Welcome back to Physics 211

Today's agenda:

- *Impulse and momentum*

Current assignments

- Reading: Chapter 10 in textbook
  - Prelecture due next Tuesday
- HW#8 due this Friday at 5 pm.

9-2.1 A crash test vehicle is equipped with two sensors: (1) one on its front bumper that measures the force of impact as a function of time F(t), and (2) one measuring the velocity v(t). How are these two related in a crash with a wall?

1. They're not related.
2. The integral (area under the curve) of v(t) is proportional to the change in the force.
3. The derivative of the velocity equals the maximum force.
4. The change in the velocity is proportional to the maximum force
5. The change in the velocity is proportional to the integral (area under the curve) of the force.

Impulse demo

- Cart equipped with force probe collides with rubber tube
- Measure force *vs.* time and momentum *vs.* time
- Find that integral of force curve is precisely the change in p!

Definitions of *impulse* and *momentum*

Impulse imparted to object 1 by object 2:

$$I_{12} = F_{12}\Delta t$$

Momentum of an object:

$$\mathbf{p} = m\mathbf{v}$$

Impulse-momentum theorem

$$\mathbf{I}_{\mathbf{net}} = \Delta\mathbf{p}$$

The net impulse imparted to an object is equal to its change in momentum.

9-2.2 Consider the **change in momentum** in these three cases:

A. A ball moving with speed *v* is brought to rest.

B. The same ball is projected from rest so that it moves with speed *v.*

C. The same ball moving with speed *v* is brought to rest and immediately projected backward with speed *v*.

In which case(s) does the ball undergo the largest magnitude of change in momentum?

1. Case A.
2. Case B.
3. Case C.
4. Cases A and B.

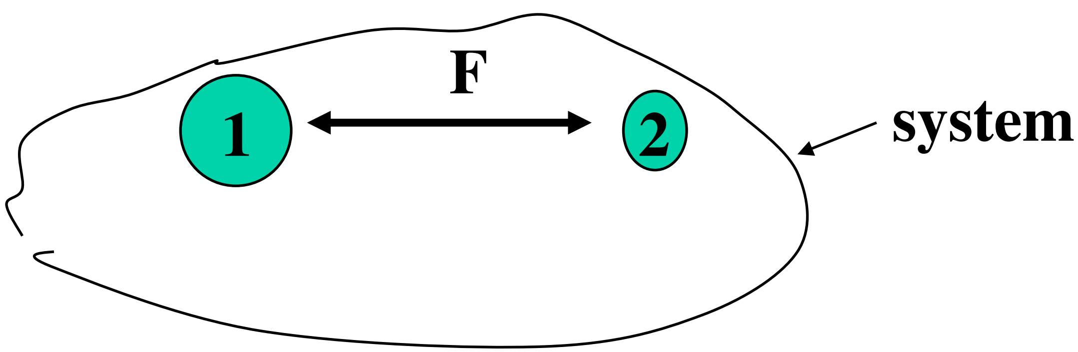
Newton's 3rd law and changes in momentum

If all external forces (weight, normal, etc.) cancel:

Conservation of momentum

- Assuming no net forces act on bodies there is no net impulse on composite system

- Therefore, no change in ***total*** momentum $\Delta(p_1 + p_2) = 0$



Conservation of momentum

(for a system consisting of two objects 1 and 2)

$$\Delta\vec{p}_1 = -\Delta\vec{p}_2$$

If the net (external) force on a system is zero, the total momentum of the system is constant.

Whenever two or more objects in an isolated system interact, the total momentum of the system remains constant

Conservation of momentum with carts

- *One cart with mass m_1 begins at rest $v_{1i} = 0$, and the other cart (with the same mass) has a velocity $v_{2i} = v$. After the two carts hit each other, what is the sum of the velocities of the two carts $v_{2f} + v_{1f}$?*

Carts Demo

9-2.3 A cart moving to the right at speed v collides with an identical stationary cart on a low-friction track. The two carts stick together after the collision and move to the right.

What is their speed after colliding?

1. $0.25\ v$
2. $0.5\ v$
3. v
4. $2v$

9-2.4 A student is sitting on a low-friction cart and is holding a medicine ball. The student then throws the ball at an angle of 60° (measured from the horizontal) with a speed of 10 m/s.

The mass of the student (with the car) is 80 kg. The mass of the ball is 4 kg.

What is the final speed of the student (with car)?

1. 0 m/s
2. 0.25 m/s
3. 0.5 m/s
4. 1 m/s

Definitions of *impulse* and *momentum*

Impulse imparted to object :

$$I_{net} = F_{net}\Delta t$$

Momentum of an object:

$$\mathbf{p} = m\mathbf{v}$$

$$\mathbf{I_{net} = \Delta p}$$

Newton's 3rd law and changes in momentum:

Last class: $$\frac{d}{dt}\left[\vec{p}_1 + \vec{p}_2\right] = \vec{F}_{12} - \vec{F}_{21} = 0$$

Conservation of momentum

(for a system consisting of two objects 1 and 2)

$$\Delta\vec{p}_1 = -\Delta\vec{p}_2$$

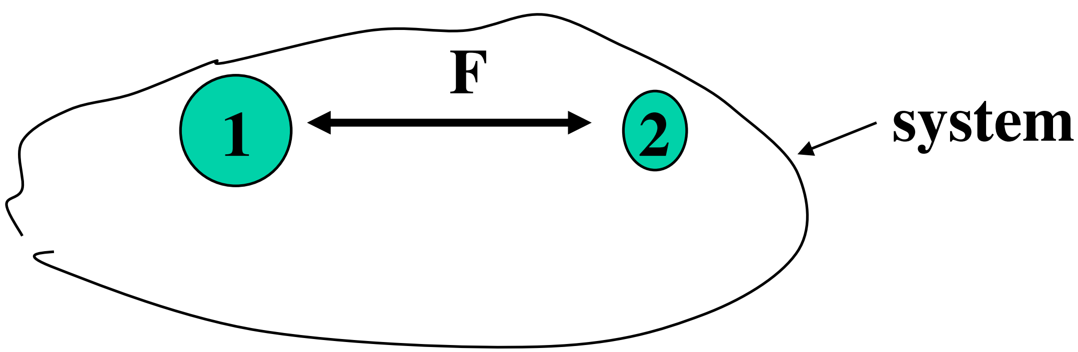
If the net (external) force on a system is zero, the total momentum of the system is constant.

Whenever two or more objects in an isolated system interact, the total momentum of the system remains constant

Another way to think about Conservation of momentum

- Assuming no net forces act on bodies there is no net impulse on composite system

- Therefore, no change in ***total*** momentum $\Delta(p_1 + p_2) = 0$



Demo: medicine ball and person on cart

Momentum is a vector!

$$\vec{p}_{A,\text{initial}} + \vec{p}_{B,\text{initial}} = \vec{p}_{A,\text{final}} + \vec{p}_{B,\text{final}}$$

- Must conserve components of momentum simultaneously

- In 2 dimensions:

Sample Problem: At the intersection of Texas Avenue and University Drive, a blue, subcompact car with mass 950 kg traveling east on University collides with a maroon pickup truck with mass 1900 kg that is traveling north on Texas and ran a red light. The two vehicles stick together as a result of the collision and, after the collision, the wreckage is sliding at 16.0 m/s in the direction 24^{o} east of north. Calculate the speed of each vehicle before the collision. The collision occurs during a heavy rainstorm; you can ignore friction forces between the vehicles and the wet road.

Kinetic Energy

- Newton's Laws are *vector* equations
- Sometimes more appropriate to consider *scalar* quantities related to speed and mass

For an object of mass m moving with speed v:

$$K = (1/2)mv^2$$

- *Energy* of motion
- scalar!
- Measured in Joules -- J

Gravitational Potential Energy

For an object of mass m near the surface of the earth:

$$U_g = mgh$$

- h is height above arbitrary reference line
- Measured in Joules -- J (like kinetic energy)

Total energy for object moving under gravity

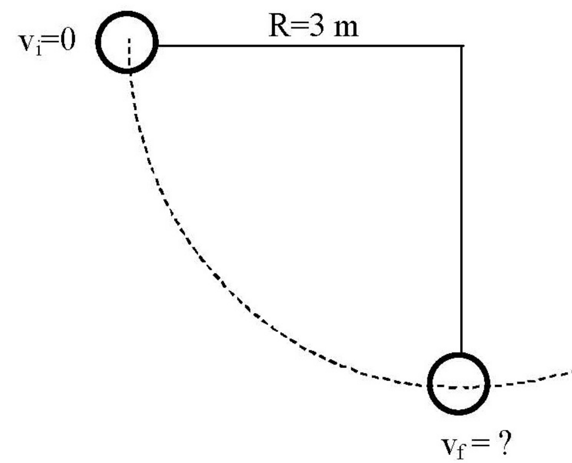
$E = U_g + K = \text{constant}$

* E is called the (mechanical) energy

* It is conserved:

$$(½)\,mv^2 + mgh = \text{constant}$$

Sample problem 2: A ball of mass m=7 kg attached to a massless string of length R=3 m is released from the position shown in the figure below. (a) Find magnitude of velocity of the ball at the lowest point on its path. (b) Find the tension in the string at that point.



Total Energy $E = U_g + K$

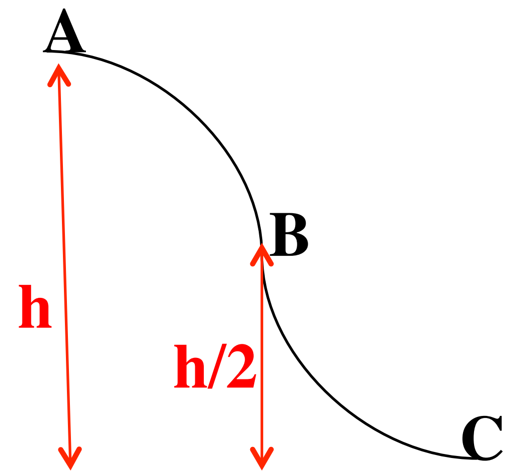
Gravitational Potential energy: for an object of mass m: $U_g = mgh$

- h is height above arbitrary reference line

Kinetic energy: For an object of mass m moving with speed v: $K = (1/2)mv^2$

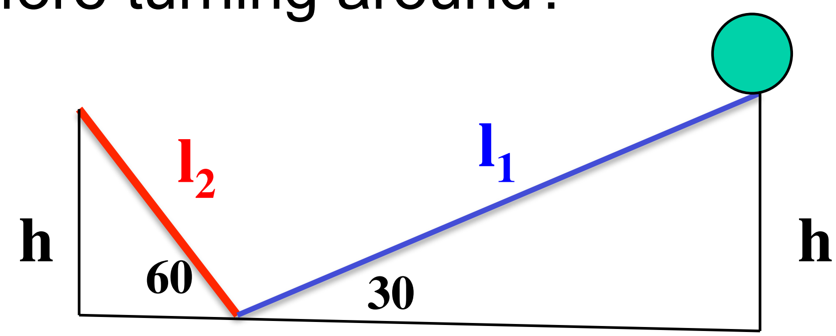
Total Energy is conserved: $E_i = E_f$

9-2.2 A ball follows the track shown in the figure, starting at A. If the velocity at B is v, then what is the velocity at C?



1. 2v
2. v
3. sqrt(2) v
4. v/sqrt(2)
5. None of the above

9-2.3: Starting from a height h, a ball rolls down a frictionless shallow ramp of length $l_1 = h/\sin(30)$ with an angle 30 degrees, and then up a steep ramp of height h with angle 60 degrees and length $l_2 = h/\sin(60)$. How far up the steep ramp does the ball go before turning around?



1. ½ l_2
2. l_1
3. $l_1 \sin(60)/\sin(30)$
4. l_2
5. None of the above

Collisions

If two objects collide and the net force exerted on the system (consisting of the two objects) is zero, the sum of their momenta is constant.

$$\vec{p}_{A,\text{initial}} + \vec{p}_{B,\text{initial}} = \vec{p}_{A,\text{final}} + \vec{p}_{B,\text{final}}$$

The sum of their kinetic energies may or may not be constant.

Elastic and inelastic collisions

- If K is conserved – collision is said to be *elastic*

e.g., cue balls on a pool table

$$K_{A,i} + K_{B,i} \qquad K_{A,f} + K_{B,f}$$

- Otherwise termed *inelastic*

  e.g., lump of putty thrown against wall

$$K_{A,i} + K_{B,i} \qquad K_{A,f} + K_{B,f}$$

- Extreme case = *completely inelastic* -- objects stick together after collision

9-2.4 Cart A moving to the right at speed v collides with an identical stationary cart (cart B) on a low-friction track. The collision is *elastic* (*i.e.,* there is no loss of kinetic energy of the system).

What is each cart's velocity after colliding (considering velocities to the right as positive)?

	Cart A	Cart B
1	$-v$	$2v$
2	$-{}^{1}/_{3}\,v$	${}^{4}/_{3}\,v$
3	0	v
4	${}^{1}/_{3}\,v$	${}^{2}/_{3}\,v$

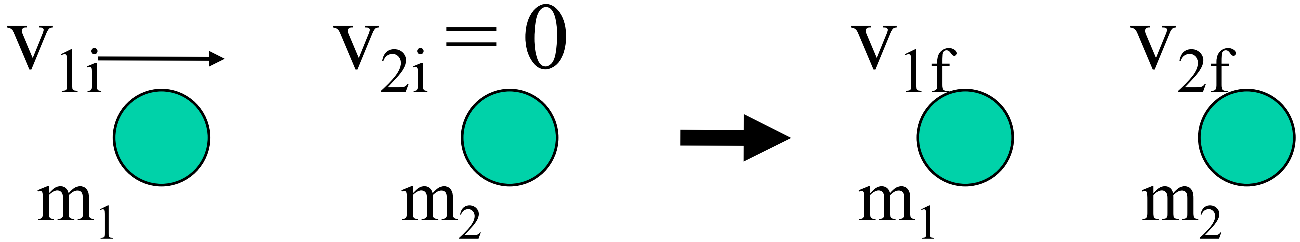
Check conservation of momentum and energy

	Cart A (m)	Cart B (m)	Final momentum	Final kin. energy
1	$-v$	$2v$		
2	$-{}^{1}/_{3}\,v$	${}^{4}/_{3}\,v$		
3	0	v		
4	${}^{1}/_{3}\,v$	${}^{2}/_{3}\,v$		

Demo

- Experiment –> zero sensors

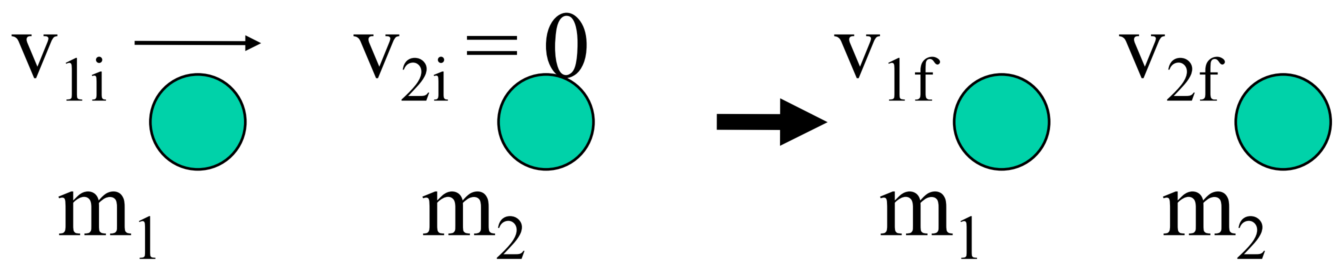
Elastic collision of two masses



Momentum → $m_1 v_{1i} + 0 = m_1 v_{1f} + m_2 v_{2f}$

Energy → $(1/2) m_1 v_{1i}^2 + 0 = (1/2) m_1 v_{1f}^2 + (1/2) m_2 v_{2f}^2$

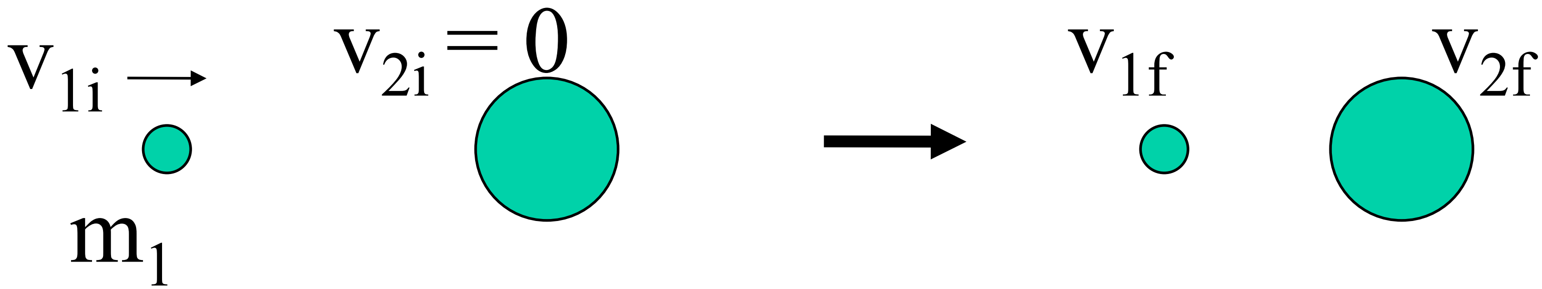
Special cases: (i) $m_1 = m_2$



$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i}$$

$$v_{2f} = \frac{2m_1}{m_1 + m_2} v_{1i}$$

Special cases: (ii) $m_1 << m_2$

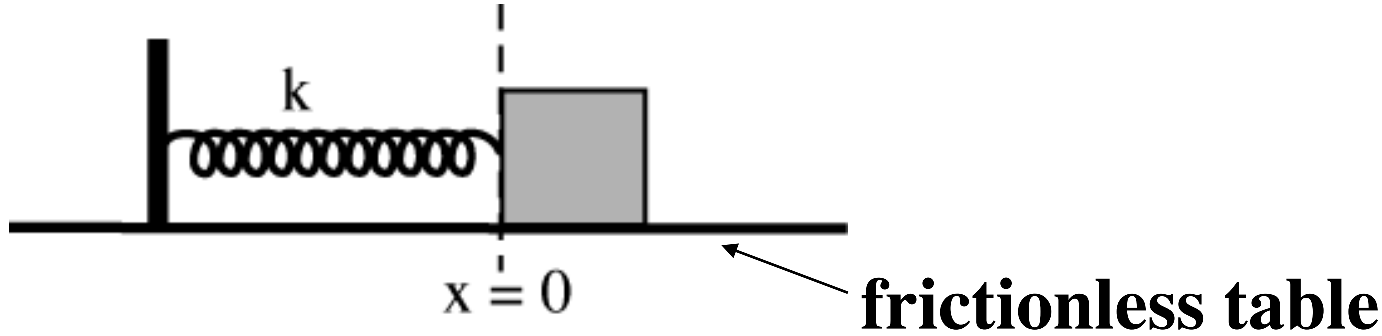


$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i}$$

$$v_{2f} = \frac{2m_1}{m_1 + m_2} v_{1i}$$

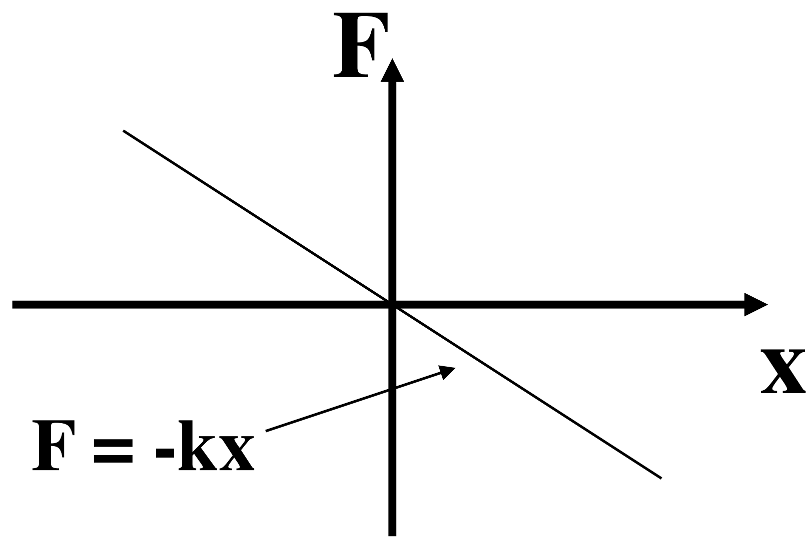
Sample problem 3: A 20 g ball of clay traveling east at 2.0 m/s collides with a 30 g ball of clay travelling 30 degrees south of west at 1.0 m/s. What are the speed and direction of the resulting 50g blob of clay?

Springs -- Elastic potential energy



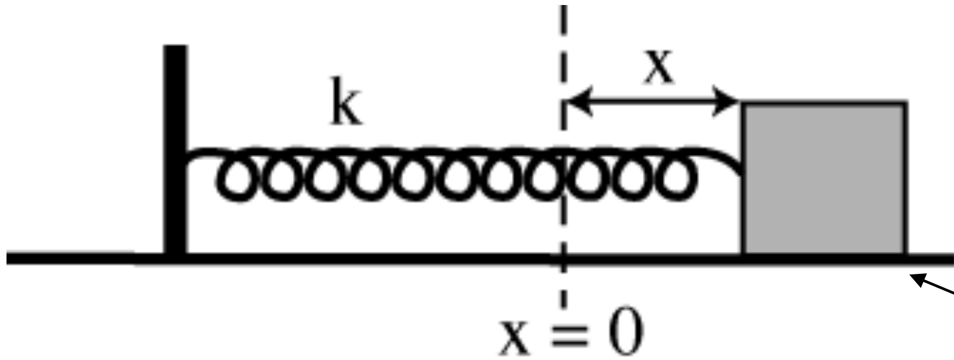
Force F = -kx (Hooke's law)

Area of triangle lying under straight line graph of F *vs*. x = (1/2)(+/-x)(-/+kx)



$$\mathbf{U_s = (1/2)kx^2}$$

(Horizontal) Spring



frictionless table

- x = displacement from relaxed state of spring

- Elastic potential energy stored in spring: $\mathbf{U_s = (1/2)kx^2}$

$$\mathbf{(1/2)kx^2 + (1/2)mv^2 = constant}$$

9.2-5 A 0.5 kg mass is attached to a spring on a horizontal frictionless table. The mass is pulled to stretch the spring 5.0 cm and is released from rest. When the mass crosses the point at which the spring is not stretched, x = 0, its speed is 20 cm/s. If the experiment is repeated with a 10.0 cm initial stretch, what speed will the mass have when it crosses x = 0 ?

1. 40 cm/s
2. 0 cm/s
3. 20 cm/s
4. 10 cm/s

Work, Energy

- Newton's Laws are ***vector*** equations

- Sometimes easier to relate speed of a particle to how far it moves under a force – a single equation can be used – need to introduce concept of ***work***

What is work?

- Assume ***constant*** force in 1D
- Consider:

$$v_F^2 = v_I^2 + 2a\ \Delta x$$

- Multiply by m/2 →

$$(1/2)mv_F^2 - (1/2)mv_I^2 = ma\Delta x$$

- But: F = ma

$$\rightarrow (1/2)mv_F^2 - (1/2)mv_I^2 = F\Delta x$$

Work-Kinetic Energy theorem (1)

$$(1/2)mv_F^2 - (1/2)mv_I^2 = Fs$$

Points: $s = \Delta x$ = displacement (for 2D)

- W = Fs → defines **work done** on particle

  = force times displacement

- K = (1/2)mv^2 → defines **kinetic energy**

  =1/2 mass times square of v

Improved definition of work

- For forces, write F → F_{AB}
- Thus W = Fs → $W_{AB} = F_{AB}\,\Delta s_A$ is **work done on A by B as A undergoes displacement Δs_A**
- Work-kinetic energy theorem:

$$W_{net,A} = \Sigma_B W_{AB} = \Delta K$$

The Work - Kinetic Energy Theorem

$$\boldsymbol{W_{\mathrm{net}} = \Delta K = K_{\mathrm{f}} - K_{\mathrm{i}}}$$

The *net work* done on an object is equal to the *change in kinetic energy* of the object.

9.2-6 Suppose a tennis ball and a bowling ball are rolling toward you. The tennis ball is moving much faster, but both have the *same momentum (mv),* and you exert the same force to stop each.

Which of the following statements is correct?

1. It takes equal distances to stop each ball.
2. It takes equal time intervals to stop each ball.
3. Both of the above.
4. Neither of the above.

9.2-7 Suppose a tennis ball and a bowling ball are rolling toward you. Both have the *same momentum (mv),* and you exert the same force to stop each.

It takes equal time intervals to stop each ball.

The distance taken for the bowling ball to stop is

1. less than.
2. equal to
3. greater than

the distance taken for the tennis ball to stop.

9.2-8 Two carts of different mass are accelerated from rest on a low-friction track by the same force for the same time interval.

Cart B has greater mass than cart A ($m_B > m_A$). The final speed of cart A is greater than that of cart B ($v_A > v_B$).

After the force has stopped acting on the carts, the kinetic energy of cart B is

1. less than the kinetic energy of cart A ($K_B < K_A$).
2. equal to the kinetic energy of cart A ($K_B = K_A$).
3. greater than the kinetic energy of cart A ($K_B > K_A$).
4. "Can't tell."