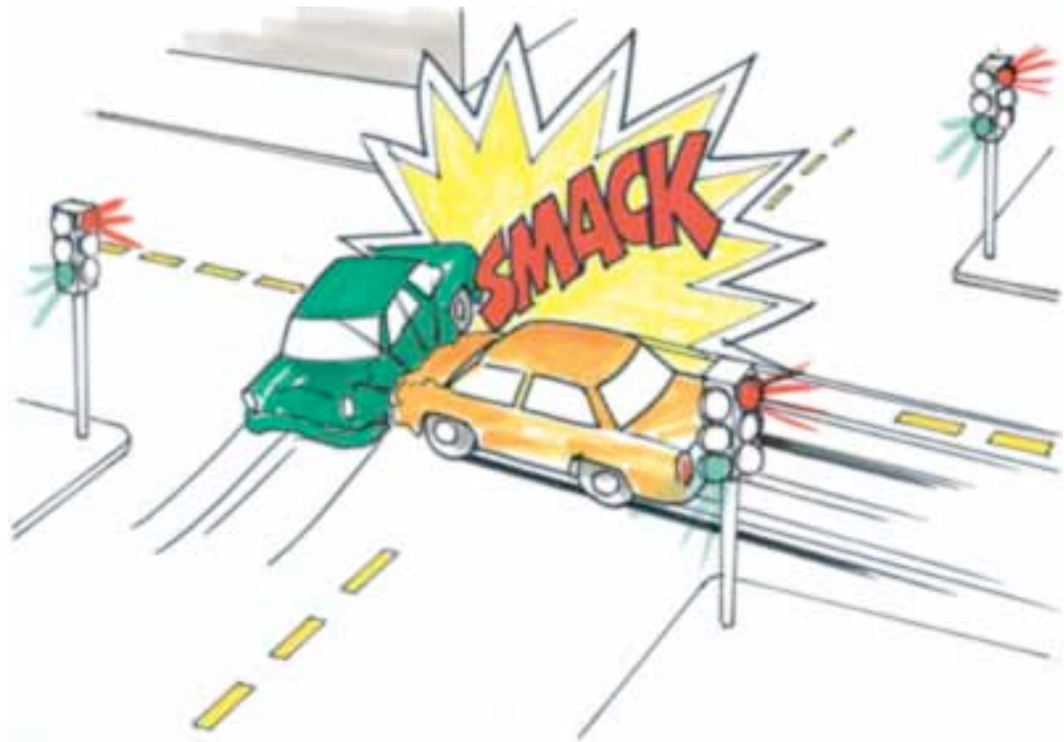
Welcome back to Physics 211

Today's agenda:

- *Impulse and momentum*
- *Energy*



Current assignments

- Reading: Finish Chapter 10 in textbook
  - Prelecture due Thursday
- HW#9 due this Friday at 5 pm.

Impulse demo

- Cart equipped with force probe collides with rubber tube
- Measure force *vs.* time and momentum *vs.* time
- Find that integral of force curve is precisely the change in p!

Sample Problem: At the intersection of Texas Avenue and University Drive, a blue, subcompact car with mass 950 kg traveling east on University collides with a maroon pickup truck with mass 1900 kg that is traveling north on Texas and ran a red light. The two vehicles stick together as a result of the collision and, after the collision, the wreckage is sliding at 16.0 m/s in the direction 24^o east of north. Calculate the speed of each vehicle before the collision. The collision occurs during a heavy rainstorm; you can ignore friction forces between the vehicles and the wet road.

Kinetic Energy

- Newton's Laws are *vector* equations
- Sometimes more appropriate to consider *scalar* quantities related to speed and mass

For an object of mass m moving with speed v:

$$K = (1/2)mv^2$$

- *Energy* of motion
- scalar!
- Measured in Joules -- J

Gravitational Potential Energy

For an object of mass m near the surface of the earth:

$$U_g = mgh$$

- h is height above arbitrary reference line
- Measured in Joules -- J (like kinetic energy)

Total energy for object moving under gravity

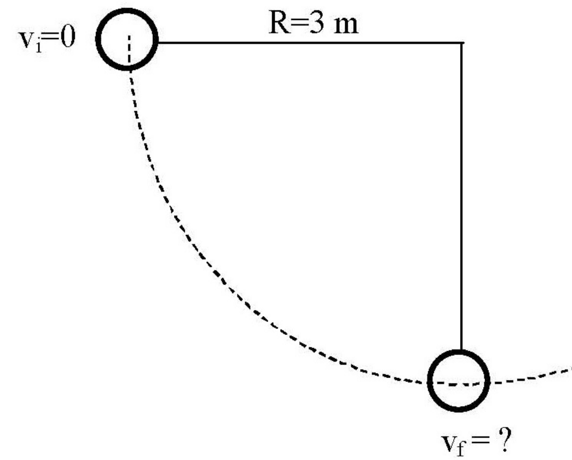
$E = U_g + K = \text{constant}$

* E is called the (mechanical) energy

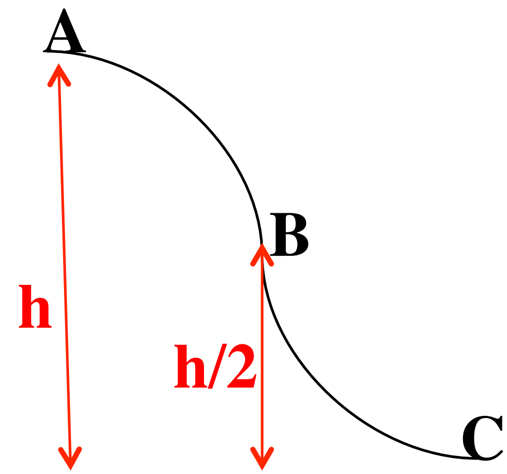
* It is conserved:

$$(1/2)\, mv^2 + mgh = \text{constant}$$

Sample problem 2: A ball of mass m=7 kg attached to a massless string of length R=3 m is released from the position shown in the figure below. (a) Find the magnitude of the velocity of the ball at the lowest point on its path. (b) Find the tension in the string at that point.

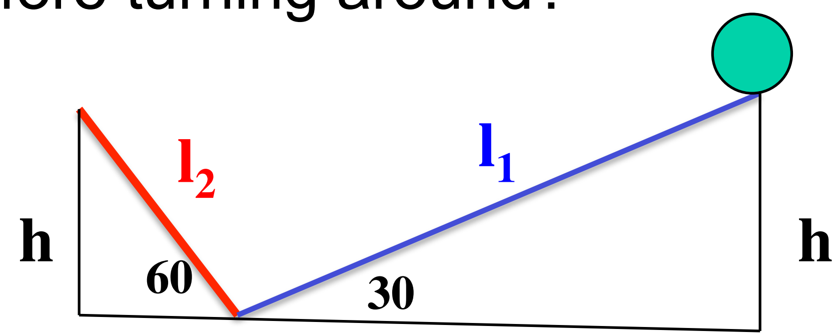


10-1.1 A ball follows the track shown in the figure, starting at A. If the velocity at B is v, then what is the velocity at C?



1. 2v
2. v
3. sqrt(2) v
4. v/sqrt(2)
5. None of the above

10-1.2: Starting from a height h, a ball rolls down a frictionless shallow ramp of length $l_1 = h/\sin(30)$ with an angle 30 degrees, and then up a steep ramp of height h with angle 60 degrees and length $l_2 = h/\sin(60)$. How far up the steep ramp does the ball go before turning around?



1. $\frac{1}{2} l_2$
2. l_1
3. $l_1 \sin(60)/\sin(30)$
4. l_2
5. None of the above

Collisions

If two objects collide and the net force exerted on the system (consisting of the two objects) is zero, the sum of their momenta is constant.

$$\vec{p}_{A,\mathrm{initial}} + \vec{p}_{B,\mathrm{initial}} = \vec{p}_{A,\mathrm{final}} + \vec{p}_{B,\mathrm{final}}$$

The sum of their kinetic energies may or may not be constant.

Elastic and inelastic collisions

- If K is conserved – collision is said to be *elastic*

e.g., cue balls on a pool table

$$K_{A,i} + K_{B,i} \qquad K_{A,f} + K_{B,f}$$

- Otherwise termed *inelastic*

  e.g., lump of putty thrown against wall

$$K_{A,i} + K_{B,i} \qquad K_{A,f} + K_{B,f}$$

- Extreme case = *completely inelastic* -- objects stick together after collision

10-1.3 Cart A moving to the right at speed *v* collides with an identical stationary cart (cart B) on a low-friction track. The collision is *elastic* (*i.e.,* there is no loss of kinetic energy of the system).

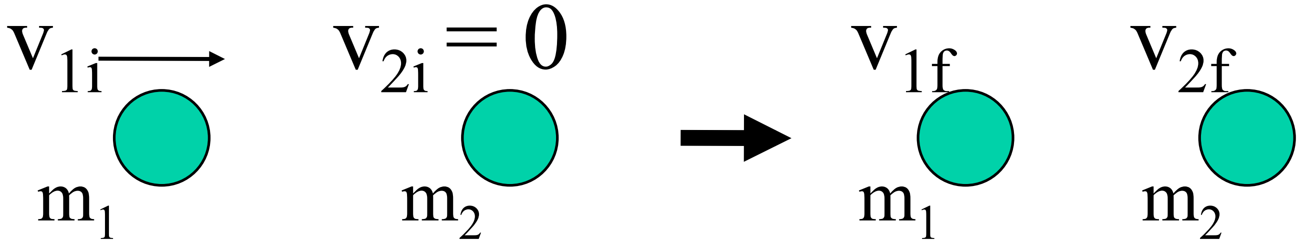
What is each cart's velocity after colliding (considering velocities to the right as positive)?

	Cart A	**Cart B**
1	$-v$	$2v$
2	$-{}^{1}/_{3}\,v$	${}^{4}/_{3}\,v$
3	0	v
4	${}^{1}/_{3}\,v$	${}^{2}/_{3}\,v$

Check conservation of momentum and energy

	Cart A (*m*)	Cart B (*m*)	Final momentum	Final kin. energy
1	$-v$	$2v$		
2	$-\,{}^{1}/_{3}\,v$	${}^{4}/_{3}\,v$		
3	0	v		
4	${}^{1}/_{3}\,v$	${}^{2}/_{3}\,v$		

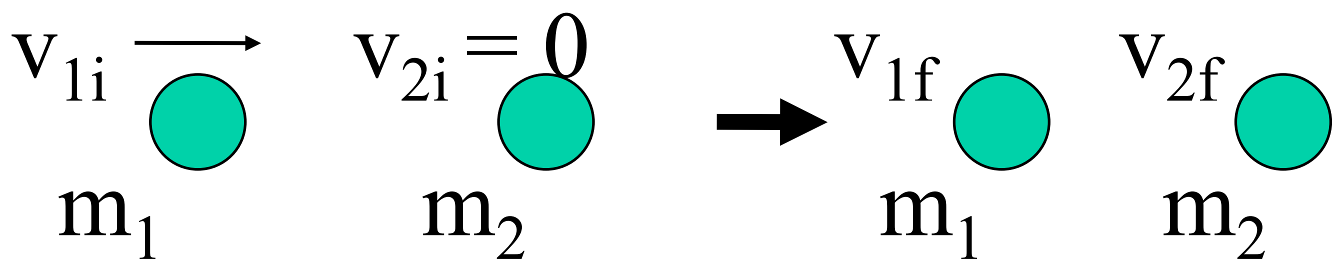
Elastic collision of two masses



Momentum → $m_1 v_{1i} + 0 = m_1 v_{1f} + m_2 v_{2f}$

Energy → $(1/2)m_1 v_{1i}^2 + 0 = (1/2)m_1 v_{1f}^2 + (1/2)m_2 v_{2f}^2$

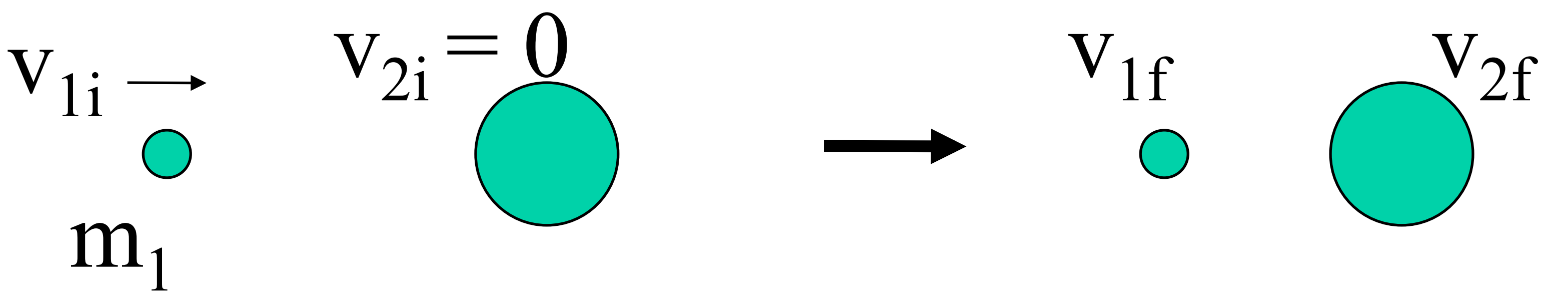
Special cases: (i) $m_1 = m_2$



$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i}$$

$$v_{2f} = \frac{2m_1}{m_1 + m_2} v_{1i}$$

Special cases: (ii) $m_1 << m_2$

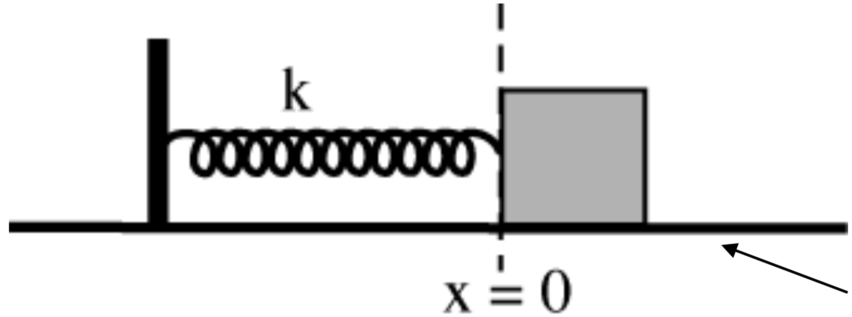


$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i}$$

$$v_{2f} = \frac{2m_1}{m_1 + m_2} v_{1i}$$

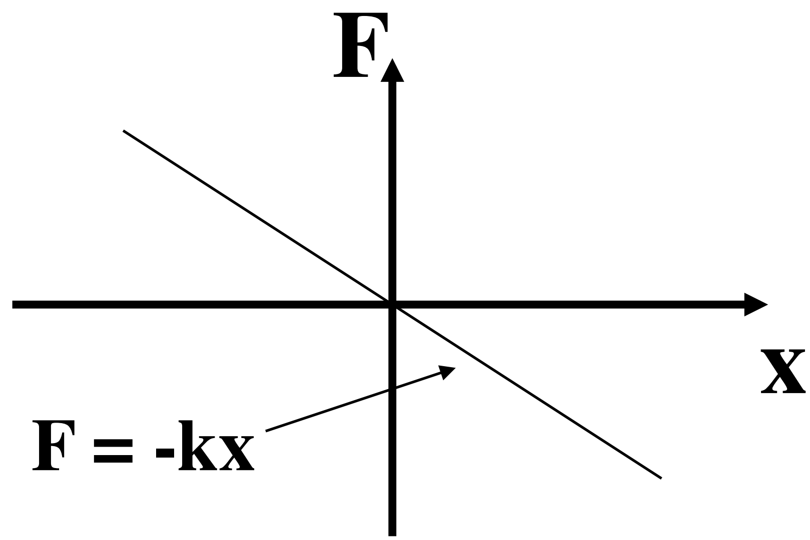
Sample problem 3: A 20 g ball of clay traveling east at 2.0 m/s collides with a 30 g ball of clay travelling 30 degrees south of west at 1.0 m/s. What are the speed and direction of the resulting 50g blob of clay?

Springs -- Elastic potential energy



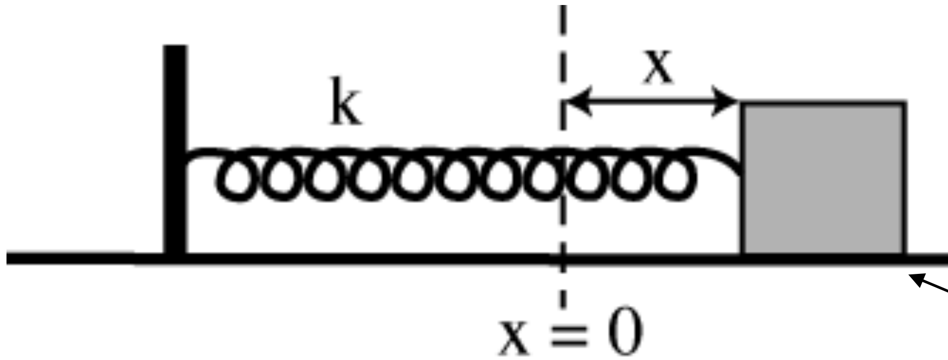
Force F = -kx (Hooke's law)

Area of triangle lying under straight line graph of F *vs*. x = (1/2)(+/-x)(-/+kx)



$$\mathbf{U_s = (1/2)kx^2}$$

(Horizontal) Spring



frictionless table

- x = displacement from relaxed state of spring

- Elastic potential energy stored in spring: $\mathbf{U_s = (1/2)kx^2}$

$$\mathbf{(1/2)kx^2 + (1/2)mv^2 = constant}$$

10.1-4 A 0.5 kg mass is attached to a spring on a horizontal frictionless table. The mass is pulled to stretch the spring 5.0 cm and is released from rest. When the mass crosses the point at which the spring is not stretched, x = 0, its speed is 20 cm/s. If the experiment is repeated with a 10.0 cm initial stretch, what speed will the mass have when it crosses x = 0 ?

1. 40 cm/s
2. 0 cm/s
3. 20 cm/s
4. 10 cm/s

Work, Energy

- Newton's Laws are ***vector*** equations

- Sometimes easier to relate speed of a particle to how far it moves under a force – a single equation can be used – need to introduce concept of ***work***

What is work?

- Assume ***constant*** force in 1D
- Consider:

$$v_F^2 = v_I^2 + 2a\ \Delta x$$

- Multiply by m/2 →

$$(1/2)mv_F^2 - (1/2)mv_I^2 = ma\Delta x$$

- But: F = ma

$$\rightarrow (1/2)mv_F^2 - (1/2)mv_I^2 = F\Delta x$$

Work-Kinetic Energy theorem (1)

$$(1/2)mv_F^2 - (1/2)mv_I^2 = Fs$$

Points: $s = \Delta x$ = displacement (for 2D)

- W = Fs → defines **work done** on particle

  = force times displacement

- K = (1/2)mv² → defines **kinetic energy**

  =1/2 mass times square of v

Improved definition of work

- For forces, write $F \rightarrow F_{AB}$
- Thus $W = Fs \rightarrow W_{AB} = F_{AB}\,\Delta s_A$ is **work done on A by B as A undergoes displacement** Δs_A
- Work-kinetic energy theorem:

$$W_{net,A} = \Sigma_B W_{AB} = \Delta K$$

The Work - Kinetic Energy Theorem

$$\boldsymbol{W_{net} = \Delta K = K_f - K_i}$$

The *net work* done on an object is equal to the *change in kinetic energy* of the object.

10.1-5 Suppose a tennis ball and a bowling ball are rolling toward you. The tennis ball is moving much faster, but both have the *same momentum (mv),* and you exert the same force to stop each.

Which of the following statements is correct?

1. It takes equal distances to stop each ball.
2. It takes equal time intervals to stop each ball.
3. Both of the above.
4. Neither of the above.

10.1-6 Suppose a tennis ball and a bowling ball are rolling toward you. Both have the *same momentum (mv),* and you exert the same force to stop each.

It takes equal time intervals to stop each ball.

The distance taken for the bowling ball to stop is

1. less than
2. equal to
3. greater than

the distance taken for the tennis ball to stop.

10.1-7 Two carts of different mass are accelerated from rest on a low-friction track by the same force for the same time interval.

Cart B has greater mass than cart A ($m_B > m_A$). The final speed of cart A is greater than that of cart B ($v_A > v_B$).

After the force has stopped acting on the carts, the kinetic energy of cart B is

1. less than the kinetic energy of cart A ($K_B < K_A$).
2. equal to the kinetic energy of cart A ($K_B = K_A$).
3. greater than the kinetic energy of cart A ($K_B > K_A$).
4. "Can't tell."

Work and Kinetic Energy in 2D

Work done on object 1 by object 2:

$$W_{(\text{on 1 by 2})} = \vec{F}_{1,2} \bullet \Delta\vec{s}_{\text{of 1}}$$

Kinetic energy of an object:

$$K = \tfrac{1}{2} m \mathbf{v}^2 \quad [\text{or:} \quad \tfrac{1}{2} m(\vec{\mathbf{v}} \bullet \vec{\mathbf{v}})]$$

Scalar (or "dot") product of vectors

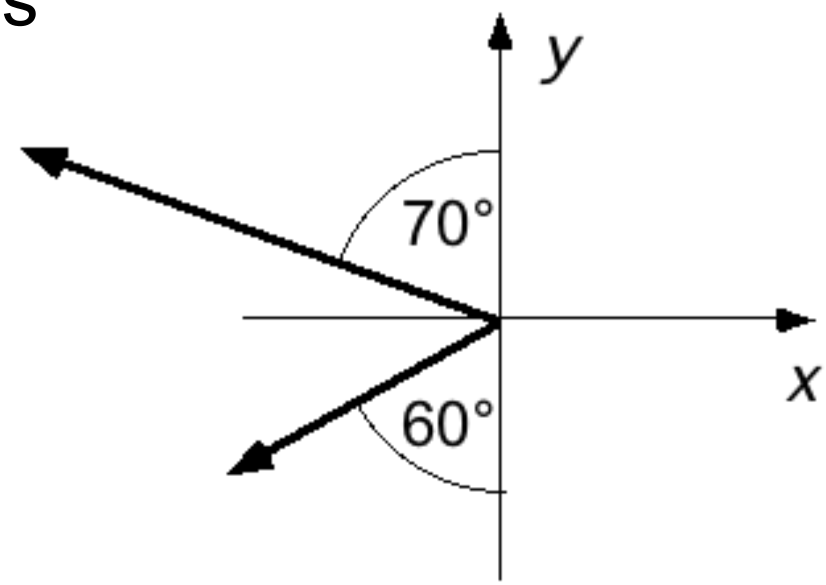
The scalar product is a way to combine two vectors to obtain a number (or *scalar*). It is indicated by a dot (•) between the two vectors.

(Note: component of A in direction n is just A•n)

$$\vec{A} \bullet \vec{B} = AB\cos\theta$$

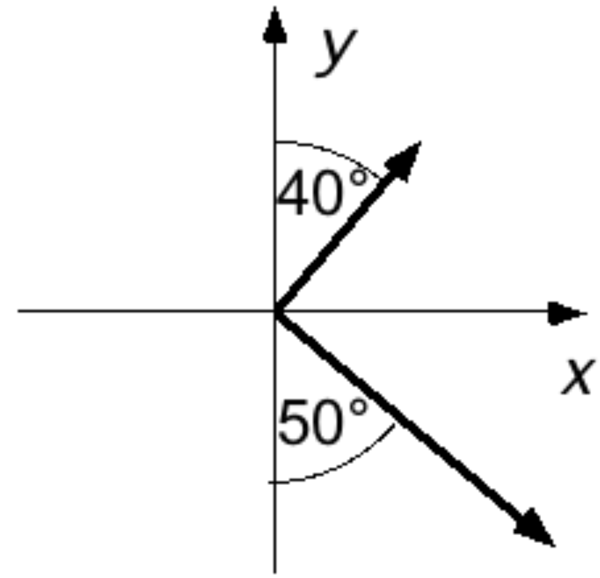
$$= A_x B_x + A_y B_y + A_z B_z$$

10-1.8 Is the scalar (“dot”) product of the two vectors



1. positive
2. negative
3. equal to zero
4. “Can’t tell.”

10-1.9 Is the scalar (“dot”) product of the two vectors

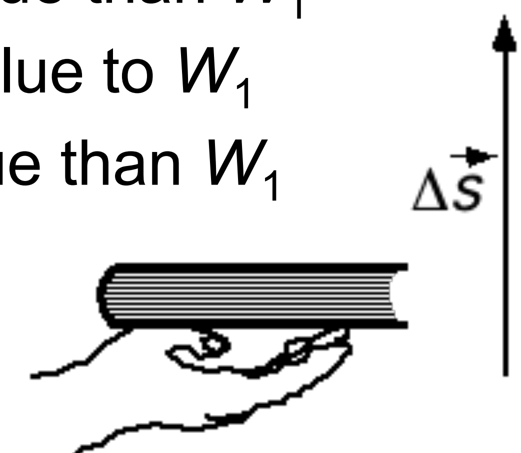


1. positive
2. negative
3. equal to zero
4. “Can’t tell.”

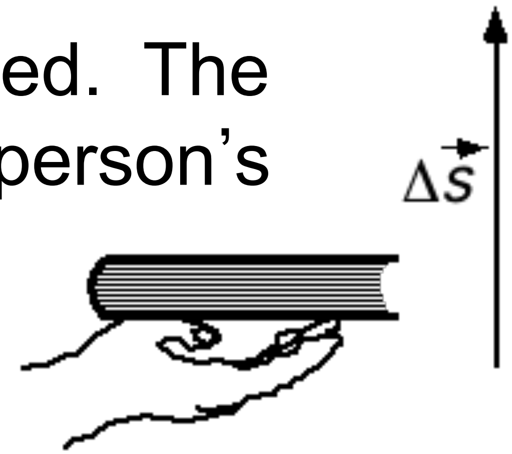
10-1.10 A person lifts a book at constant speed. Since the force exerted on the book by the person's hand is in the same direction as the displacement of the book, the work (W_1) done on the book by the person's hand is positive.

The work done on the book by the earth is:

1. negative and equal in absolute value to W_1
2. negative and less in absolute value than W_1
3. positive and equal in absolute value to W_1
4. positive and less in absolute value than W_1



A person lifts a book at constant speed. The work (W_1) done on the book by the person's hand is positive.



Work done on the book by the earth:

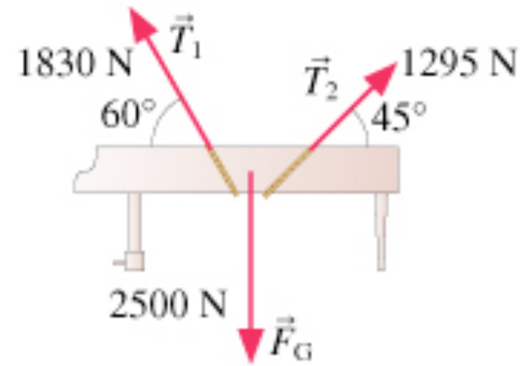
Net work done on the book:

Change in kinetic energy of the book:

10-1.11 A person carries a book horizontally at constant speed. The work done on the book by the person's hand is

1. positive
2. negative
3. equal to zero
4. "Can't tell."

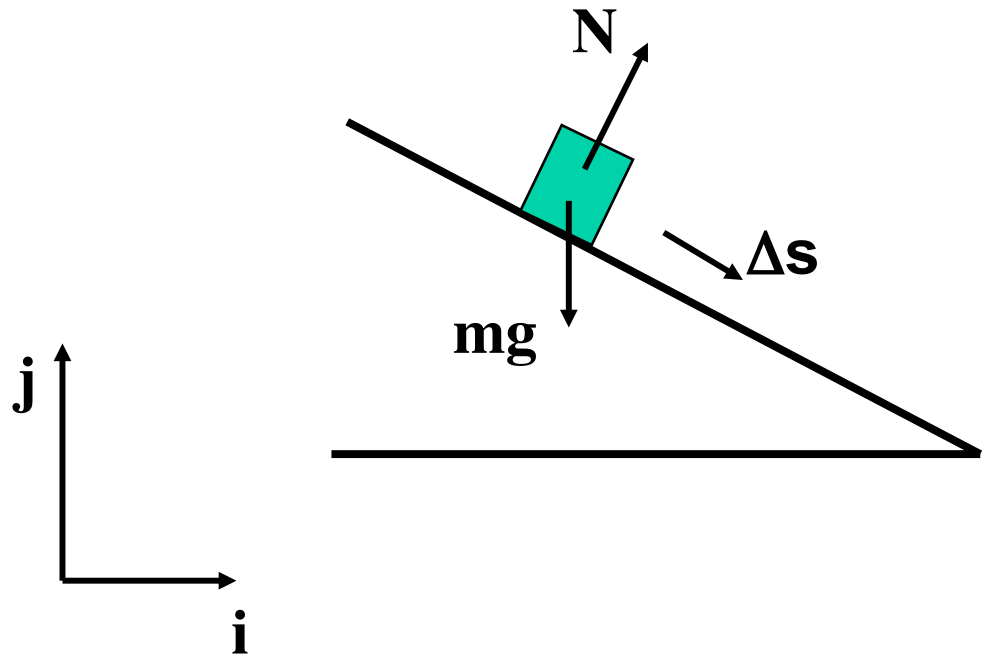
Sample problem: The two ropes seen in the figure are used to lower a 255 kg piano 5.1 m from a second-story window to the ground. How much work is done by each of the three forces?



Solution

- Which forces do work on block?
- Which, if any, are constant?
- What is **F•Δs** for motion?

Work done by gravity



Work $W = -mg\ \mathbf{j}\bullet\Delta s$

Therefore,

$$W = -mg\Delta h$$

N does no work!

Work done on an object by gravity

$$W_{(on \text{ object } by \text{ earth})} = -\, m\, g\, \Delta h,$$

where $\Delta h = h_{\text{final}} - h_{\text{initial}}$ is the change in height.

Object moves	Change in height Δh	Work done on object by earth
upward	positive	negative
downward	negative	positive

Defining gravitational potential energy

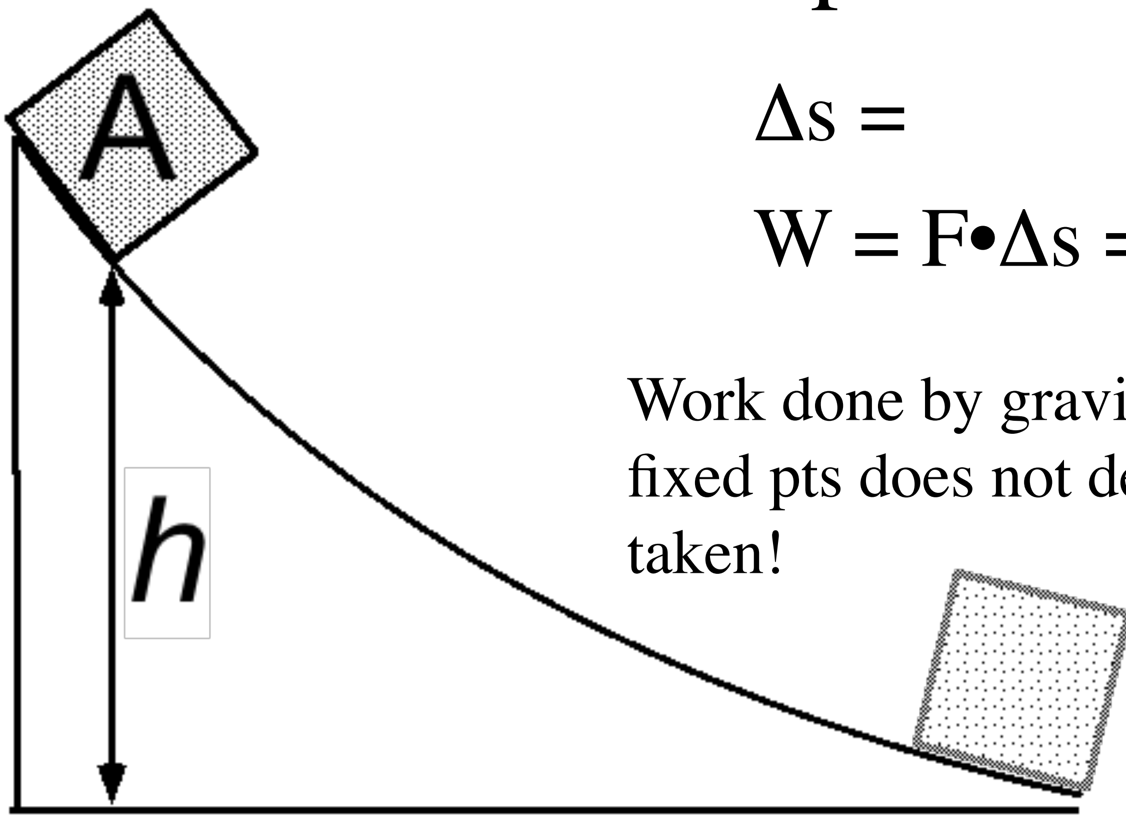
$$W_{\text{(on obj. by earth)}} = \Delta K$$

$$0 = \Delta K - W_{\text{(on obj. by earth)}}$$

$$0 = \Delta K + \Delta U_{\text{g}}$$

The *change* in gravitational potential energy of the object -earth system is just another name for the negative value of the work done on an object by the earth.

Curved ramp



$\Delta s =$

$W = F \bullet \Delta s =$

Work done by gravity between 2 fixed pts does not depend on path taken!

Work done by gravitational force in moving some object along any path is *independent* of the path depending *only* on the change in vertical height