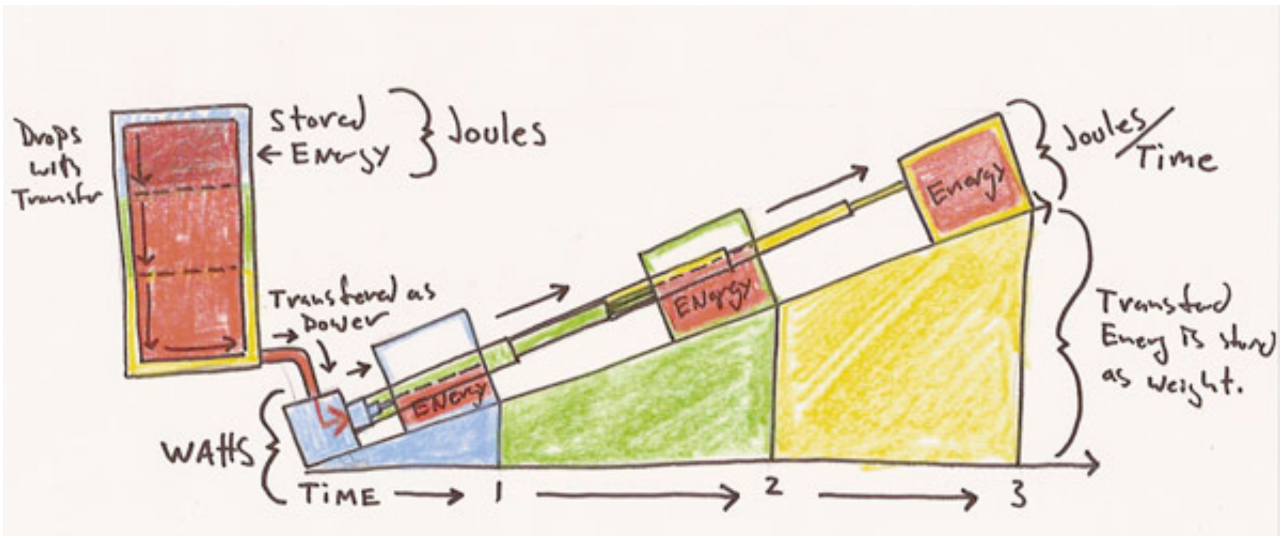
Welcome back to Physics 211

Today's agenda:

Energy



Current assignments

- HW#9 due this Friday at 5 pm.

Prelecture assignment due next Tuesday

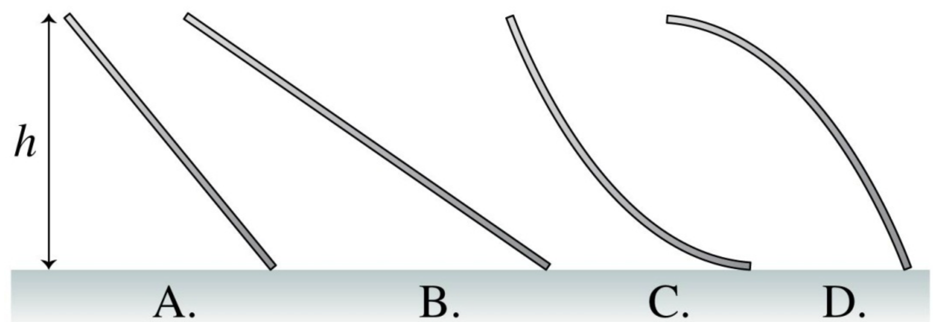
10-2.1: A small child slides down the four frictionless slides A–D. Rank in order, from largest to smallest, her speeds at the bottom.

A. $v_D > v_A > v_B > v_C$

B. $v_D > v_A = v_B > v_C$

C. $v_C > v_A > v_B > v_D$

D. $v_A = v_B = v_C = v_D$



Collisions

If two objects collide and the net force exerted on the system (consisting of the two objects) is zero, the sum of their momenta is constant.

$$\vec{p}_{A,\text{initial}} + \vec{p}_{B,\text{initial}} = \vec{p}_{A,\text{final}} + \vec{p}_{B,\text{final}}$$

The sum of their kinetic energies may or may not be constant.

Elastic and inelastic collisions

- If K is conserved – collision is said to be *elastic*

e.g., cue balls on a pool table

$$K_{A,i} + K_{B,i} \qquad K_{A,f} + K_{B,f}$$

- Otherwise termed *inelastic*

  e.g., lump of putty thrown against wall

$$K_{A,i} + K_{B,i} \qquad K_{A,f} + K_{B,f}$$

- Extreme case = *completely inelastic* -- objects stick together after collision

10-2.2 Cart A moving to the right at speed *v* collides with an identical stationary cart (cart B) on a low-friction track. The collision is *elastic* (*i.e.*, there is no loss of kinetic energy of the system).

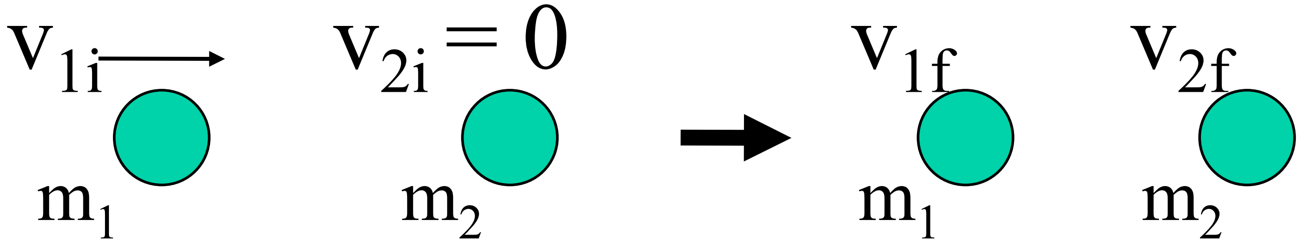
What is each cart's velocity after colliding (considering velocities to the right as positive)?

	Cart A	**Cart B**
1	$-v$	$2v$
2	$-\frac{1}{3}v$	$\frac{4}{3}v$
3	0	v
4	$\frac{1}{3}v$	$\frac{2}{3}v$

Check conservation of momentum and energy

	Cart A ***(m)***	**Cart B** ***(m)***	**Final momentum**	**Final kin. energy**
1	$-v$	$2v$		
2	$-{}^{1}/_{3}\,v$	${}^{4}/_{3}\,v$		
3	0	v		
4	${}^{1}/_{3}\,v$	${}^{2}/_{3}\,v$		

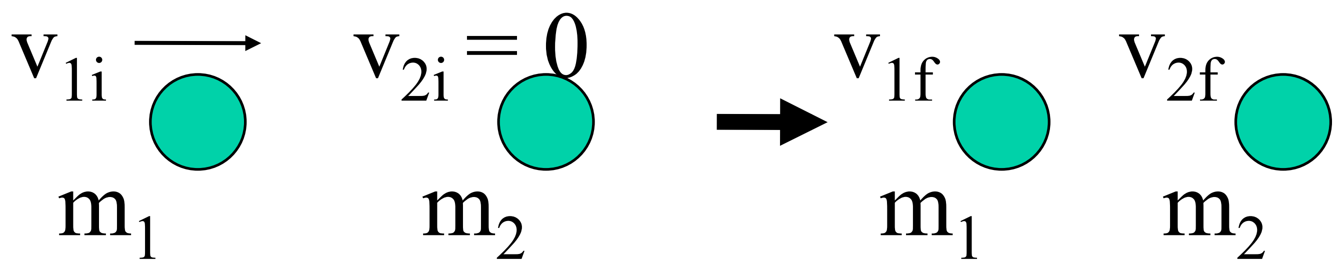
Elastic collision of two masses



Momentum → $m_1 v_{1i} + 0 = m_1 v_{1f} + m_2 v_{2f}$

Energy → $(1/2)m_1 v_{1i}^2 + 0 = (1/2)m_1 v_{1f}^2 + (1/2)m_2 v_{2f}^2$

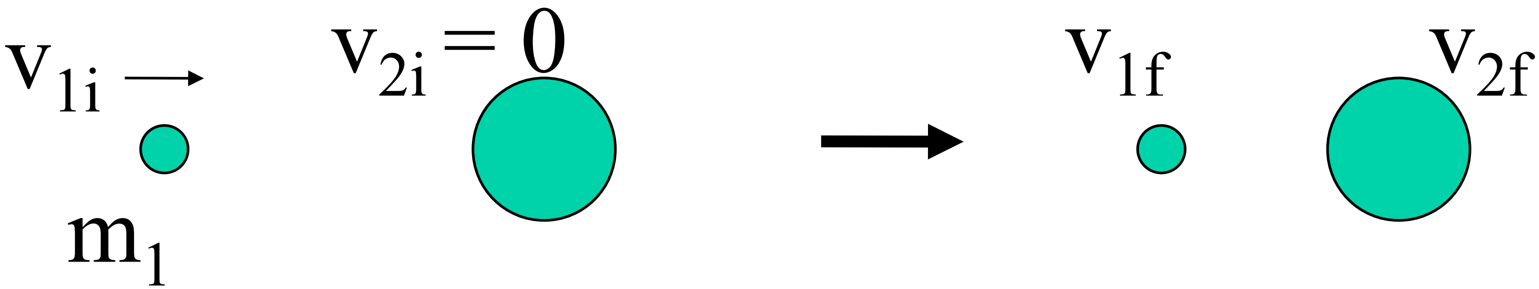
Special cases: (i) $m_1 = m_2$



$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i}$$

$$v_{2f} = \frac{2m_1}{m_1 + m_2} v_{1i}$$

Special cases: (ii) $m_1 << m_2$

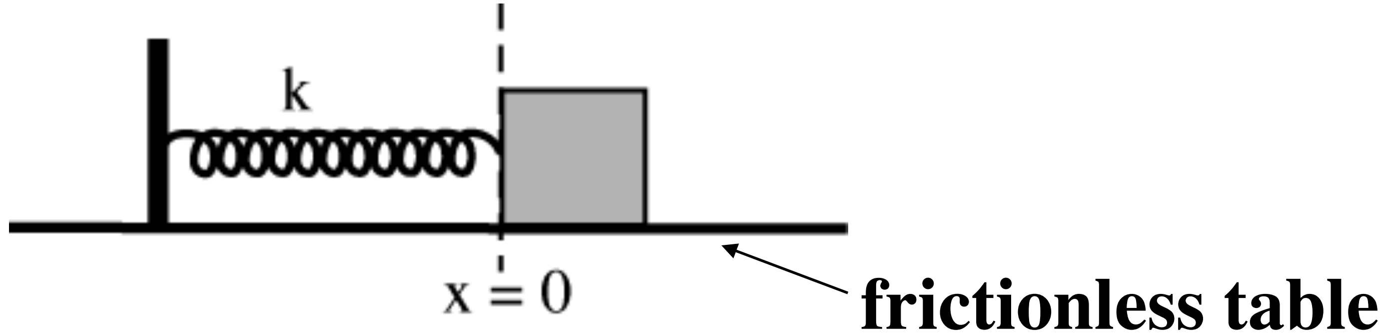


$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i}$$

$$v_{2f} = \frac{2m_1}{m_1 + m_2} v_{1i}$$

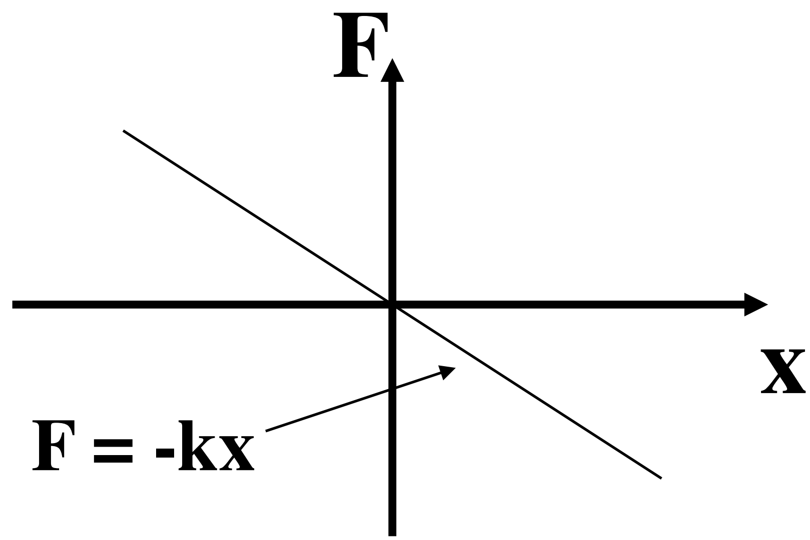
Sample problem 3: A 20 g ball of clay traveling east at 2.0 m/s collides with a 30 g ball of clay travelling 30 degrees south of west at 1.0 m/s. What are the speed and direction of the resulting 50g blob of clay?

Springs -- Elastic potential energy



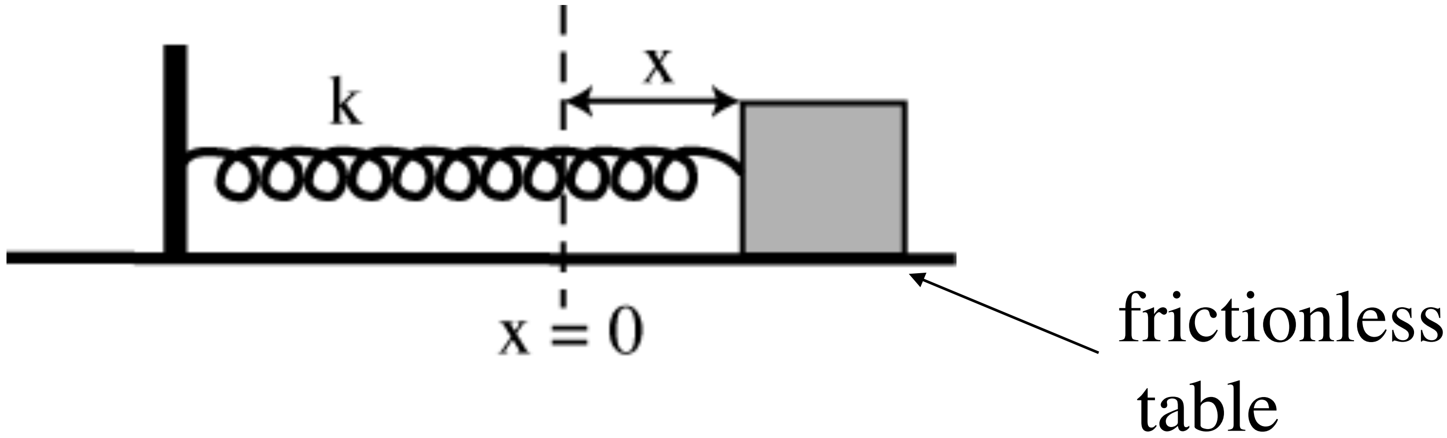
Force F = -kx (Hooke's law)

Area of triangle lying under straight line graph of F *vs*. x = (1/2)(+/-x)(-/+kx)



$$\mathbf{U_s = (1/2)kx^2}$$

(Horizontal) Spring



- x = displacement from relaxed state of spring

- Elastic potential energy stored in spring: $\mathbf{U_s = (1/2)kx^2}$

$$\mathbf{(1/2)kx^2 + (1/2)mv^2 = constant}$$

Work, Energy

- Newton's Laws are ***vector*** equations

- Sometimes easier to relate speed of a particle to how far it moves under a force – a single equation can be used – need to introduce concept of ***work***

What is work?

- Assume ***constant*** force in 1D

- Consider:

$$v_F^2 = v_I^2 + 2a\ \Delta x$$

- Multiply by m/2 →

$$(1/2)mv_F^2 - (1/2)mv_I^2 = ma\Delta x$$

- But: F = ma

$$\rightarrow (1/2)mv_F^2 - (1/2)mv_I^2 = F\Delta x$$

Work-Kinetic Energy theorem (1D)

$$(1/2)mv_F^2 - (1/2)mv_I^2 = Fs$$

- *Define s* = Δx = displacement
- W = Fs → defines **work done** on particle

  = force times displacement

- K = $(1/2)mv^2$ → defines **kinetic energy**

  =1/2 mass times square of v

Improved definition of work (many forces)

- For forces, write F → F_{AB}

- Thus $W = Fs \rightarrow W_{AB} = F_{AB}\,\Delta s_A$ is **work done on A by B as A undergoes displacement** Δs_A

- Work-kinetic energy theorem:

$$W_{net,A} = \Sigma_B W_{AB} = \Delta K$$

The Work - Kinetic Energy Theorem

$$\boldsymbol{W_{\mathrm{net}} = \Delta K = K_{\mathrm{f}} - K_{\mathrm{i}}}$$

The *net work* done on an object is equal to the *change in kinetic energy* of the object.

10.2-3 Suppose a tennis ball and a bowling ball are rolling toward you. The tennis ball is moving much faster, but both have the *same momentum (mv),* and you exert the same force to stop each.

Which of the following statements is correct?

1. It takes equal distances to stop each ball.
2. It takes equal time intervals to stop each ball.
3. Both of the above.
4. Neither of the above.

10-2.4 Suppose a tennis ball and a bowling ball are rolling toward you. Both have the *same momentum (mv),* and you exert the same force to stop each.

It takes equal time intervals to stop each ball.

The distance taken for the bowling ball to stop is

1. less than.
2. equal to
3. greater than

the distance taken for the tennis ball to stop.

Work and Kinetic Energy in 2D

Work done on object 1 by object 2:

$$W_{(\text{on 1 by 2})} = \vec{F}_{1,2} \bullet \Delta\vec{s}_{\text{of 1}}$$

Kinetic energy of an object:

$$K = \tfrac{1}{2} m v^2 \quad [\text{or:} \quad \tfrac{1}{2} m(\vec{v} \bullet \vec{v})]$$

Scalar (or “dot”) product of vectors

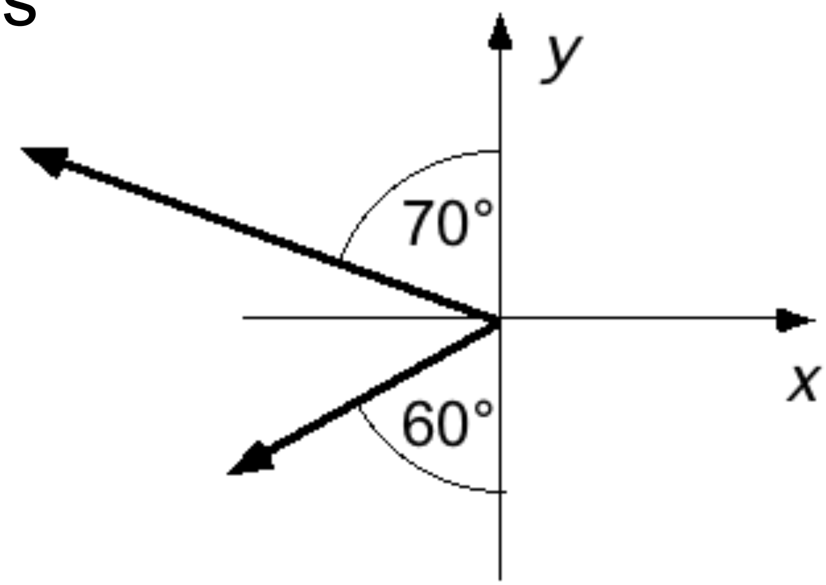
The scalar product is a way to combine two vectors to obtain a number (or *scalar*). It is indicated by a dot (•) between the two vectors.

(Note: component of A in direction n is just A•n)

$$\vec{A} \bullet \vec{B} = AB\cos\theta$$

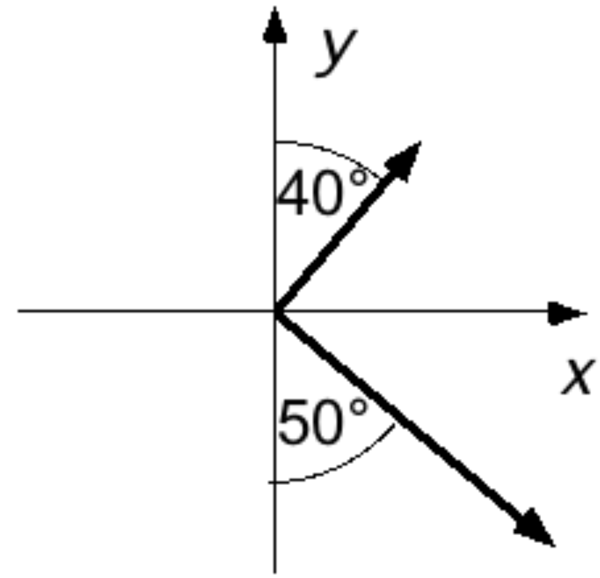
$$= A_x B_x + A_y B_y + A_z B_z$$

10-2.3 Is the scalar ("dot") product of the two vectors



1. positive
2. negative
3. equal to zero
4. "Can't tell."

10-2.4 Is the scalar (“dot”) product of the two vectors

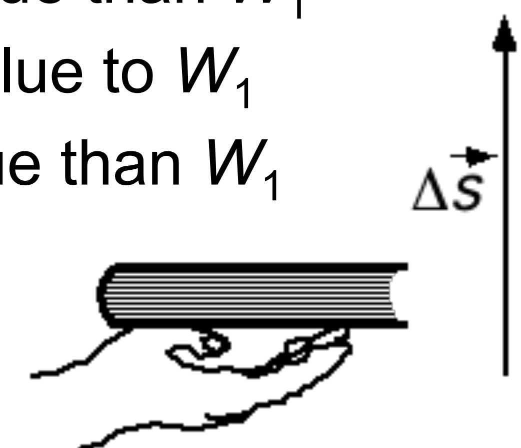


1. positive
2. negative
3. equal to zero
4. “Can’t tell.”

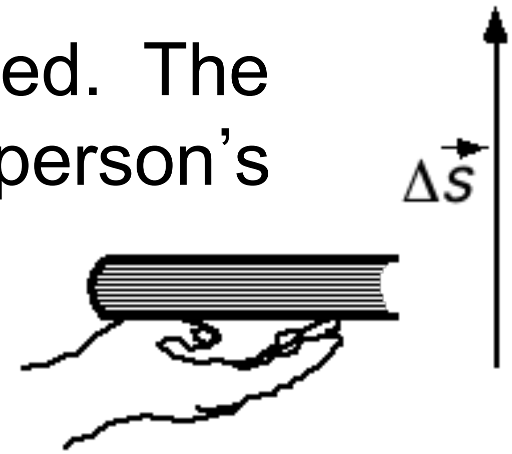
10-2.5 A person lifts a book at constant speed. Since the force exerted on the book by the person's hand is in the same direction as the displacement of the book, the work (W_1) done on the book by the person's hand is positive.

The work done on the book by the earth is:

1. negative and equal in absolute value to W_1
2. negative and less in absolute value than W_1
3. positive and equal in absolute value to W_1
4. positive and less in absolute value than W_1



A person lifts a book at constant speed. The work (W_1) done on the book by the person's hand is positive.



Work done on the book by the earth:

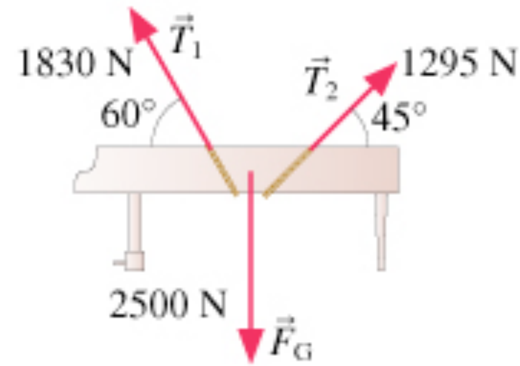
Net work done on the book:

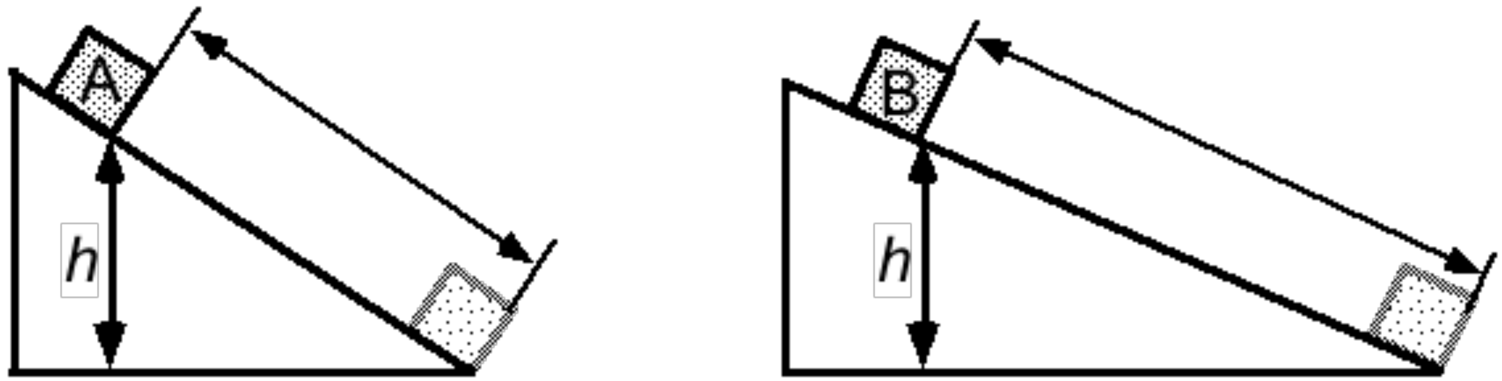
Change in kinetic energy of the book:

10-2.6 A person carries a book horizontally at constant speed. The work done on the book by the person's hand is

1. positive
2. negative
3. equal to zero
4. "Can't tell."

Sample problem: The two ropes seen in the figure are used to lower a 255 kg piano 5.1 m from a second-story window to the ground. How much work is done by each of the three forces?





9-2.7 Two identical blocks slide down two frictionless ramps. Both blocks start from the same height, but block A is on a steeper incline than block B.

The speed of block A at the bottom of its ramp is

1. less than the speed of block B.
2. equal to the speed of block B.
3. greater than the speed of block B.
4. "Can't tell."

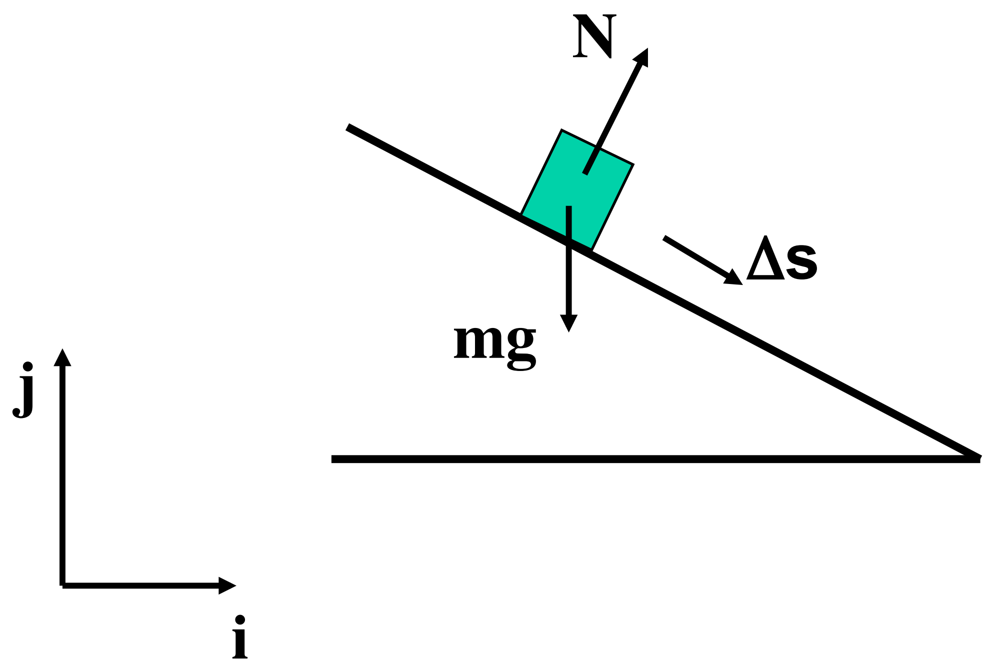
Solution

- Which forces do work on block?

- Which, if any, are constant?

- What is **F•Δs** for motion?

Work done by gravity



Work W = -mg j•Δs

Therefore,

$$W = -mg\Delta h$$

N does no work!

Work done on an object by gravity

$$W_{(\textit{on} \text{ object } \textit{by} \text{ earth})} = -\, m\, g\, \Delta h,$$

where $\Delta h = h_{\text{final}} - h_{\text{initial}}$ is the change in height.

Object moves	Change in height Δh	Work done on object by earth
upward	positive	negative
downward	negative	positive

Defining gravitational potential energy

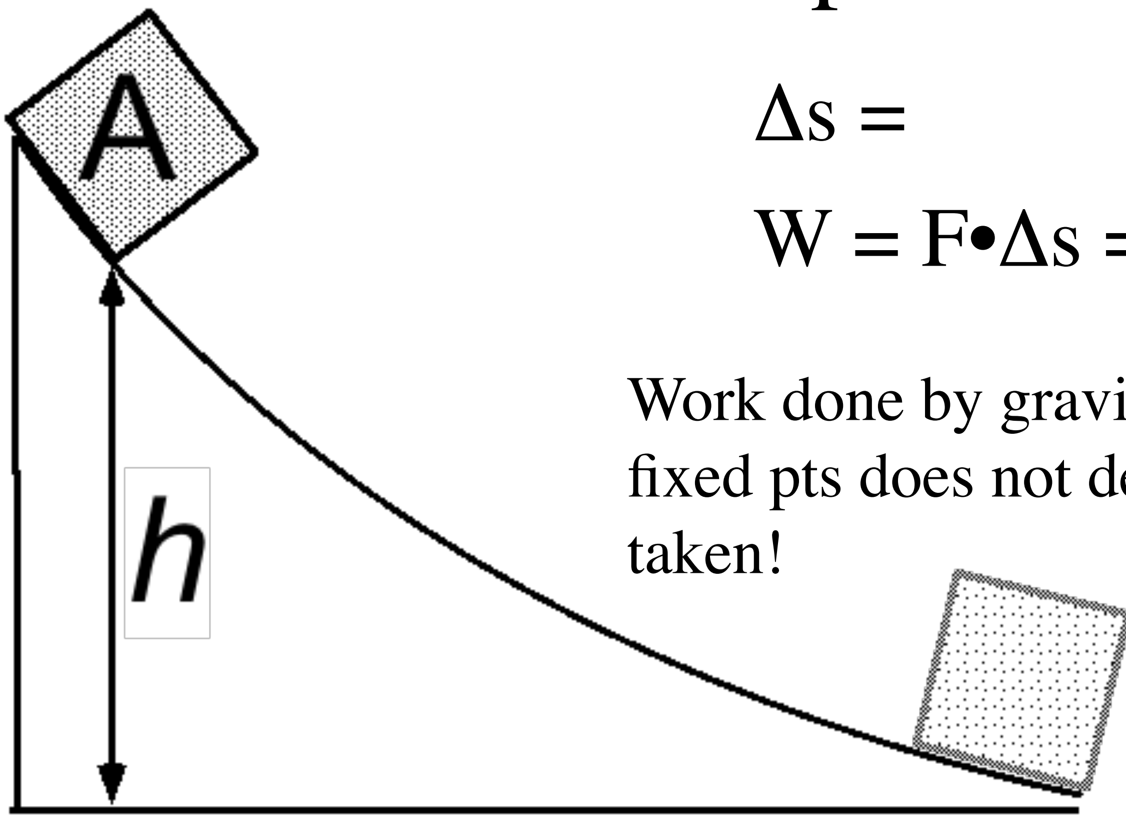
$$W_{\text{(on obj. by earth)}} = \Delta K$$

$$0 = \Delta K - W_{\text{(on obj. by earth)}}$$

$$0 = \Delta K + \Delta U_{\text{g}}$$

The *change* in gravitational potential energy of the object -earth system is just another name for the negative value of the work done on an object by the earth.

Curved ramp



$\Delta s =$

$W = F \bullet \Delta s =$

Work done by gravity between 2 fixed pts does not depend on path taken!

Work done by gravitational force in moving some object along any path is *independent* of the path depending *only* on the change in vertical height

Conservative forces

- If the work done by some force (e.g. gravity) does **not depend on path** the force is called **conservative**.

- Then gravitational potential energy U_g only depends on (vertical) position of object $U_g = U_g(h)$

- Elastic forces also conservative – elastic potential energy $U = (1/2)kx^2$...

Many forces

- For a particle which is subject to several (conservative) forces F_1, F_2 …

  $$\mathbf{E = (1/2)mv^2 + U_1 + U_2 + \ldots} \text{ is constant}$$

- Principle called

  Conservation of total mechanical energy

Nonconservative forces

- friction, air resistance, plastic deformation...

- Potential energies can only be defined for conservative forces

Nonconservative forces

- Can do work, but cannot be represented by a potential energy function
- Total mechanical energy can now change due to work done by nonconservative force
- For example, frictional force leads to decrease of total mechanical energy -- energy converted to heat, or internal energy
- Total energy = total mech. energy + internal energy *-- is conserved*

Conservation of ***total*** energy

The total energy of an object or system is said to be conserved if the sum of **all** energies (including those outside of mechanics that have not yet been discussed) never changes.

This is believed always to be true.

Summary

- Total (mechanical) energy of an isolated system is constant in time.

- Must be ***no*** non-conservative forces

- Must sum over ***all*** conservative forces

A 5.00-kg package slides 1.50 m down a long ramp that is inclined at 12.0° below the horizontal. The coefficient of kinetic friction between the package and the ramp is $\mu_k = 0.310$. Calculate: (a) the work done on the package by friction; (b) the work done on the package by gravity; (c) the work done on the package by the normal force; (d) the total work done on the package. (e) If the package has a speed of 2.20 m/s at the top of the ramp, what is its speed after sliding 1.50 m down the ramp?