

$$P_i = P_f, \quad KE_i = KE_f$$

$$m_1 V_{1i} + 0 = m_1 V_{1f} + m_2 V_{2f}$$

$$\frac{1}{2} m_1 V_{1i}^2 + 0 = \frac{1}{2} m_1 V_{1f}^2 + \frac{1}{2} m_2 V_{2f}^2$$

$$m_1 V_{1f} = \frac{m_1}{m_1} V_{1i} - \frac{m_2}{m_1} V_{2f} \Rightarrow V_{1f} = V_{1i} - \frac{m_2}{m_1} V_{2f}$$

$$V_{1f}^2 = V_{1i}^2 - 2 \frac{m_2}{m_1} V_{1i} V_{2f} + \left(\frac{m_2}{m_1}\right)^2 V_{2f}^2$$

$$\frac{1}{2} m_1 V_{1i}^2 = \frac{1}{2} m_1 \left( V_{1i}^2 - 2 \frac{m_2}{m_1} V_{1i} V_{2f} + \left(\frac{m_2}{m_1}\right)^2 V_{2f}^2 \right) + \frac{1}{2} m_2 V_{2f}^2$$

$$\cancel{\frac{1}{2} m_1 V_{1i}^2} = \cancel{\frac{1}{2} m_1 V_{1i}^2} - m_2 V_{1i} V_{2f} + \frac{1}{2} \frac{m_2^2}{m_1} V_{2f}^2 + \frac{1}{2} m_2 V_{2f}^2$$

$$0 = -m_2 V_{1i} V_{2f} + \frac{1}{2} \frac{m_2^2}{m_1} V_{2f}^2 + \frac{1}{2} m_2 V_{2f}^2$$

$$0 = -m_2 V_{1i} + \frac{1}{2} \frac{m_2^2}{m_1} V_{2f} + \frac{1}{2} m_2 V_{2f}$$

$$m_2 V_{1i} = \left( \frac{1}{2} \frac{m_2^2}{m_1} + \frac{1}{2} m_2 \right) V_{2f}$$

$$m_2 V_{1i} = \left( \frac{m_2^2 + m_1 m_2}{2 m_1} \right) V_{2f}$$

$$\frac{2 m_1 m_2}{m_2^2 + m_1 m_2} V_{1i} = V_{2f}$$

$$V_{2f} = \frac{2 m_1}{m_1 + m_2} V_{1i}$$

$$V_{1f} = V_{1i} - \frac{m_2}{m_1} V_{2f}$$

$$V_{1f} = V_{1i} - \frac{m_2}{m_1} \frac{2m_1}{(m_1 + m_2)} V_{1i}$$

$$V_{1f} = V_{1i} \left( 1 - \frac{2m_2}{m_1 + m_2} \right) = V_{1i} \left( \frac{m_1 + m_2 - 2m_2}{m_1 + m_2} \right)$$

$$V_{1f} = \frac{m_1 - m_2}{m_1 + m_2} V_{1i}$$