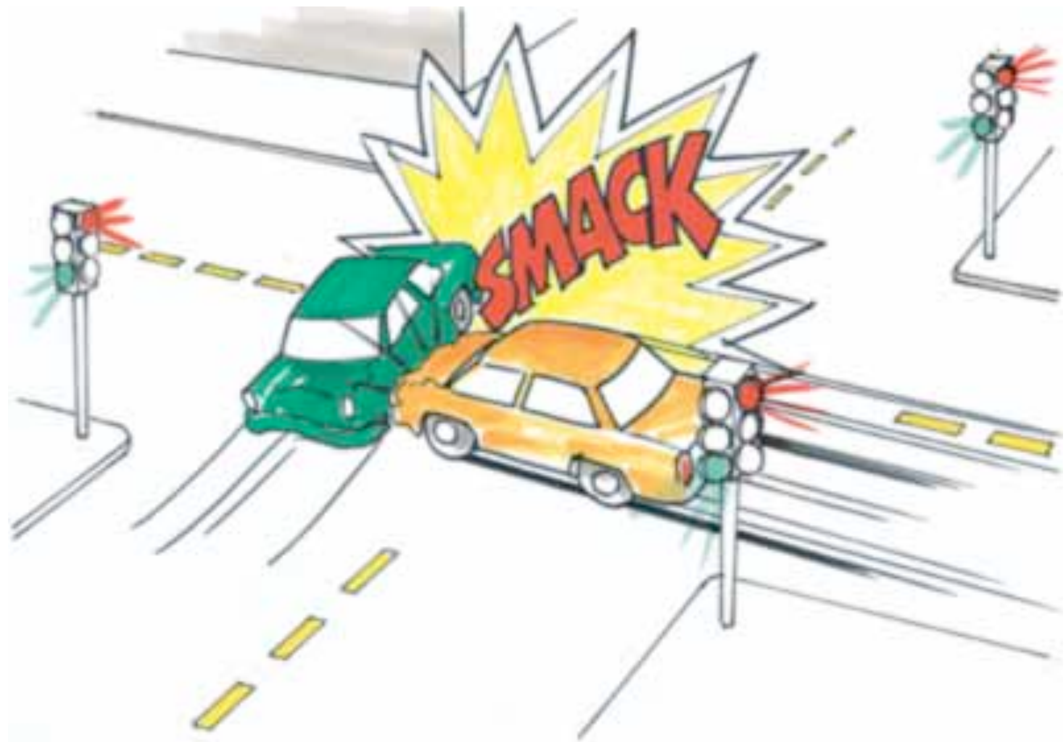
Welcome back to Physics 211

Today's agenda:

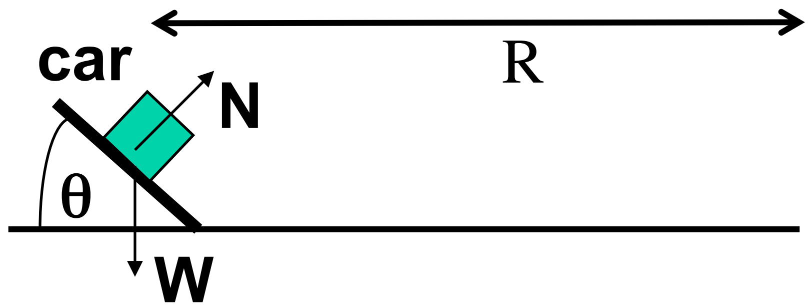
- *Circular motion*
- *Impulse and momentum*



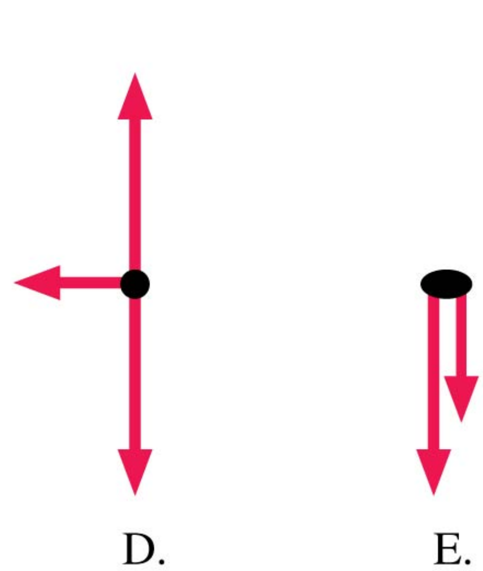
Current assignments

- Reading: Chapter 9 in textbook
  - Prelecture due next Thursday
- HW#8 due NEXT Friday (extension!) at 5 pm.
- Midterm 2 on Tuesday, Oct 21:
  - Chapters 4.4-7
  - Newton's Laws, circular motion kinematics, relative motion

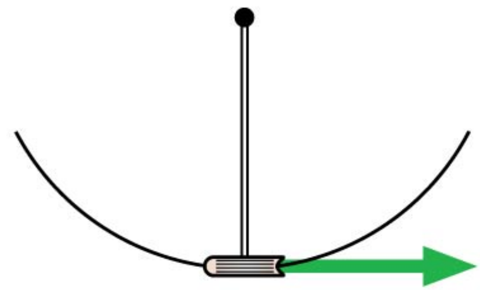
Sample problem: A 1000 kg car is going around a banked, **frictionless** circular track with radius 100 m and bank angle of 10 degrees. How fast should the car go so that it doesn't slide off the track?

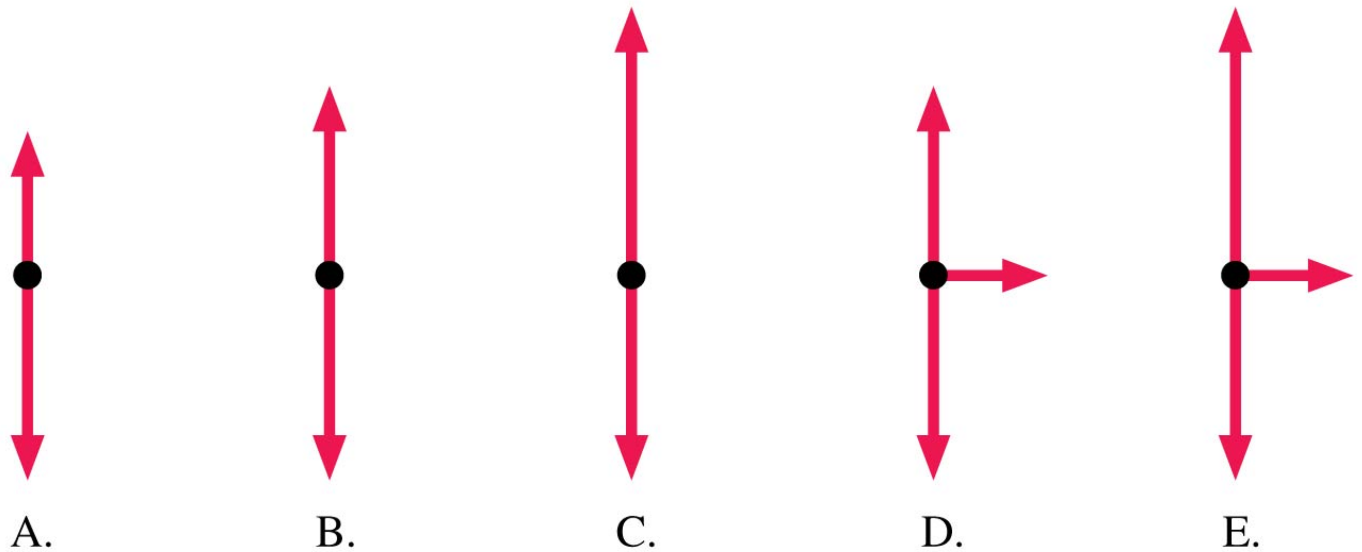


8-2.1 A roller coaster car does a loop-the-loop. Which of the free-body diagrams shows the forces on the car at the top of the loop? Rolling friction can be neglected.



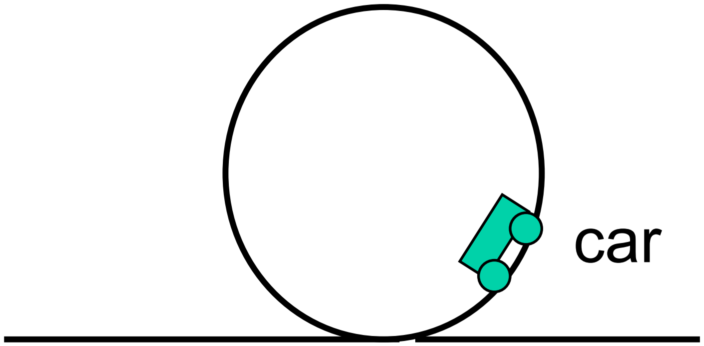
8-2.2 A physics textbook swings back and forth as a pendulum. Which is the correct free-body diagram when the book is at the bottom and moving to the right?



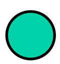


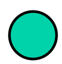
Demo: Motion on loop-the-loop

What is normal force on car at top and bottom of loop?
Neglect friction; assume moves with speed v_B at bottom and v_T at top



At bottom





At top

Demo – swinging water bucket

- Does the water fall out?

- What is the FBD for the water at the top of the swing?

Demo – swinging water bucket

- So why doesn't the water fall out?

- What if $mv^2/R < mg$ for the water?
- This is like a satellite in orbit around the earth

Forces in circular motion summary:

- Draw a free body diagram
- Sum the forces as usual
  - There IS NOT AN “EXTRA” centripetal force
  - find F_{NET}(radial) and F_{NET}(other)
  - Velocity is NOT a force
- THEN figure out what F_{NET} (radial) has to be: in uniform circular motion
  - F_{NET}(radial) = ma
  - $a = v^2/r$

Impulse

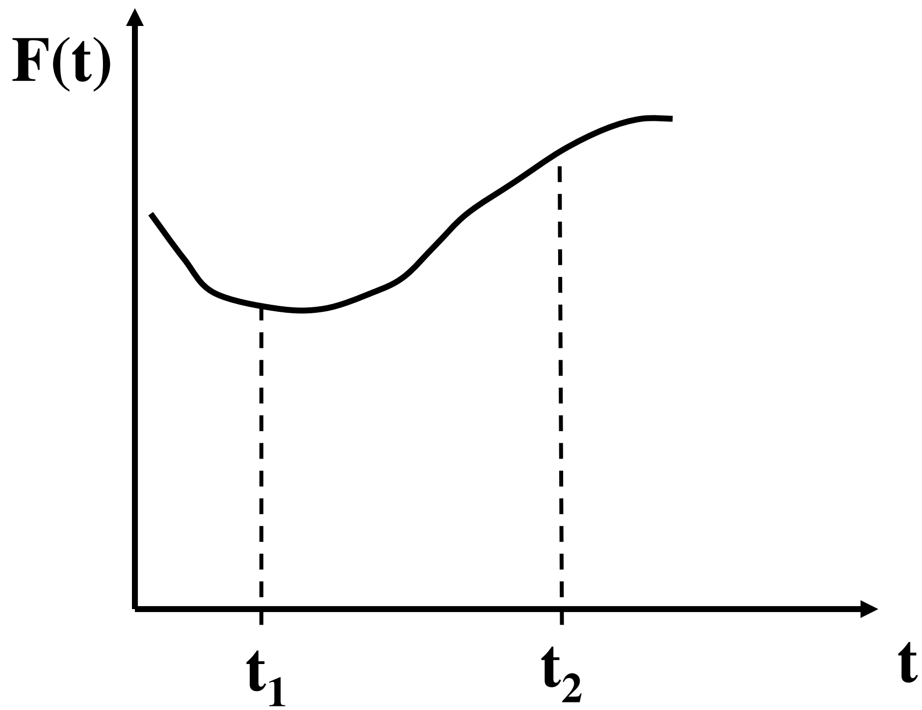
- Constant force F_{12} acting on object 1 due to object 2 for a time Δt yields an **impulse**

$$I_{12} = F_{12}\,\Delta t$$

- In general, for a time varying force need to use this for small Δt and add:

$$I = \sum F(t)\,\Delta t =$$

Impulse for time varying forces



*** area under curve equals impulse**

Impulse → change in momentum

- Consider first constant forces ...

- Constant acceleration equation:

$$v_f = v_i + at$$

↓

$$mv_f - mv_i = ma\Delta t =$$

- If we call p = mv ***momentum*** we see that

$$\Delta p =$$

8-2.3 A crash test vehicle is equipped with two sensors: (1) one on its front bumper that measures the force of impact as a function of time F(t), and (2) one measuring the velocity v(t). How are these two related in a crash with a wall?

1. They're not related.
2. The integral (area under the curve) of v(t) is proportional to the change in the force.
3. The derivative of the velocity equals the maximum force.
4. The change in the velocity is proportional to the maximum force
5. The change in the velocity is proportional to the integral (area under the curve) of the force.

Impulse demo

- Cart equipped with force probe collides with rubber tube
- Measure force *vs.* time and momentum *vs.* time
- Find that integral of force curve is precisely the change in p!

Definitions of *impulse* and *momentum*

Impulse imparted to object 1 by object 2:

$$I_{12} = F_{12}\Delta t$$

Momentum of an object:

$$\mathbf{p} = m\mathbf{v}$$

Impulse-momentum theorem

$$\mathbf{I}_{\mathbf{net}} = \Delta \mathbf{p}$$

The net impulse imparted to an object is equal to its change in momentum.

8-2.4 Consider the **change in momentum** in these three cases:

A. A ball moving with speed *v* is brought to rest.

B. The same ball is projected from rest so that it moves with speed *v.*

C. The same ball moving with speed *v* is brought to rest and immediately projected backward with speed *v*.

In which case(s) does the ball undergo the largest magnitude of change in momentum?

1. Case A.
2. Case B.
3. Case C.
4. Cases A and B.

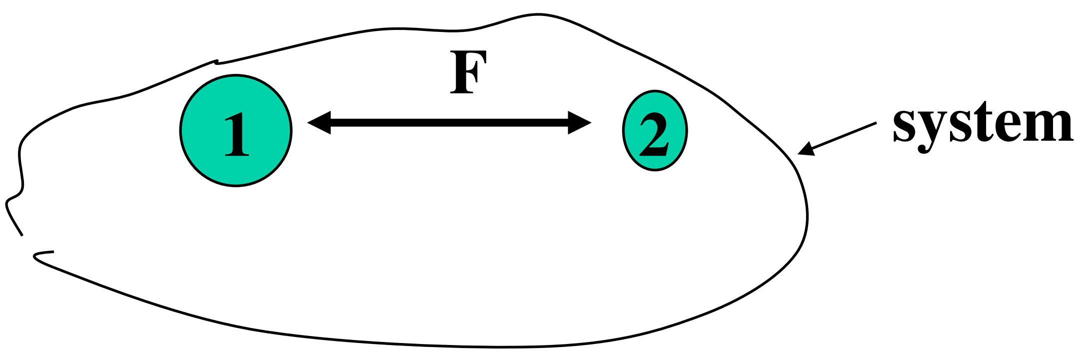
Newton's 3rd law and changes in momentum

If all external forces (weight, normal, etc.) cancel:

Conservation of momentum

- Assuming no net forces act on bodies there is no net impulse on composite system

- Therefore, no change in ***total*** momentum $\Delta(p_1 + p_2) = 0$



Conservation of momentum

(for a system consisting of two objects 1 and 2)

$$\Delta\vec{p}_1 = -\Delta\vec{p}_2$$

If the net (external) force on a system is zero, the total momentum of the system is constant.

Whenever two or more objects in an isolated system interact, the total momentum of the system remains constant

Conservation of momentum with carts

- *One cart with mass m_1 begins at rest $v_{1i} = 0$, and the other cart (with the same mass) has a velocity $v_{2i} = v$. After the two carts hit each other, what is the sum of the velocities of the two carts $v_{2f} + v_{1f}$?*

Demo

- Experiment –> zero sensors

8-2.5 A cart moving to the right at speed v collides with an identical stationary cart on a low-friction track. The two carts stick together after the collision and move to the right.

What is their speed after colliding?

1. 0.25 v
2. 0.5 v
3. v
4. $2v$

8-2.6 A student is sitting on a low-friction cart and is holding a medicine ball. The student then throws the ball at an angle of 60° (measured from the horizontal) with a speed of 10 m/s.

The mass of the student (with the car) is 80 kg. The mass of the ball is 4 kg.

What is the final speed of the student (with car)?

1. 0 m/s
2. 0.25 m/s
3. 0.5 m/s
4. 1 m/s

Definitions of *impulse* and *momentum*

Impulse imparted to object :

$$I_{net} = F_{net}\Delta t$$

Momentum of an object:

$$\mathbf{p} = m\mathbf{v}$$

$$\mathbf{I_{net} = \Delta p}$$

Newton's 3rd law and changes in momentum:

Last class: $$\frac{d}{dt}\left[\vec{p}_1 + \vec{p}_2\right] = \vec{F}_{12} - \vec{F}_{21} = 0$$

Conservation of momentum

(for a system consisting of two objects 1 and 2)

$$\Delta\vec{p}_1 = -\Delta\vec{p}_2$$

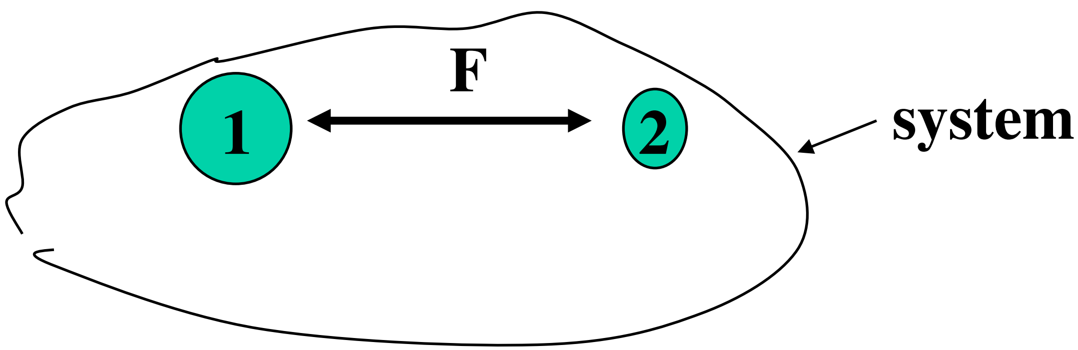
If the net (external) force on a system is zero, the total momentum of the system is constant.

Whenever two or more objects in an isolated system interact, the total momentum of the system remains constant

Another way to think about Conservation of momentum

- Assuming no net forces act on bodies there is no net impulse on composite system

- Therefore, no change in ***total*** momentum $\Delta(p_1 + p_2) = 0$



Demo: medicine ball and person on cart

Momentum is a vector!

$$\vec{p}_{A,\text{initial}} + \vec{p}_{B,\text{initial}} = \vec{p}_{A,\text{final}} + \vec{p}_{B,\text{final}}$$

- Must conserve components of momentum simultaneously

- In 2 dimensions:

Sample Problem: At the intersection of Texas Avenue and University Drive, a blue, subcompact car with mass 950 kg traveling east on University collides with a maroon pickup truck with mass 1900 kg that is traveling north on Texas and ran a red light. The two vehicles stick together as a result of the collision and, after the collision, the wreckage is sliding at 16.0 m/s in the direction 24^o east of north. Calculate the speed of each vehicle before the collision. The collision occurs during a heavy rainstorm; you can ignore friction forces between the vehicles and the wet road.

Kinetic Energy

- Newton's Laws are *vector* equations
- Sometimes more appropriate to consider *scalar* quantities related to speed and mass

For an object of mass m moving with speed v:

$$K = (1/2)mv^2$$

- *Energy* of motion
- scalar!
- Measured in Joules -- J

Gravitational Potential Energy

For an object of mass m near the surface of the earth:

$$U_g = mgh$$

- h is height above arbitrary reference line
- Measured in Joules -- J (like kinetic energy)

Total energy for object moving under gravity

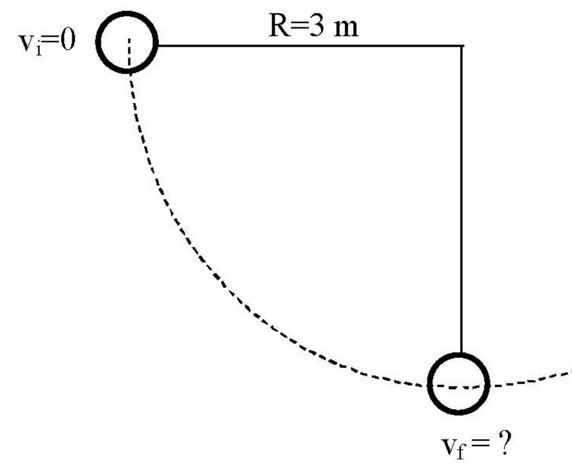
$E = U_g + K = \text{constant}$

* E is called the (mechanical) energy

* It is conserved:

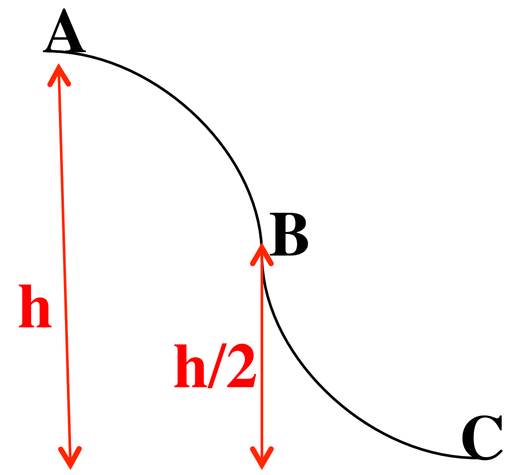
$$(½)\, mv^2 + mgh = \text{constant}$$

Sample problem: A ball of mass m=7 kg attached to a massless string of length R=3 m is released from the position shown in the figure below. (a) Find magnitude of velocity of the ball at the lowest point on its path. (b) Find the tension in the string at that point.



8-2.6 If the velocity at B is v, then what is the velocity at C?

1. 2v
2. v
3. sqrt(2) v
4. v/sqrt(2)
5. None of the above



Collisions

If two objects collide and the net force exerted on the system (consisting of the two objects) is zero, the sum of their momenta is constant.

$$\vec{p}_{A,\text{initial}} + \vec{p}_{B,\text{initial}} = \vec{p}_{A,\text{final}} + \vec{p}_{B,\text{final}}$$

The sum of their kinetic energies may or may not be constant.

Elastic and inelastic collisions

- If K is conserved – collision is said to be *elastic*

e.g., cue balls on a pool table

$$K_{A,i} + K_{B,i} \qquad K_{A,f} + K_{B,f}$$

- Otherwise termed *inelastic*

  e.g., lump of putty thrown against wall

$$K_{A,i} + K_{B,i} \qquad K_{A,f} + K_{B,f}$$

- Extreme case = *completely inelastic* -- objects stick together after collision

8-2.7 Cart A moving to the right at speed *v* collides with an identical stationary cart (cart B) on a low-friction track. The collision is *elastic* (*i.e.*, there is no loss of kinetic energy of the system).

What is each cart's velocity after colliding (considering velocities to the right as positive)?

	Cart A	Cart B
1	$-v$	$2v$
2	$-\frac{1}{3}v$	$\frac{4}{3}v$
3	0	v
4	$\frac{1}{3}v$	$\frac{2}{3}v$

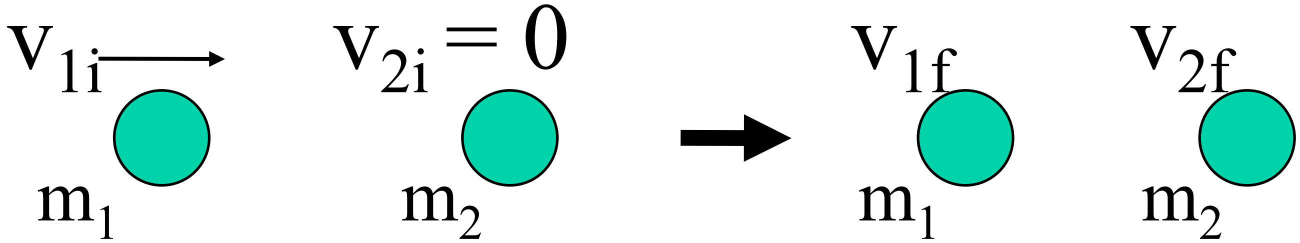
Check conservation of momentum and energy

	Cart A *(m)*	Cart B *(m)*	Final momentum	Final kin. energy
1	$-v$	$2v$		
2	$-\,{}^{1}/_{3}\,v$	${}^{4}/_{3}\,v$		
3	0	v		
4	${}^{1}/_{3}\,v$	${}^{2}/_{3}\,v$		

Demo

- Experiment –> zero sensors

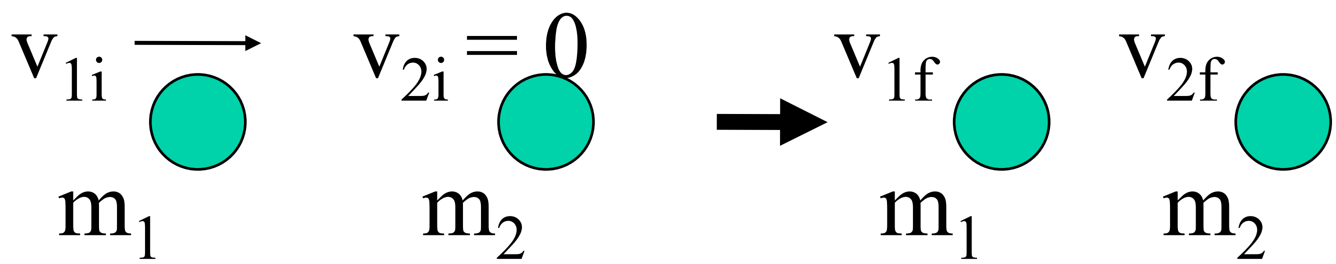
Elastic collision of two masses



Momentum → $m_1 v_{1i} + 0 = m_1 v_{1f} + m_2 v_{2f}$

Energy → $(1/2)m_1 v_{1i}^2 + 0 = (1/2)m_1 v_{1f}^2 + (1/2)m_2 v_{2f}^2$

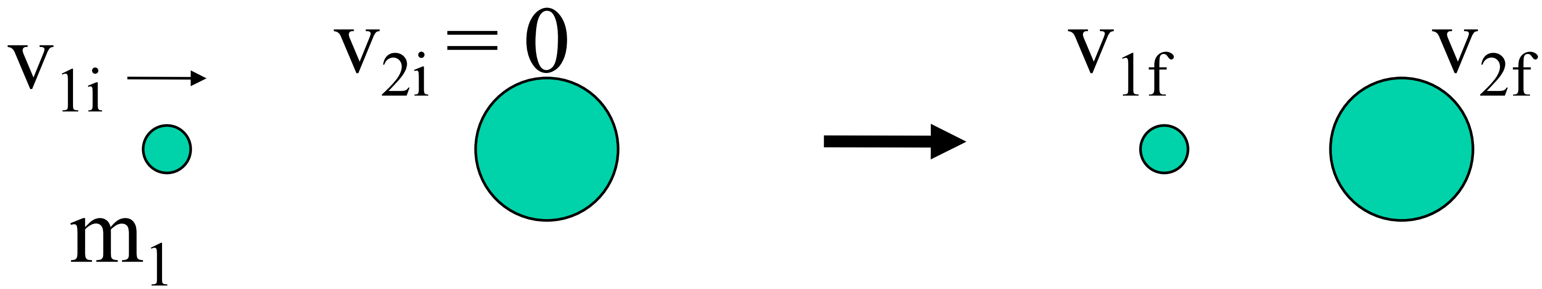
Special cases: (i) $m_1 = m_2$



$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i}$$

$$v_{2f} = \frac{2m_1}{m_1 + m_2} v_{1i}$$

Special cases: (ii) $m_1 << m_2$



$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i}$$

$$v_{2f} = \frac{2m_1}{m_1 + m_2} v_{1i}$$

Relative velocities for elastic collisions

- For example, velocity of m_2 relative to $m_1 = v_2 - v_1$
- With m_2 stationary initially, $v_{2f} - v_{1f} = v_{1i}$
- In general, for *elastic collisions*, relative velocity has same magnitude before and after collision

$$v_{2f} - v_{1f} = -(v_{2i} - v_{1i})$$