories of a projectile launched at different angles with a speed of 99 m/s.



- This distance is sometimes called the *range* of a projectile.
- Example 4.5 from your textbook shows:

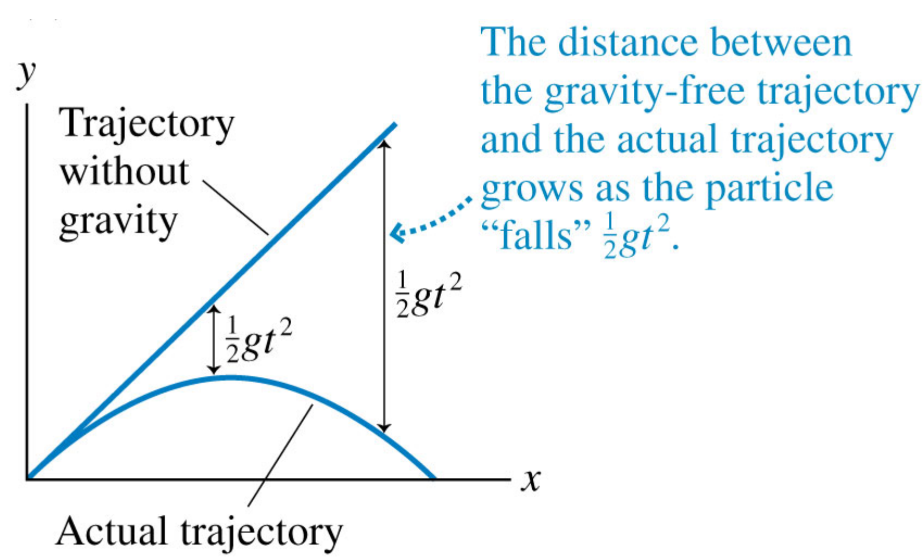
$$\text{distance} = \frac{{v_0}^2 \sin(2\theta)}{g}$$

- The maximum distance occurs for $\theta = 45°$.

Hunter and “coconut” demo

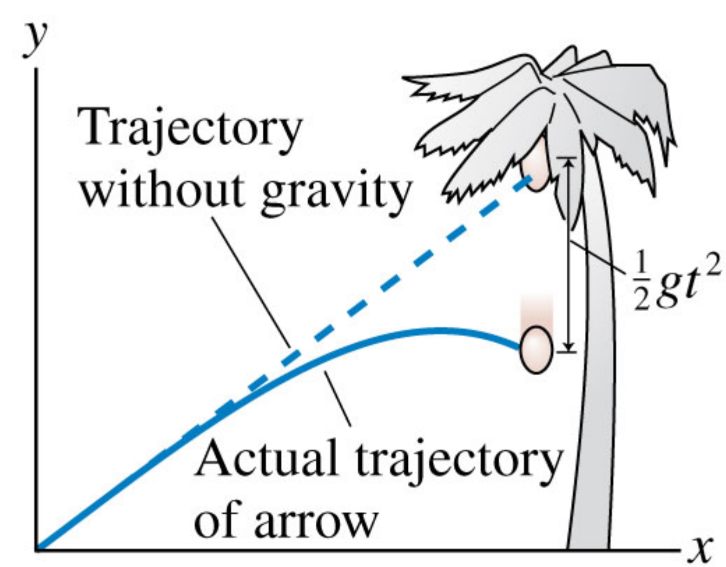
A hunter in the jungle wants to shoot down a "coconut" that is hanging from the branch of a tree. He points his arrow directly at the coconut, but the coconut falls from the branch at the *exact* instant the hunter shoots the arrow. Does the arrow hit the coconut?

- Without gravity, the arrow would follow a straight line.
- Because of gravity, the arrow at time *t* has "fallen" a distance $\frac{1}{2}gt^2$ below this line.
- The separation grows as $\frac{1}{2}gt^2$, giving the trajectory its parabolic shape.

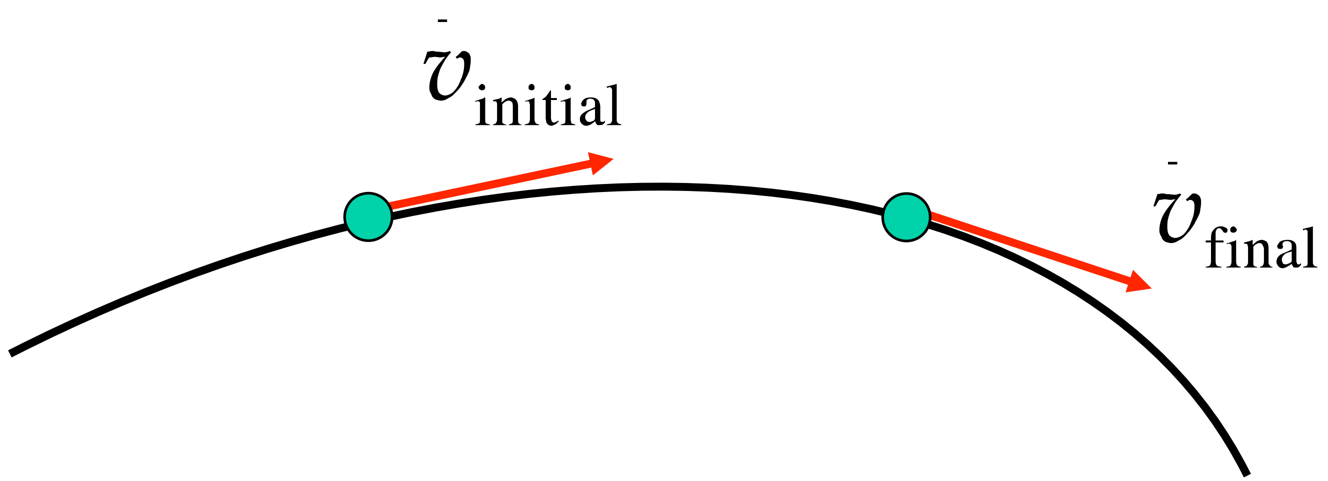


A hunter in the jungle wants to shoot down a coconut that is hanging from the branch of a tree. He points his arrow directly at the coconut, but the coconut falls from the branch at the *exact* instant the hunter shoots the arrow. Does the arrow hit the coconut?

- Had the coconut stayed on the tree, the arrow would have curved under its target as gravity causes it to fall a distance $\frac{1}{2}gt^2$ below the straight line.
- But $\frac{1}{2}gt^2$ is also the distance the coconut falls while the arrow is in flight.
- So *yes*, the arrow hits the coconut!

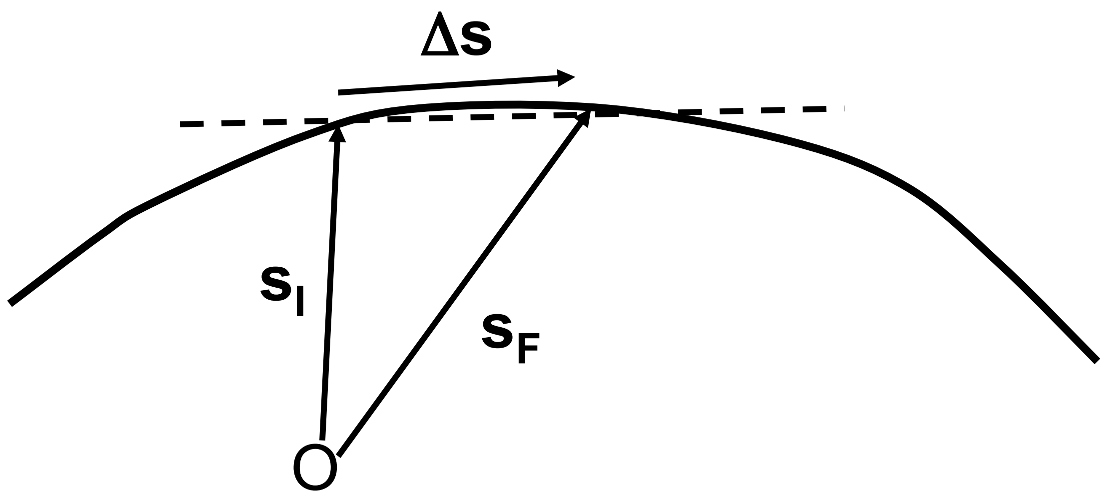


Motion on a curved path at constant speed



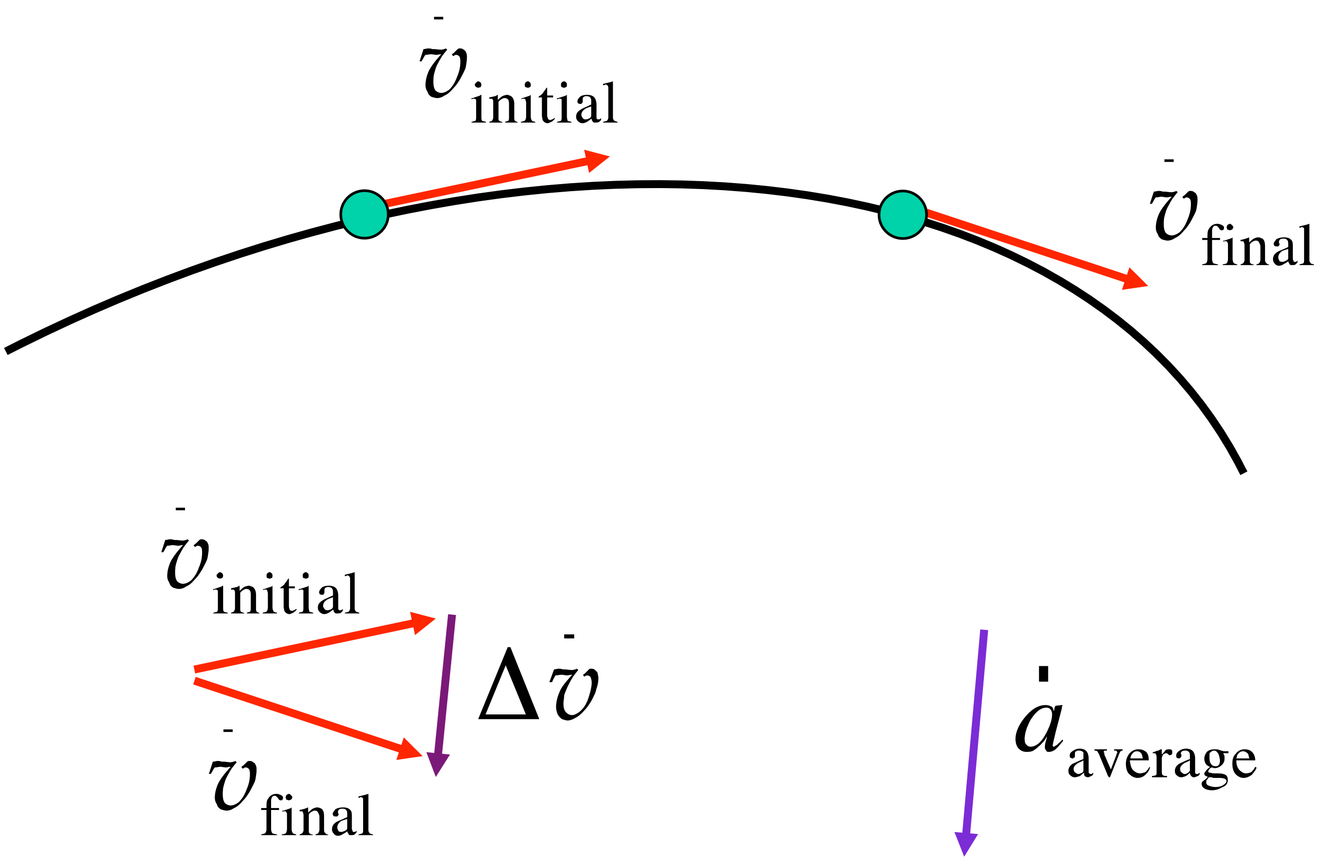
Is the acceleration of the object equal to zero?

Velocity is tangent to path



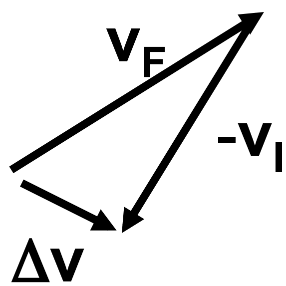
$\mathbf{v} = \Delta\mathbf{s}/\Delta t$ lies along dotted line. As $\Delta t \to 0$ direction of **v** is **tangent** to path

Motion on a curved path at constant speed

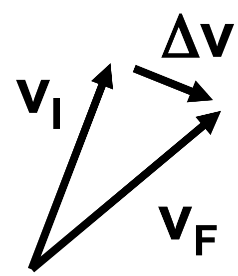


Subtracting vectors

Recall that $\mathbf{v_F} + (-\mathbf{v_I}) = \Delta\mathbf{v}$



same as



For an object moving at constant *speed* along a curved path, the acceleration is ***not*** zero.

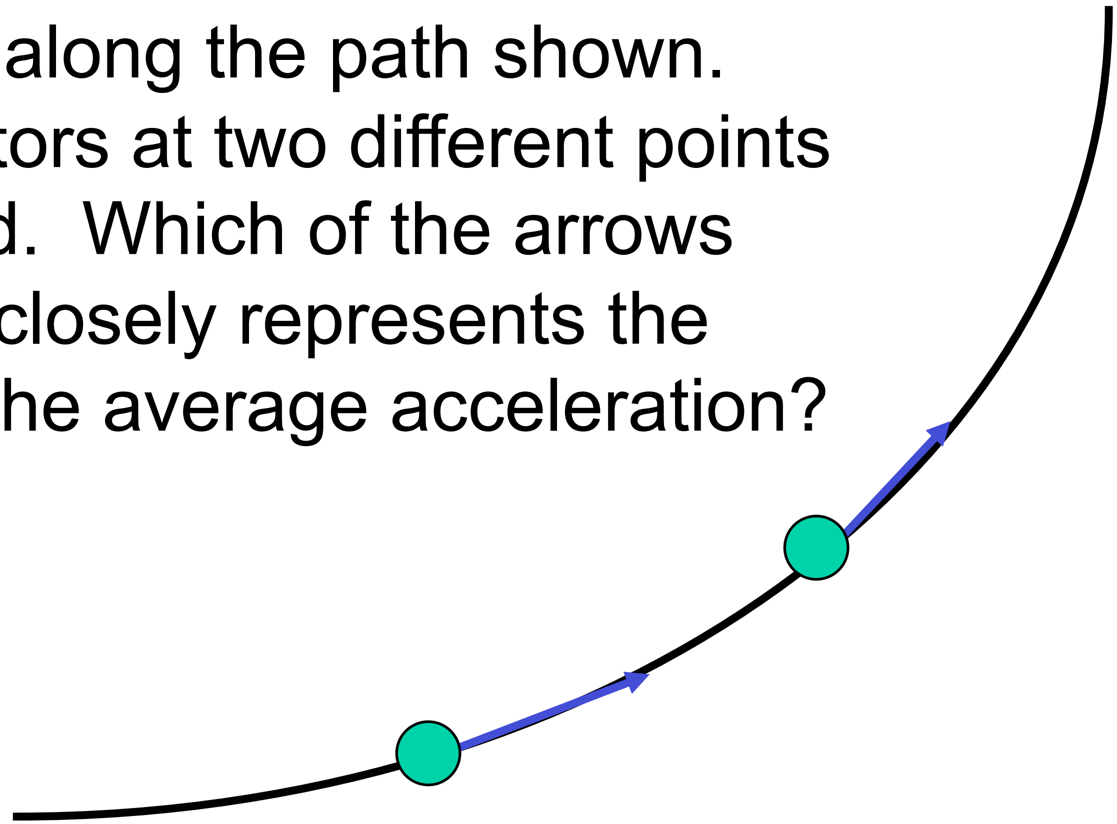
Reading assignment

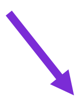
- Circular motion
- 4.5 – 4.7 in textbook

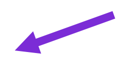
For which of the following motions of a car does the net change in velocity vector have the greatest magnitude? (All motions occur at the same constant speed.)

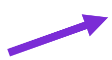
1. A 90° right turn at constant speed
2. A U-turn at constant speed
3. A 270° turn on a highway on -ramp
4. The change in velocity is zero for all three motions.

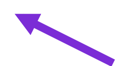
A car moves along the path shown. Velocity vectors at two different points are sketched. Which of the arrows below most closely represents the direction of the average acceleration?



1. 

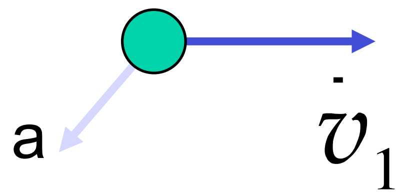
2. 

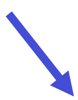
3. 

4. 

A child is riding a bicycle on a level street. The velocity and acceleration vectors of the child at a given time are shown.

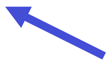
Which of the following velocity vectors may represent the velocity at a later time?



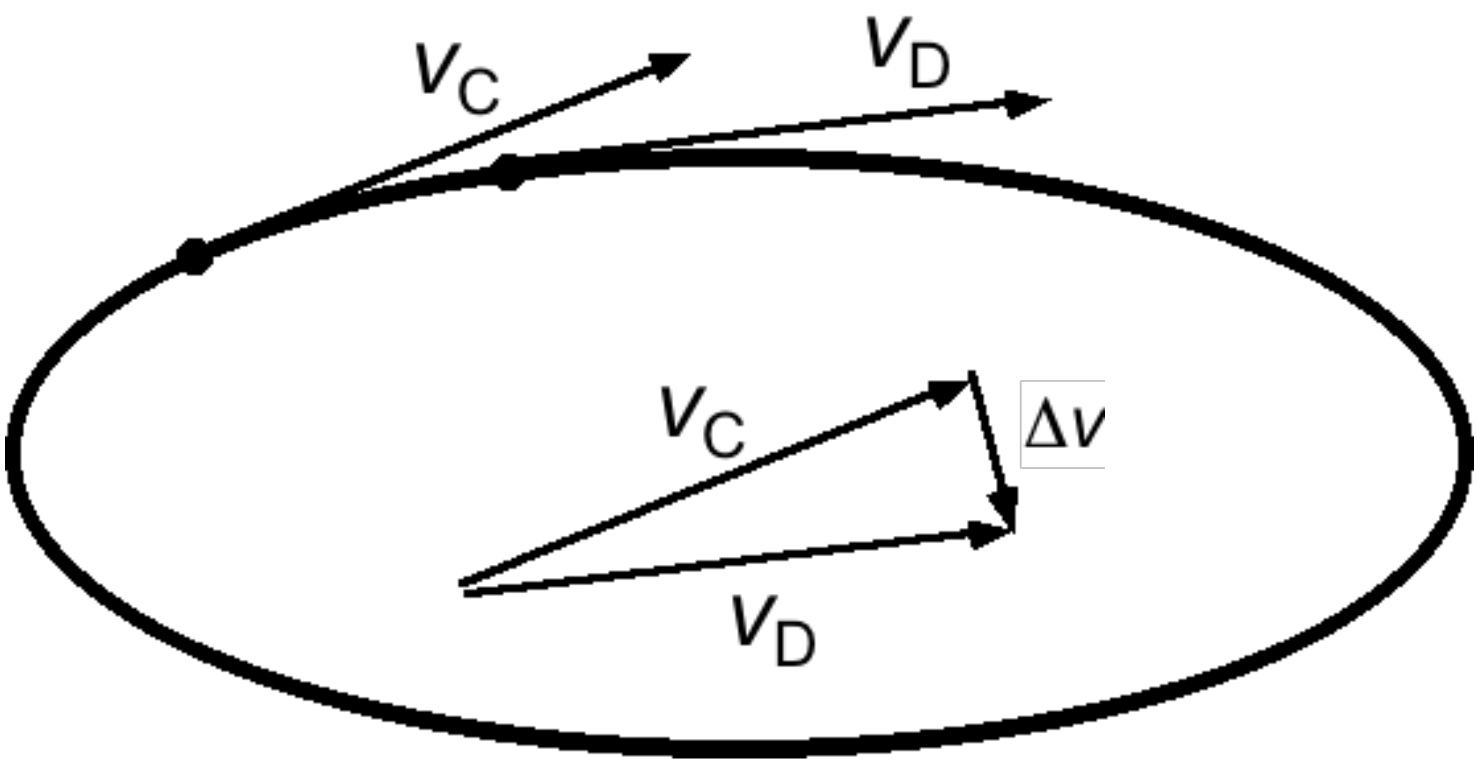
1. 

2. 

3. 

4. 

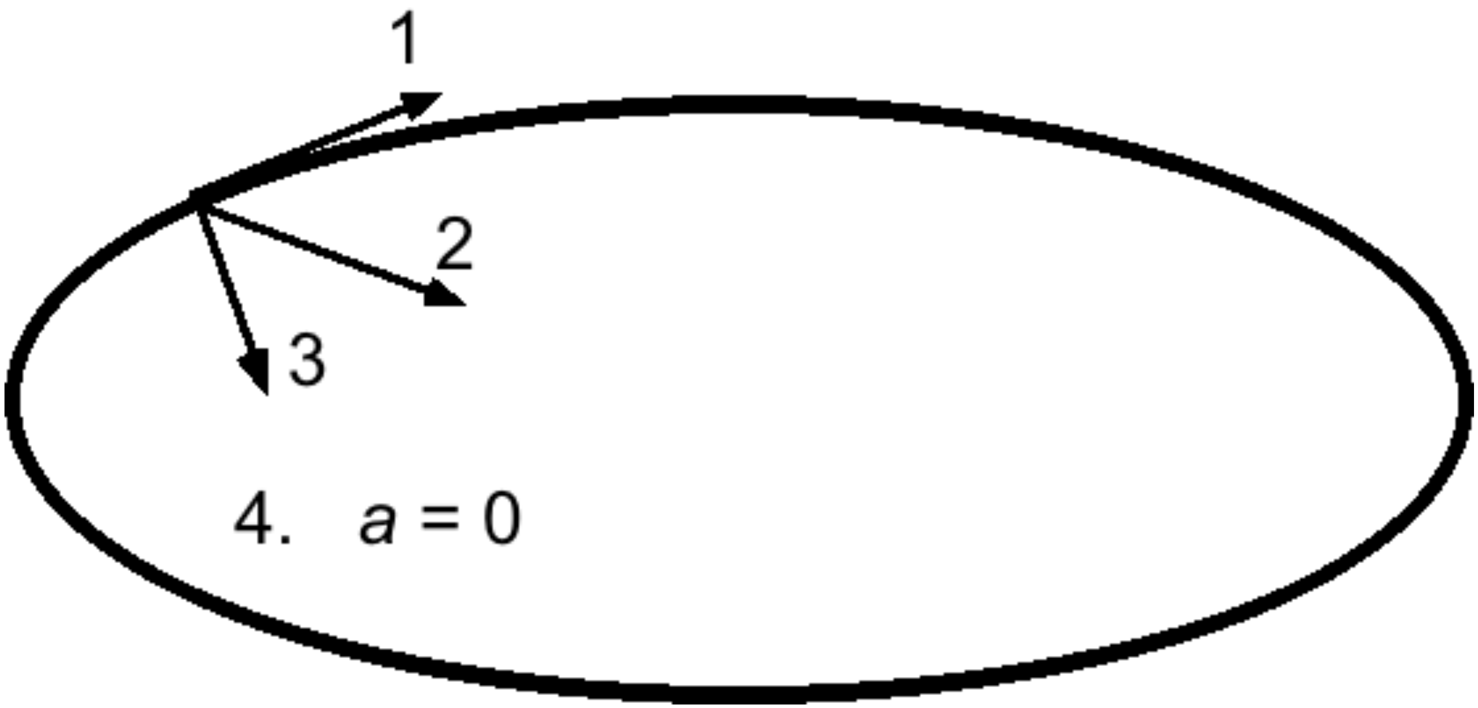
Biker moving around oval at *constant* speed



As point D *is moved closer to* C*, angle approaches 90°.*

A biker is riding at **constant** speed clockwise on the oval track shown below.

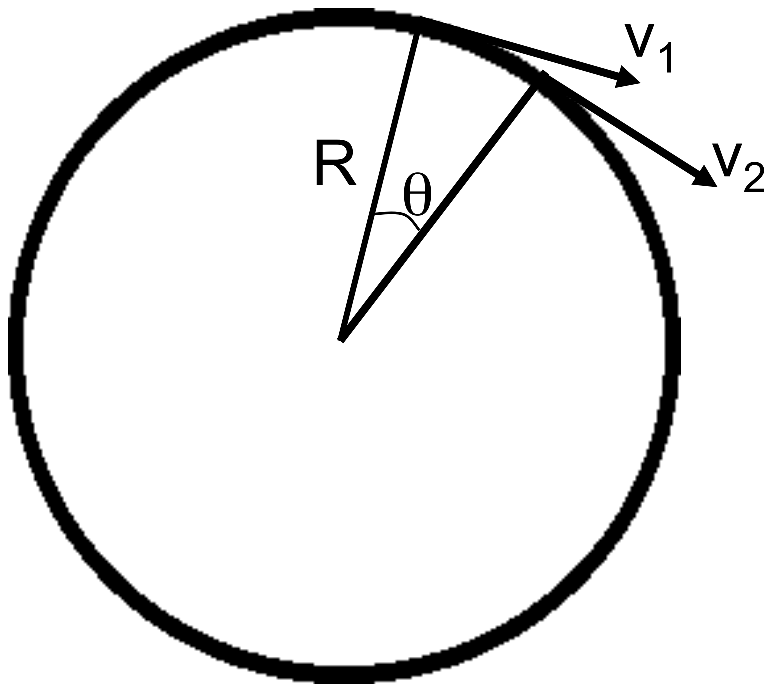
Which vector correctly describes the *acceleration* at the point indicated?

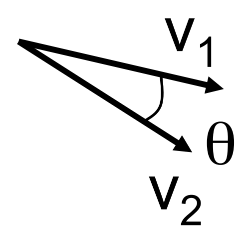


Summary

- For motion at constant speed, instantaneous acceleration vector is perpendicular to velocity vector
- Points ``inward''
- What is the magnitude of the acceleration vector?

Acceleration vectors for ball swung in a horizontal circle at *constant* speed v





What is the magnitude of the acceleration?

$$|a| = v^2/R$$

Exam 1: in two weeks! (9/25)

- In Stolkin (here!) at the usual lecture time
- Material covered:
  - **Textbook** chapters 1 - 4
  - **Lectures** up through 9/18 (slides online)
  - **Wed/Fri Workshop activities**
  - **Homework assignments**

- *Work on more practice exam problems in recitation workshop*