Welcome back to Physics 211

Today’s agenda:

More projectile motion

Exam 1: in one week! (9/23)

- In Stolkin (here!) at the usual lecture time
- Material covered:
  - **Textbook** chapters 1 – 4.3
  - **Lectures** up through 9/16 (slides online)
  - **Wed/Fri Workshop activities**
  - **Homework assignments**

- ***Work on more practice exam problems in recitation workshop***

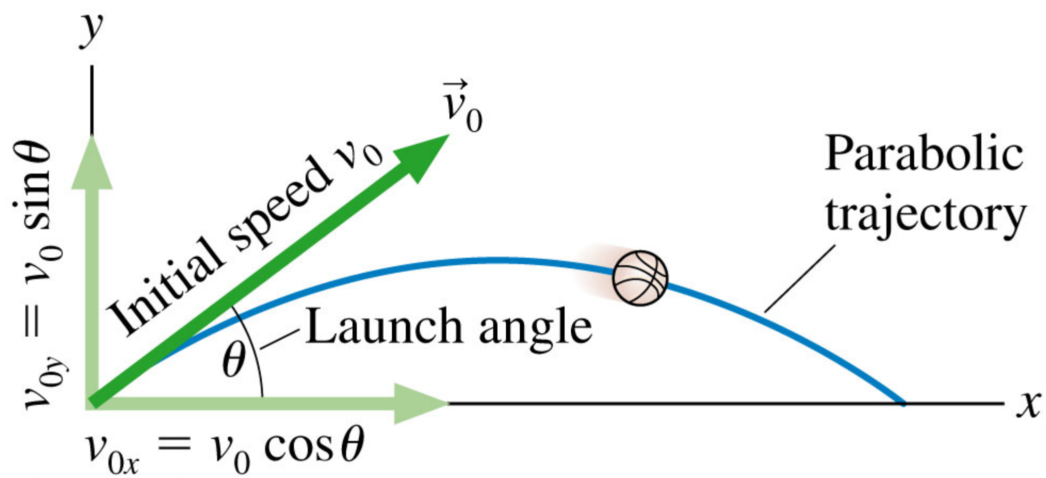
Upcoming assignments

HW#4: Due Friday 5 pm online and to TA

Prelecture 4-2: for Thursday

Review for Exam 1, next Tuesday,
Sep 23 !

- The start of a projectile's motion is called the *launch*.
- The angle θ of the initial velocity v_0 above the *x*-axis is called the **launch angle**.



- The initial velocity vector can be broken into components.

$$v_{0x} = v_0 \cos\theta$$

$$v_{0y} = v_0 \sin\theta$$

where v_0 is the initial speed.

Motion under gravity

y- motion

$a_y = -g$

$v_y = v_{0y} - gt$

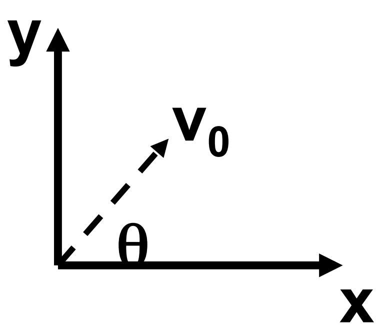
$y = y_0 + v_{0y}t - (1/2)gt^2$

x- motion

$a_x = 0$

$v_x = v_{0x}$

$x = x_0 + v_{0x}t$

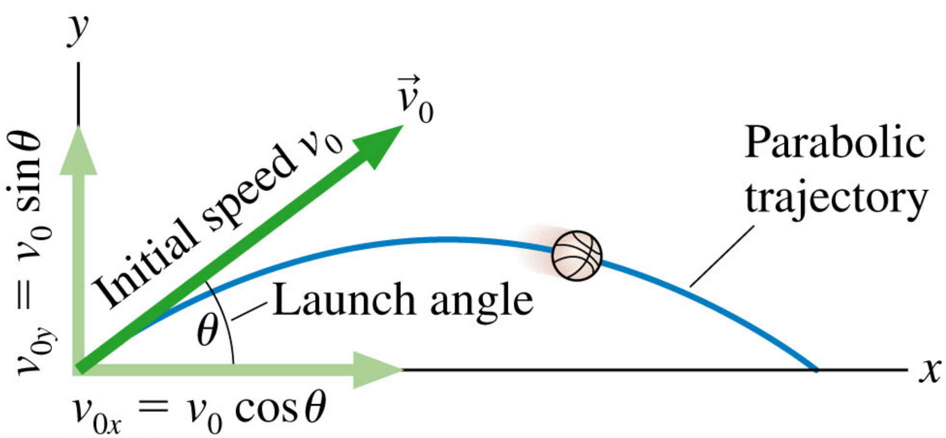


$v_{0y} = v_0\sin(\theta)$

$v_{0x} = v_0\cos(\theta)$

Projectile motion…

- Gravity acts downward.
- Therefore, a projectile has no horizontal acceleration.
- Thus:



$$a_x = 0$$
$$a_y = -g$$
(projectile motion)

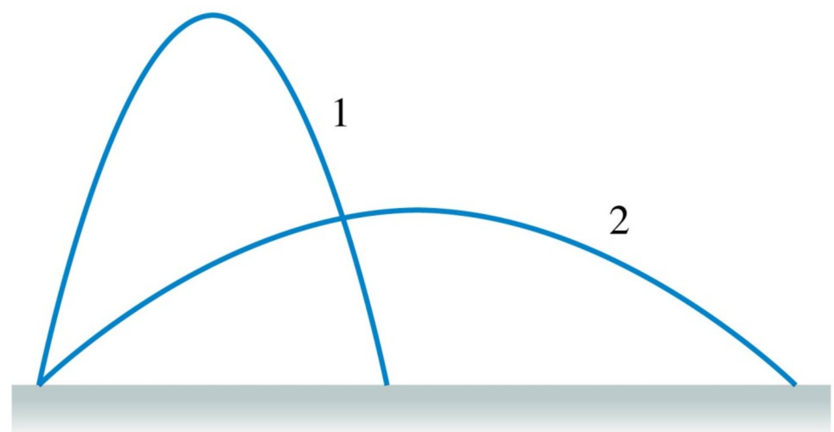
- The vertical component of acceleration a_y is $-g$ of free fall.
- The horizontal component of a_x is zero.
- Projectiles are in free fall.

Projectile question

- A ball is thrown at 45^o to vertical with a speed of 7 m/s. Assuming g=10 m/s^2, how far away does the ball land?

Clicker 4-1.1

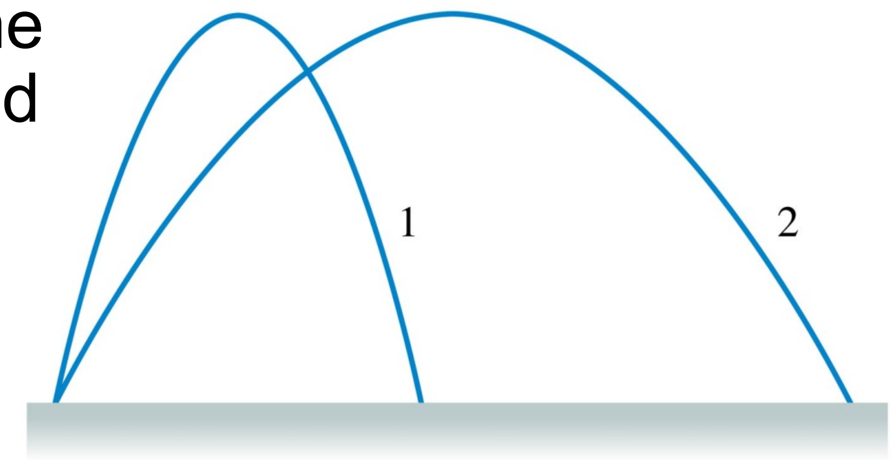
Projectiles 1 and 2 are launched over level ground with the same speed but at different angles. Which hits the ground first? Ignore air resistance.



1. Projectile 1 hits first.
2. Projectile 2 hits first.
3. They hit at the same time.
4. There's not enough information to tell.

Clicker 4-1.2

Projectiles 1 and 2 are launched over level ground with different speeds. Both reach the same height. Which hits the ground first? Ignore air resistance.



1. Projectile 1 hits first.
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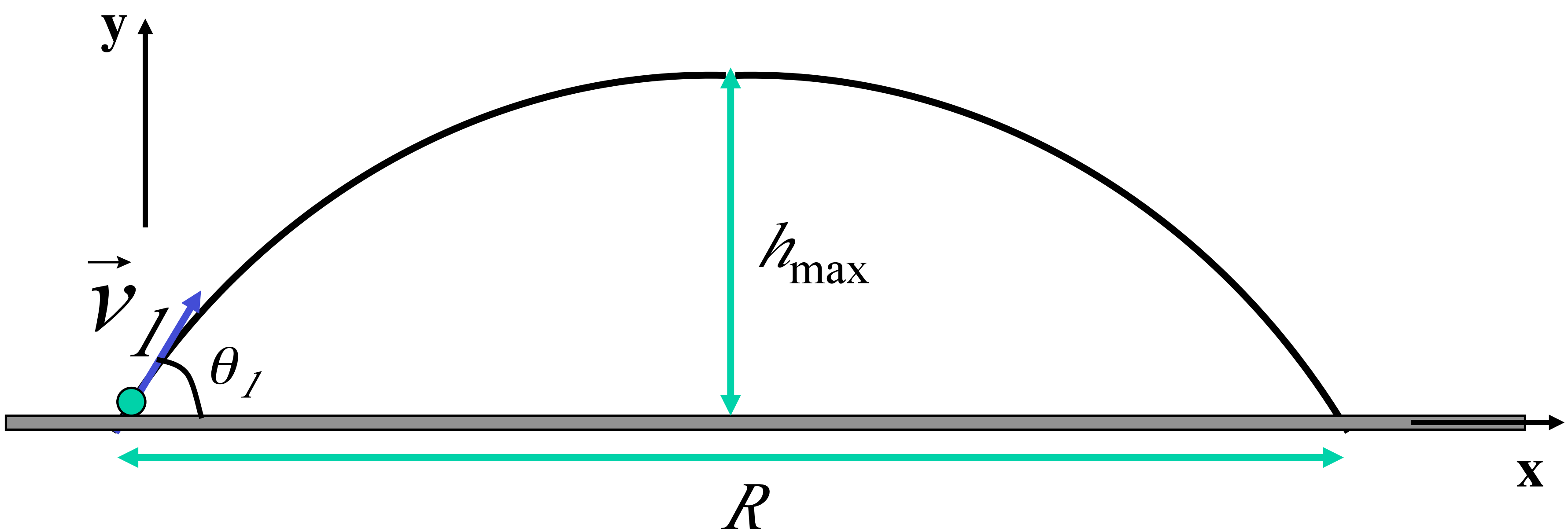
Projectile motion



R : when is y=0 ?

t[v_{y1}-(1/2)gt] = 0

i.e., T = (2v)sinθ/g → R (x-eqn.) → h_{max} (y-eqn.)

Maximum height and range (only works on FLAT ground)

$$2\cos\theta\sin\theta = \sin(2\theta)$$

$$R = \frac{v_1^2 \sin\left(2\theta_1\right)}{g}$$

Challenge: Derive the formula for the maximum height:

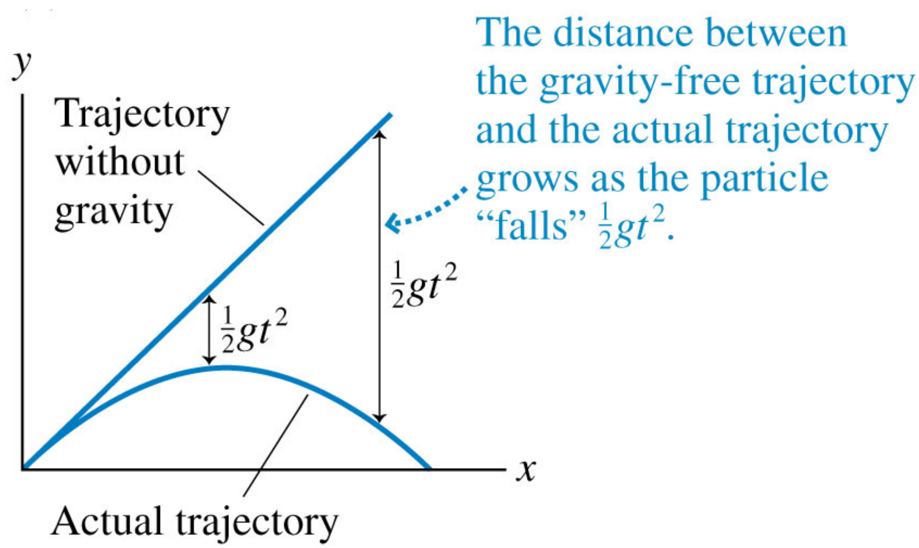
$$h_{\max} = \frac{v_0^2 \sin^2\theta}{2g}$$

Hunter and mascot demo

Reasoning About Projectile Motion

A hunter in the jungle wants to shoot down a "coconut" that is hanging from the branch of a tree. He points his arrow directly at the coconut, but the coconut falls from the branch at the *exact* instant the hunter shoots the arrow. Does the arrow hit the coconut?

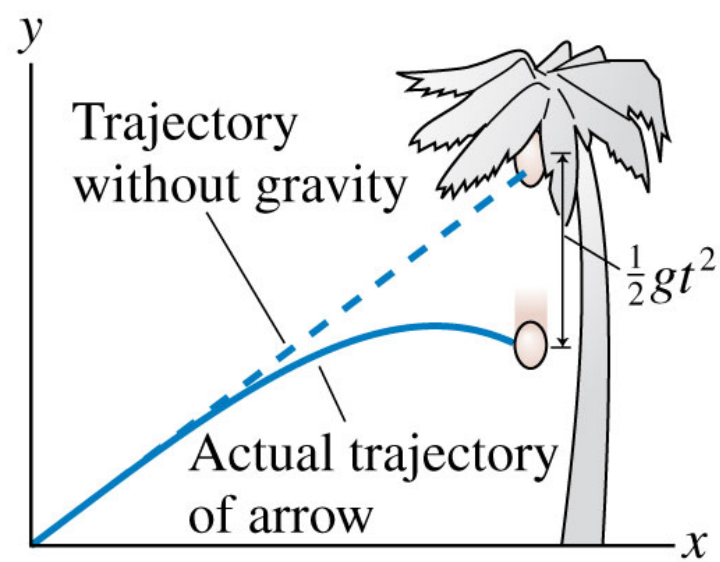
- Without gravity, the arrow would follow a straight line.
- Because of gravity, the arrow at time *t* has "fallen" a distance $\frac{1}{2}gt^2$ below this line.
- The separation grows as $\frac{1}{2}gt^2$, giving the trajectory its parabolic shape.



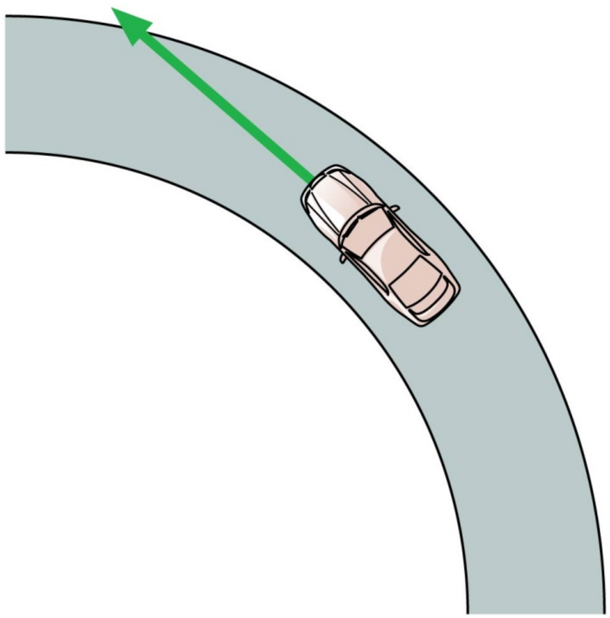
Reasoning About Projectile Motion

A hunter in the jungle wants to shoot down a coconut that is hanging from the branch of a tree. He points his arrow directly at the coconut, but the coconut falls from the branch at the *exact* instant the hunter shoots the arrow. Does the arrow hit the coconut?

- Had the coconut stayed on the tree, the arrow would have curved under its target as gravity causes it to fall a distance $\frac{1}{2}gt^2$ below the straight line.
- But $\frac{1}{2}gt^2$ is also the distance the coconut falls while the arrow is in flight.
- So *yes*, the arrow hits the coconut!

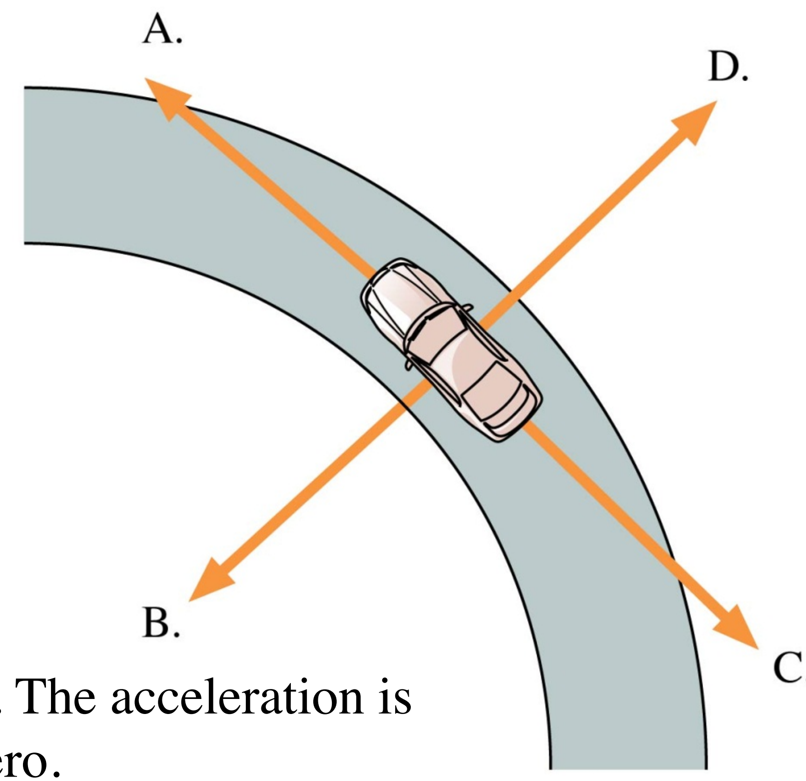


Clicker 4-1.3: A car is traveling around a curve at a steady 45 mph. Is the car accelerating?



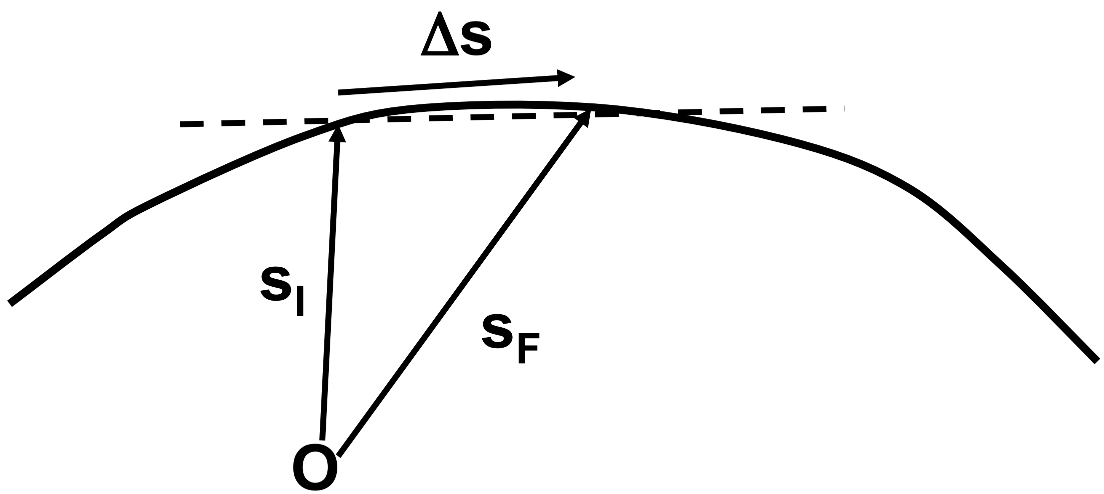
1. Yes
2. No

4-1.4: A car is traveling around a curve at a steady 45 mph. Which vector shows the direction of the car's acceleration?



E. The acceleration is zero.

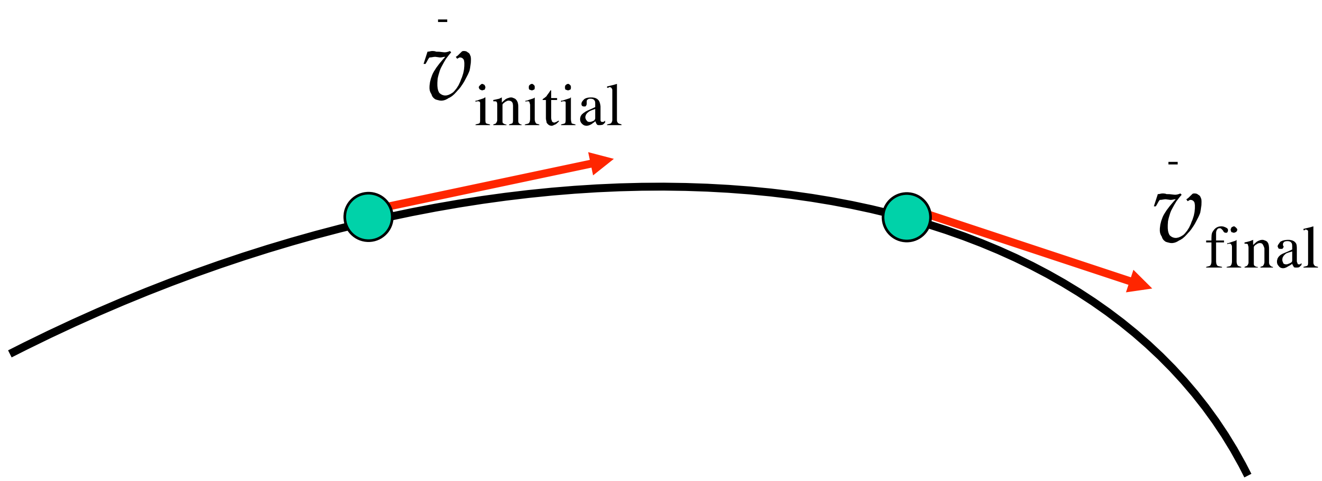
Velocity is tangent to path



v = Δs/Δt lies along dotted line. As Δt → 0 direction of v is tangent to path

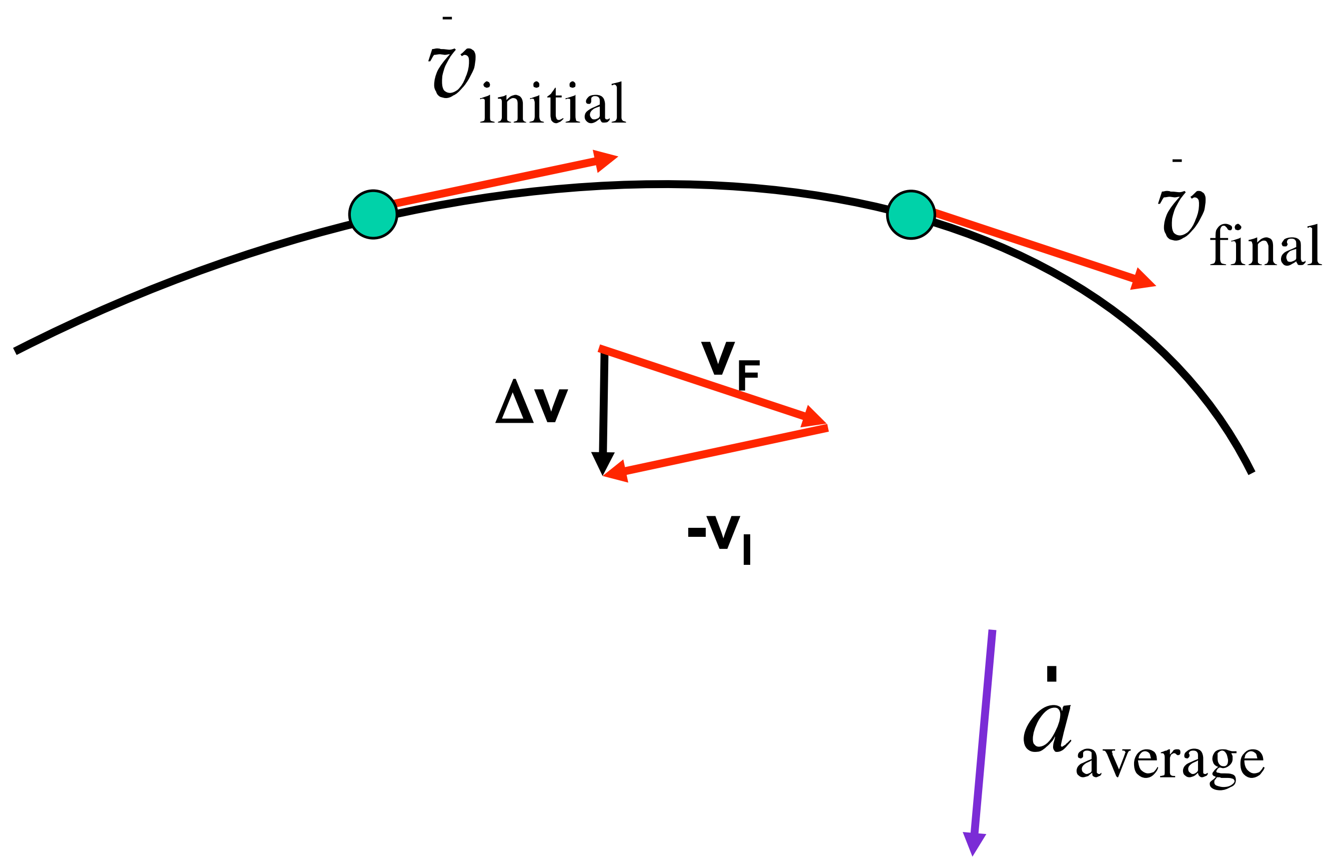
Dog on a string demo

Motion on a curved path at constant speed

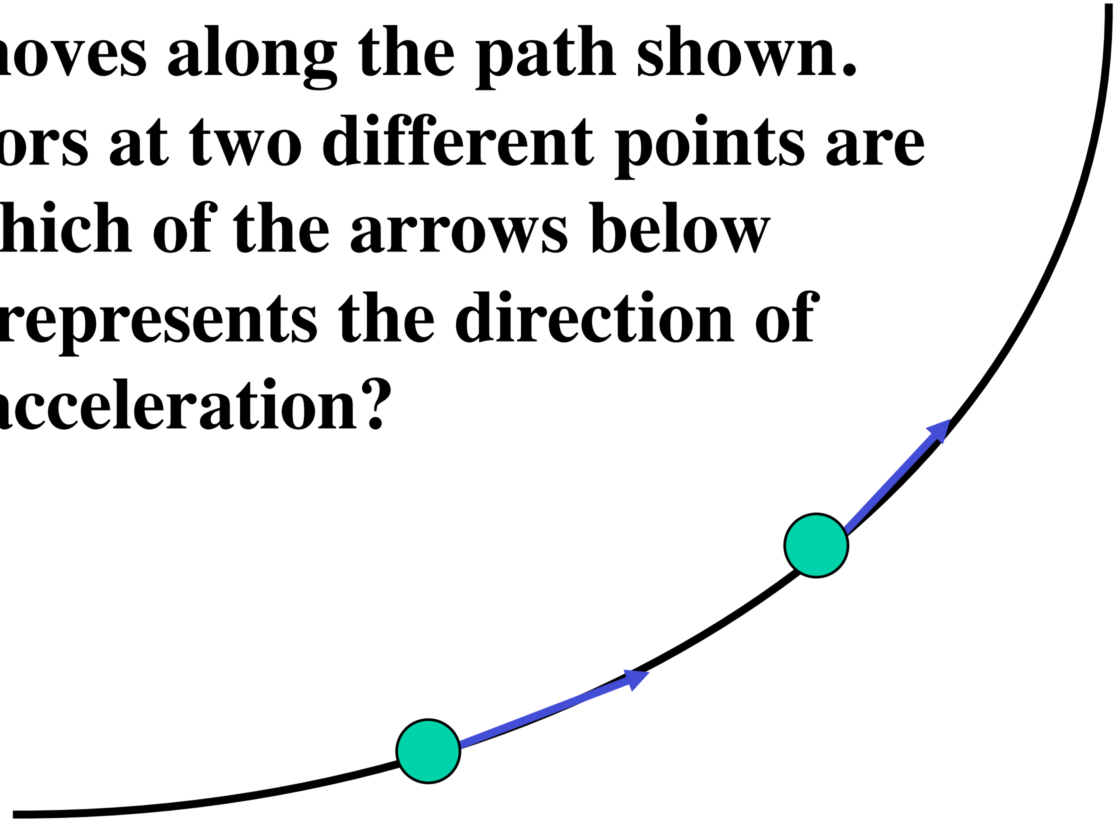


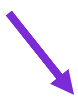
Is the acceleration of the object equal to zero?

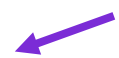
Motion on a curved path at constant speed

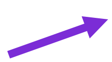


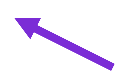
4-1. 5 A car moves along the path shown. Velocity vectors at two different points are sketched. Which of the arrows below most closely represents the direction of the average acceleration?



1. 

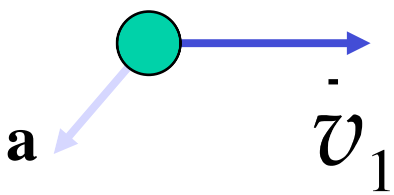
2. 

3. 

4. 

4-1.6 A child is riding a bicycle on a level street. The velocity and acceleration vectors of the child at a given time are shown.

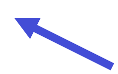
Which of the following velocity vectors may represent the velocity at a later time?



A.

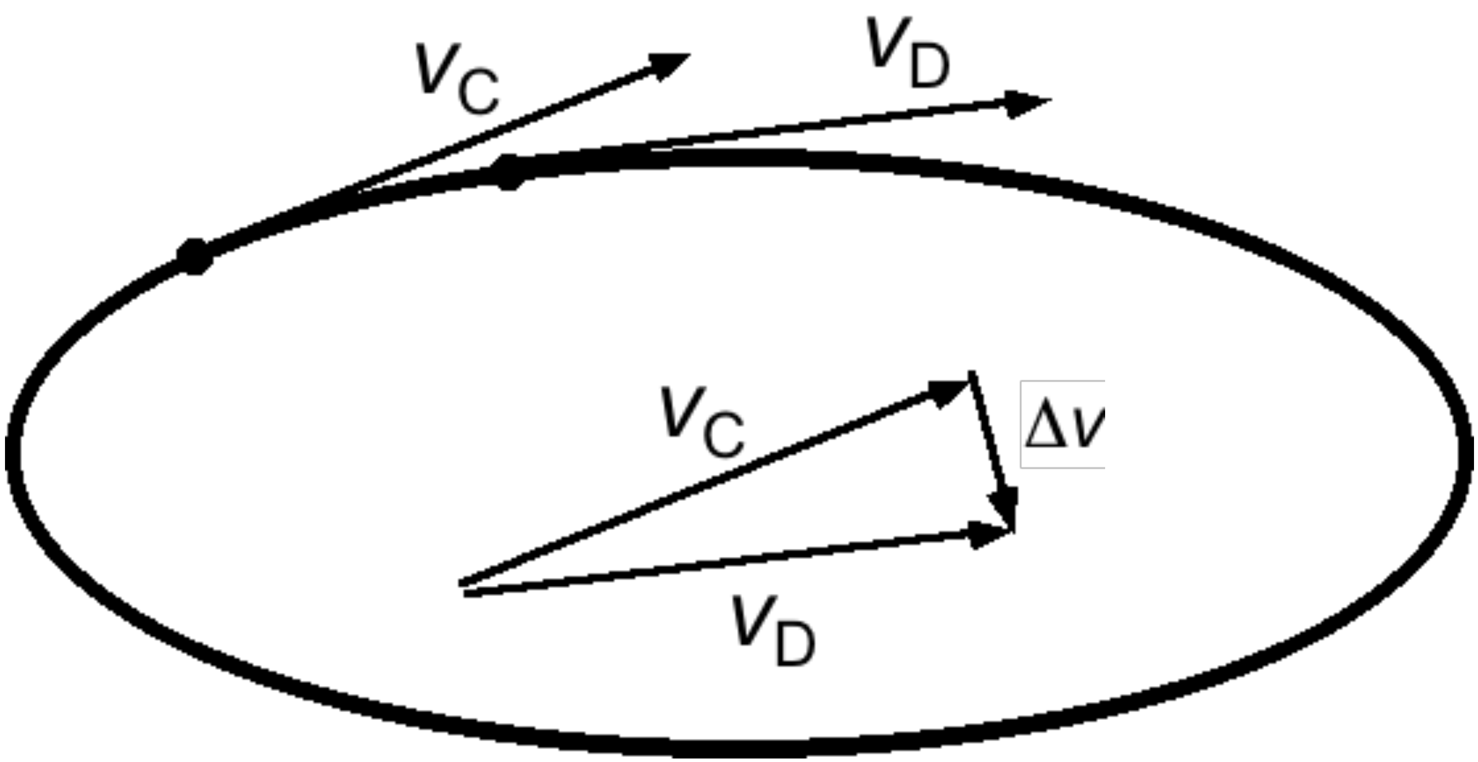
B.

C.



D.

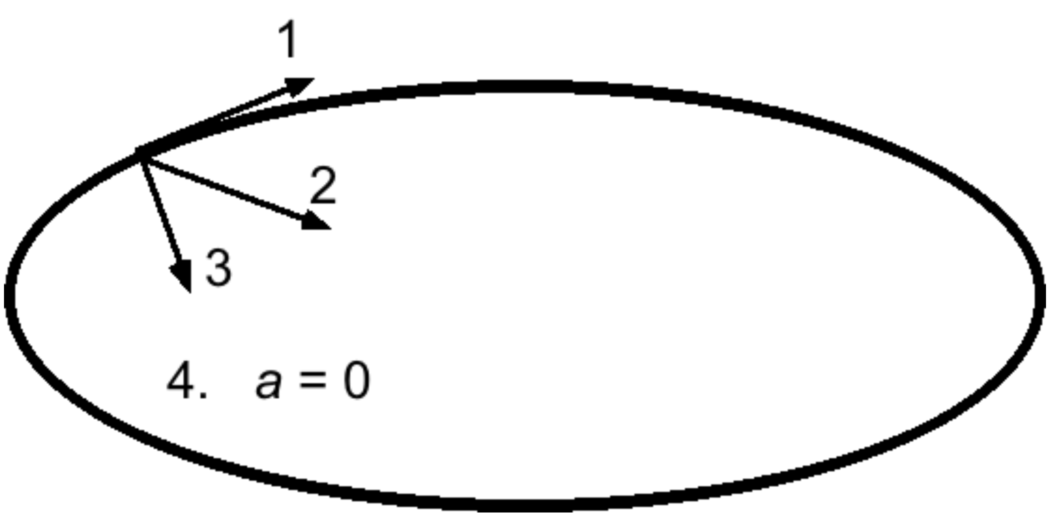
Biker moving around oval at *constant* speed



As point D *is moved closer to* C, *angle approaches 90°.*

4-1. 7 A biker is riding at constant speed clockwise on the oval track shown below.

Which vector correctly describes the *acceleration* at the point indicated?



Summary

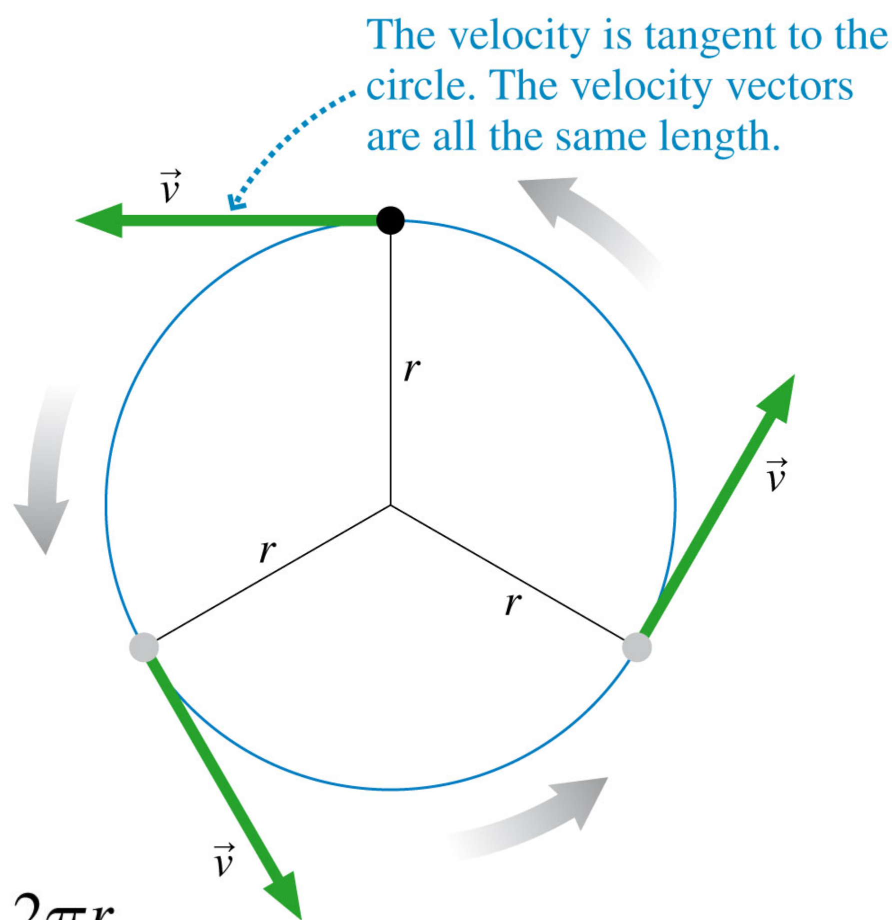
- For motion at constant speed, instantaneous acceleration vector is perpendicular to velocity vector
- Points ``inward''
- What is the **magnitude** of the acceleration vector?

Circular Motion

- Consider a ball on a roulette wheel.
- It moves along a circular path of radius r.
- Other examples of circular motion are a satellite in an orbit or a ball on the end of a string.
- Circular motion is an example of two-dimensional motion in a plane.

Uniform Circular Motion

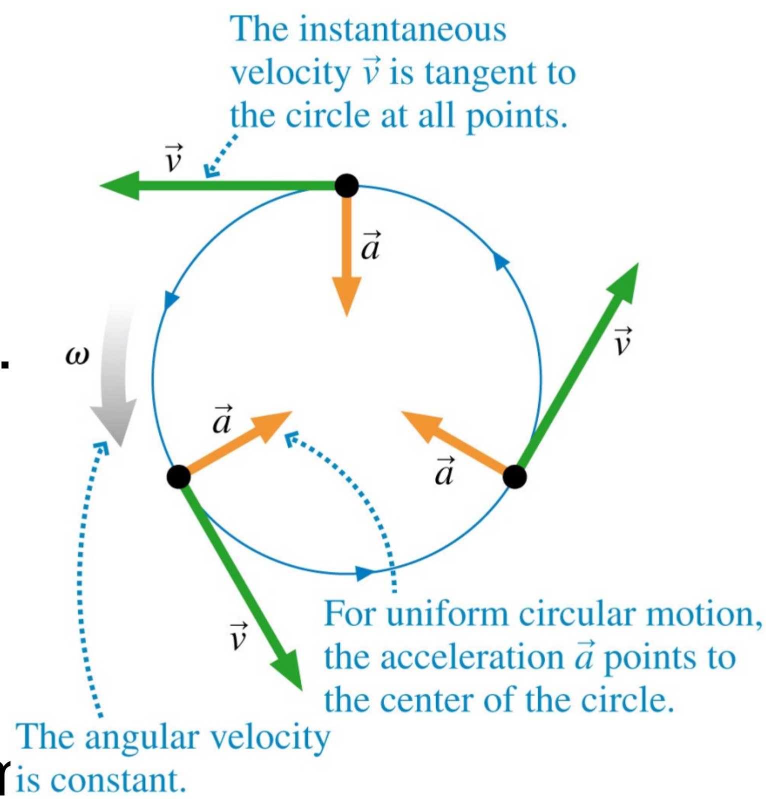
- particle that moves at *constant speed* around a circle of radius r.
- The time interval to complete one revolution is called the period, T.
- The period T is related to the speed v:



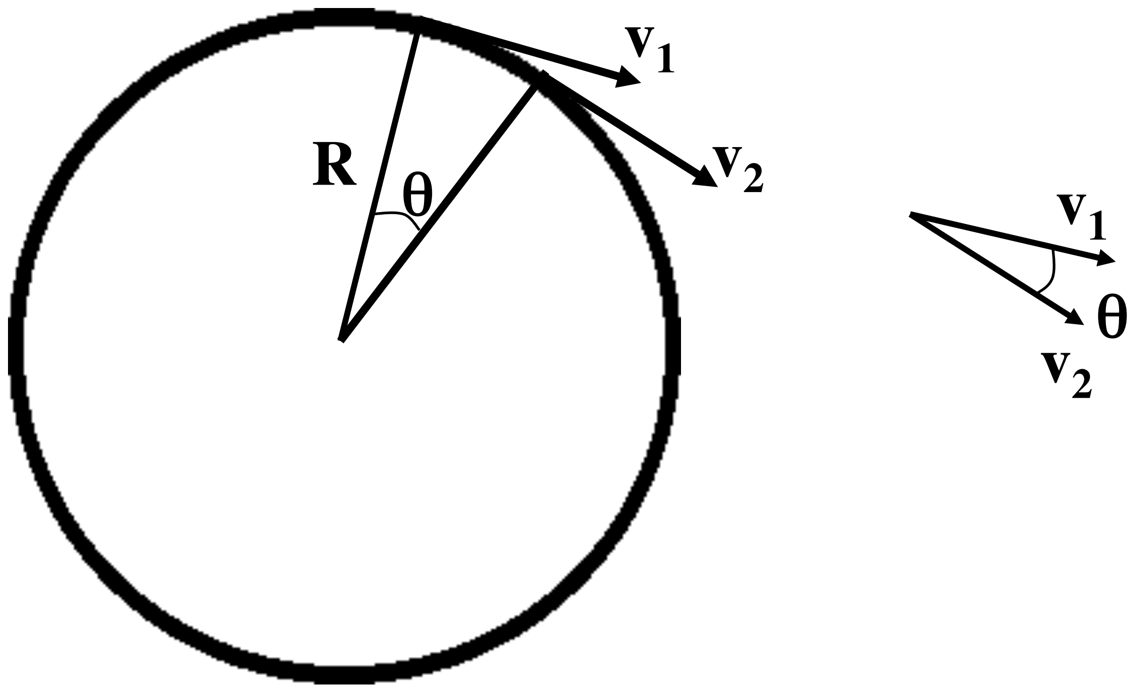
$$v = \frac{1 \text{ circumference}}{1 \text{ period}} = \frac{2\pi r}{T}$$

Centripetal Acceleration

- In uniform circular motion, although the speed is constant, there is a centripetal acceleration because the *direction* of the velocity vector is always changing.
- The direction of the centripetal acceleration is toward the center of the circle.
- The magnitude of the centripetal acceleration is constant for uniform circular motion.



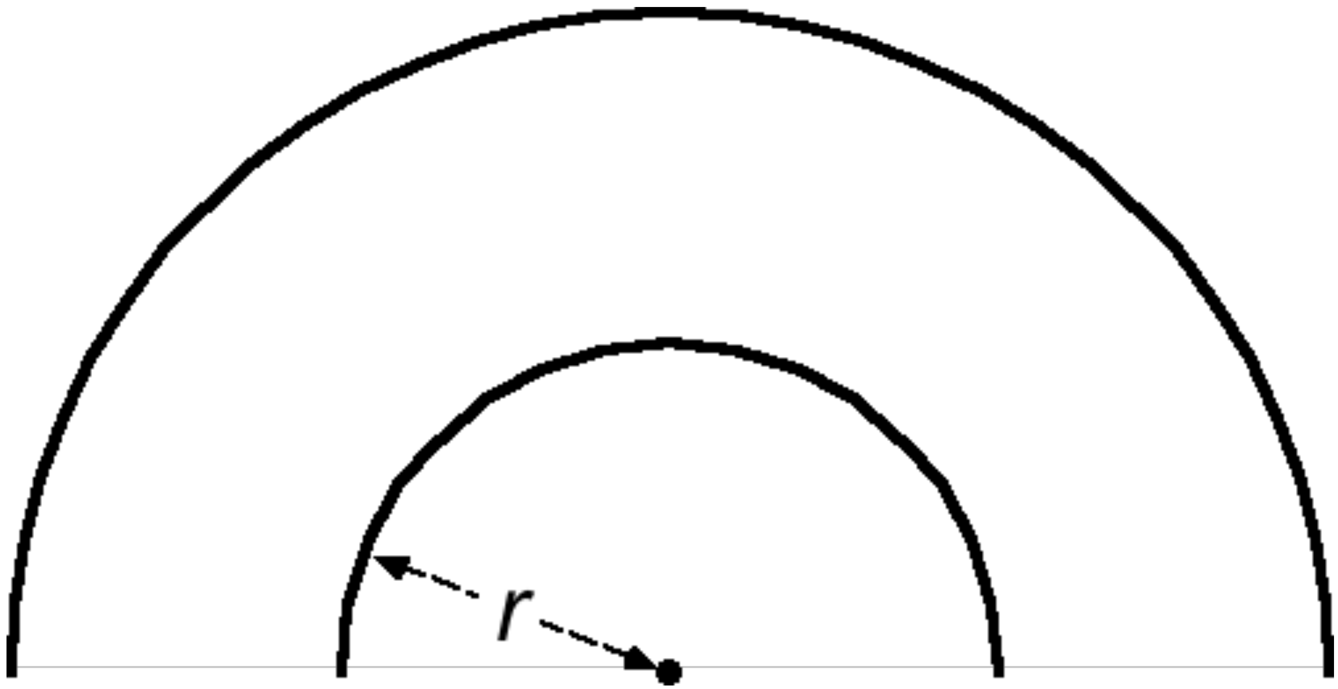
Acceleration vectors for ball swung in a horizontal circle at *constant* speed v



What is the magnitude of the acceleration?

$$|a| = v^2/R$$

Acceleration of object moving at constant speed on a circular path: $a = \frac{v^2}{r}$



Acceleration depends on radius of circle.

4-1.8 Two cars are moving at different constant speeds on a curved road. One after the other, they are passing the same point on the road: Car A at 18 mph; car B at 36 mph. If car A's acceleration is 2 m/s^2, car B's acceleration is:

A. 1 m/s^2

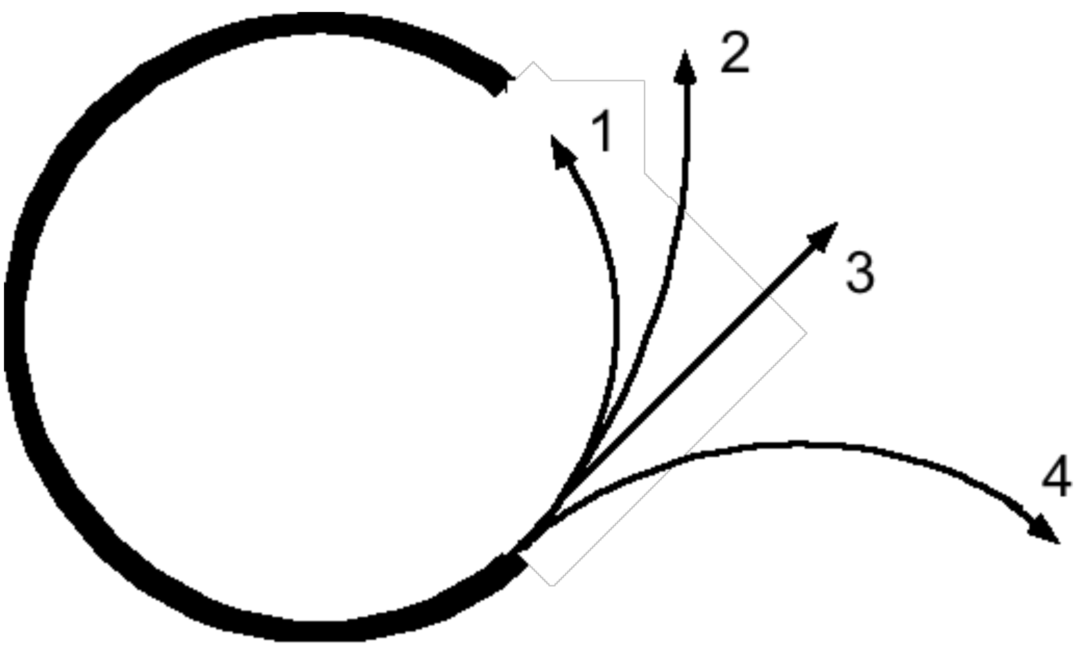
B. 2 m/s^2

C. 4 m/s^2

D. 8 m/s^2

4-1.9 A ball is rolling counter-clockwise at constant speed on a circular track. One quarter of the track is removed.

What path will the ball follow after reaching the end of the track?



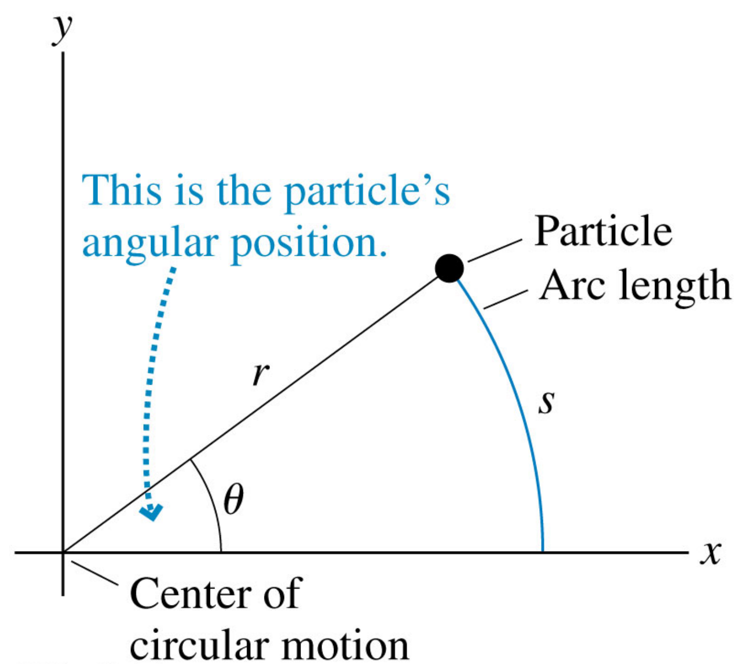
Angular Position

- The angle may be measured in degrees, revolutions (rev) or **radians** (rad), that are related by:

- Radians are the best! Because:

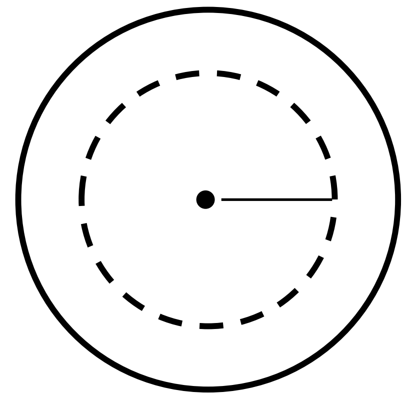
$$1 \text{ rev} = 360° = 2\pi \text{ rad}$$

$$s = r\theta \qquad \text{(with } \theta \text{ in rad)}$$



Rotations about fixed axis

- Linear speed: $v = (2\pi r)/T = \omega r$. Quantity ω is called ***angular velocity***

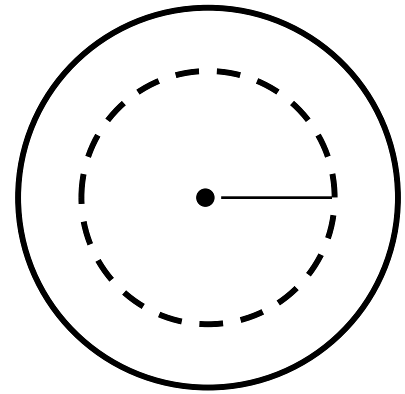


- ω is a vector! Use right hand rule to find direction of ω.

- Angular acceleration $\alpha = \Delta\omega/\Delta t$ is also a vector!
  - ω and α *parallel* → angular speed increasing
  - ω and α *antiparallel* → angular speed decreasing

Relating linear and angular kinematics

- Linear speed: $v = (2\pi r)/T = \omega r$



- Tangential acceleration: $a_{tan} = r\alpha$
- Radial acceleration: $a_{rad} = v^2/r = \omega^2 r$

$$\omega = \omega_0 + \alpha t$$

$$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$$

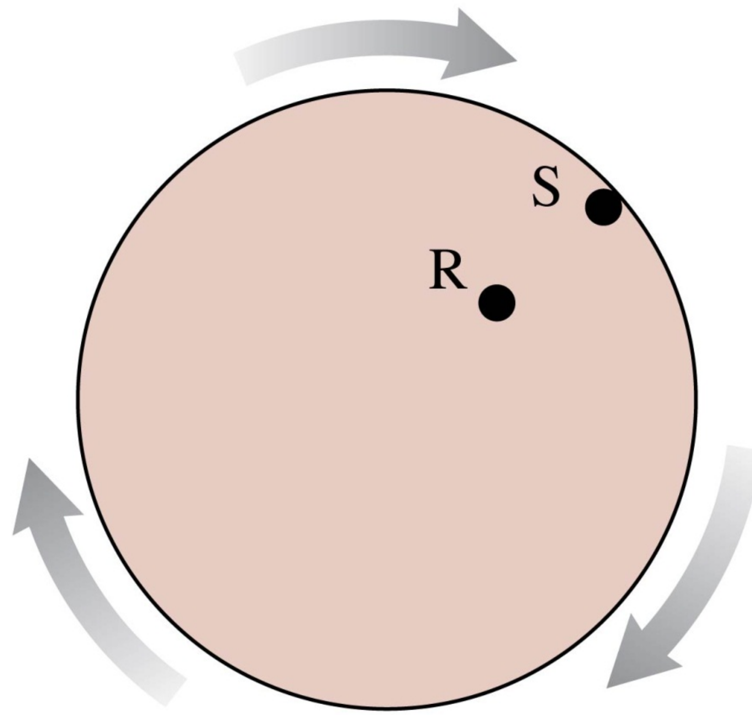
$$\omega^2 = \omega_0^2 + 2\alpha\Delta\theta$$

Sample problem

Taken directly from DIYelectriccar.com: I want to build an electric car that can go 60 mph. (This means that you want the edge of the tire to go 60 mph.) If my tires are 15” in diameter, how many rpms (revolutions per minute) must the motor be able to produce on the tire axels?

Clicker 4-1.10

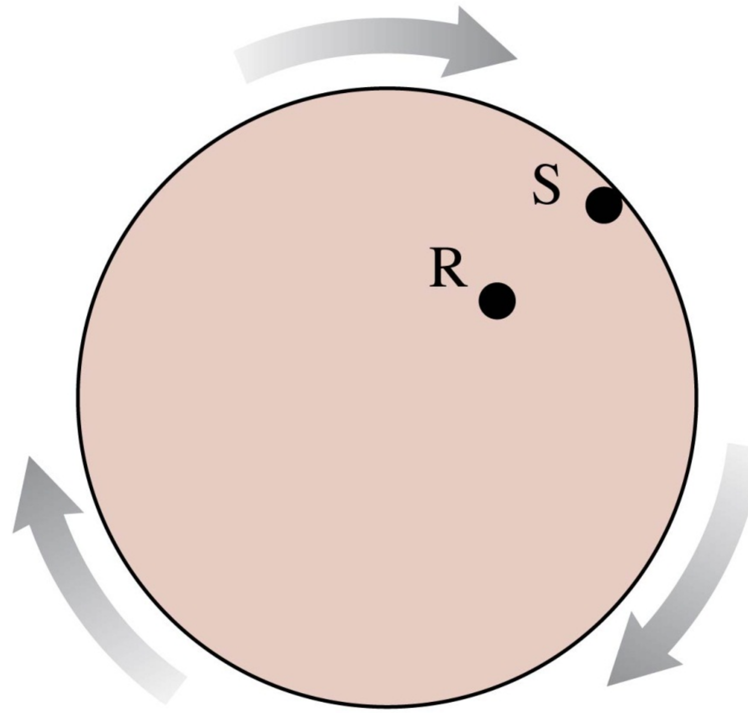
Rasheed and Sofia are riding a merry-go-round that is spinning steadily. Sofia is twice as far from the axis as is Rasheed. Sofia's angular velocity is ______ that of Rasheed.



1. half
2. the same as
3. twice
4. four times
5. We can't say without knowing their radii.

Clicker 4-1.9

Rasheed and Sofia are riding a merry-go-round that is spinning steadily. Sofia is twice as far from the axis as is Rasheed. Sofia's velocity is ______ that of Rasheed.

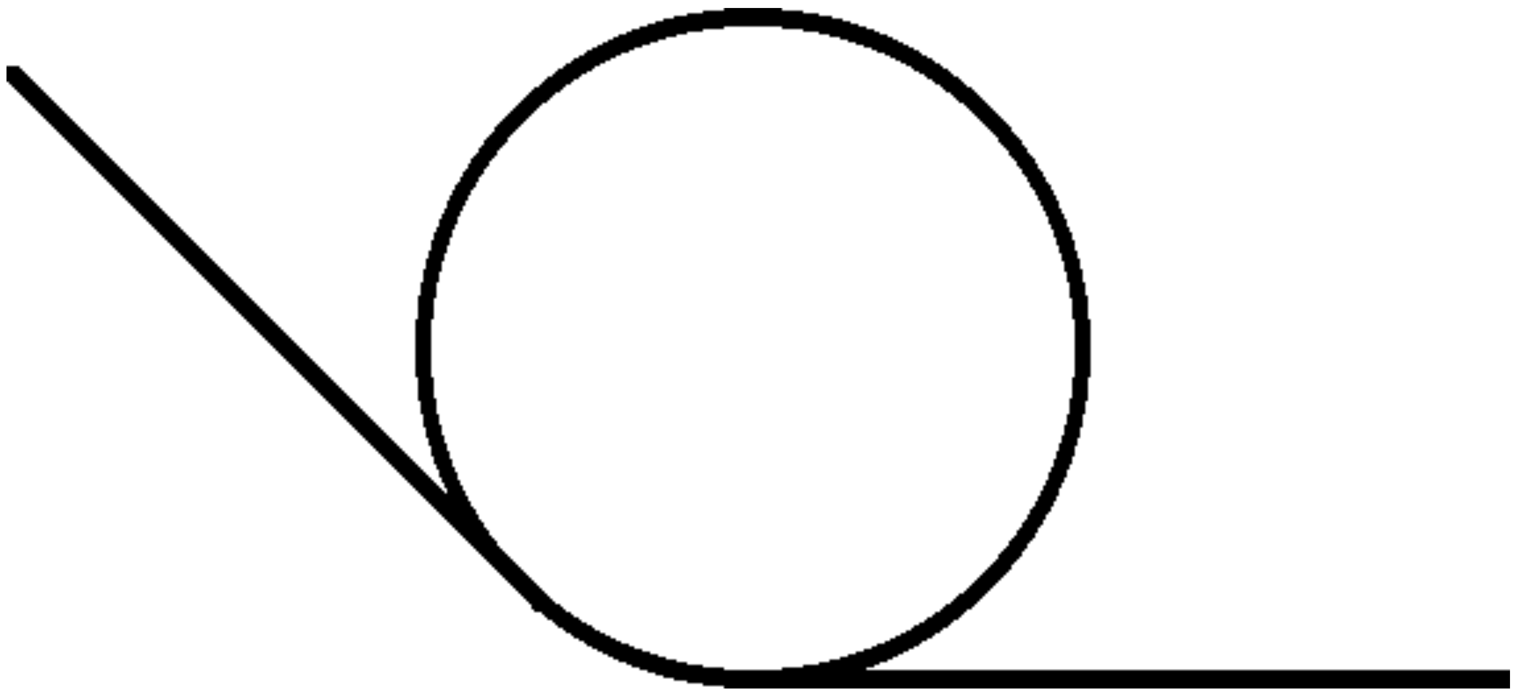


1. half
2. the same as
3. twice
4. four times
5. We can't say without knowing their radii.

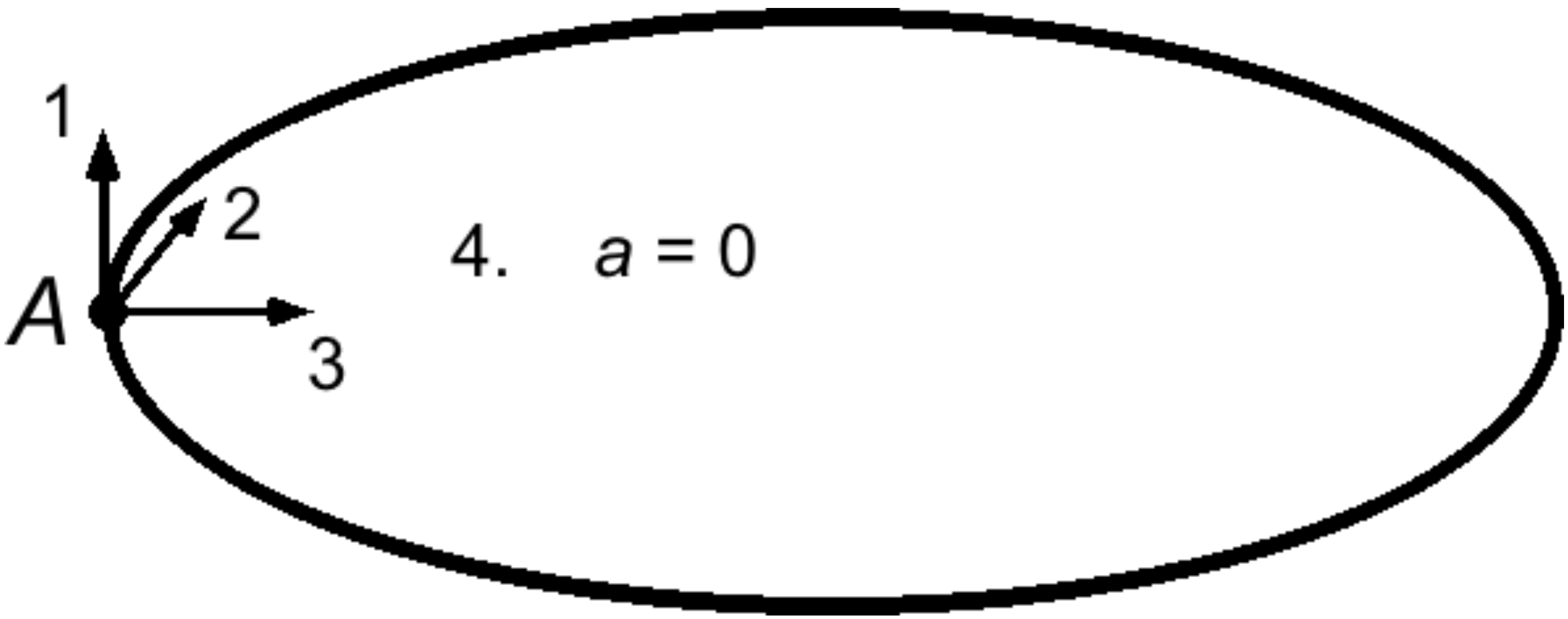
Reading assignment

- Relative motion
- 4.4 in textbook
- Review for Exam 1 !

Ball going through loop-the-loop



Acceleration vector for object speeding up from rest at point *A* ?



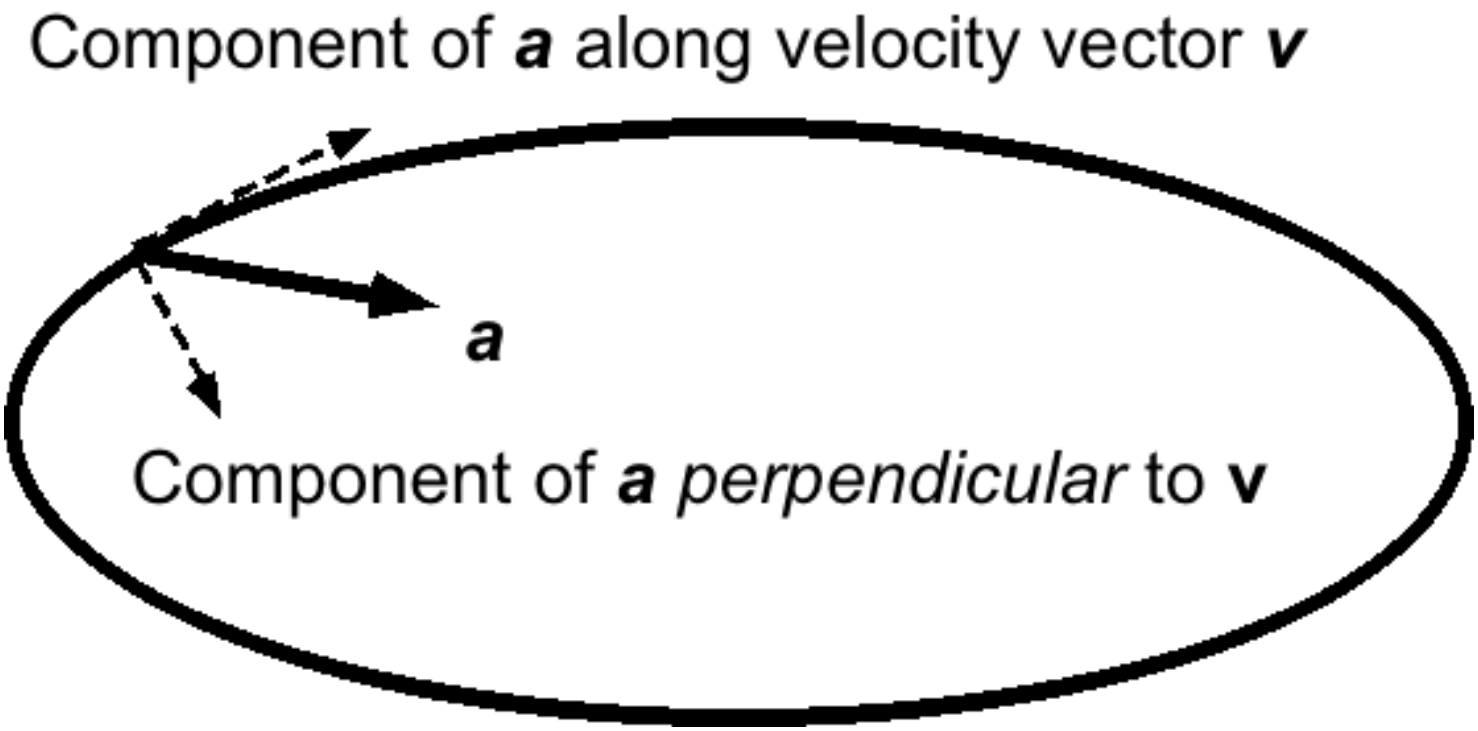
What if the speed is changing?

- Consider acceleration for object on curved path *starting from rest*
- Initially, $v^2/r = 0$, so no radial acceleration
- But a is not zero! It must be **parallel** to velocity

Acceleration vectors for object speeding up:

Tangential and radial components

(or parallel and perpendicular)



Sample problem

A Ferris wheel with diameter 14.0 m, which rotates counter-clockwise, is just starting up. At a given instant, a passenger on the rim of the wheel and passing through the lowest point of his circular motion is moving at 3.00 m/s and is gaining speed at a rate of 0.500 m/s^2. (a) Find the magnitude and the direction of the passenger's acceleration at this instant. (b) Sketch the Ferris wheel and passenger showing his velocity and acceleration vectors.

What is the magnitude of the acceleration of an object moving at *constant speed* if the path is curved but *not* a circle?

$$a = \frac{v^2}{r}$$

"*r*" is the *radius of curvature* of the path at a given point

Summary

Components of acceleration vector:

- Parallel to direction of velocity:

  (Tangential acceleration)

  – "How much does speed of the object increase?"

- Perpendicular to direction of velocity:

  (Radial acceleration)

  – "How quickly does the object turn?"