Welcome back to Physics 211

Today's agenda:

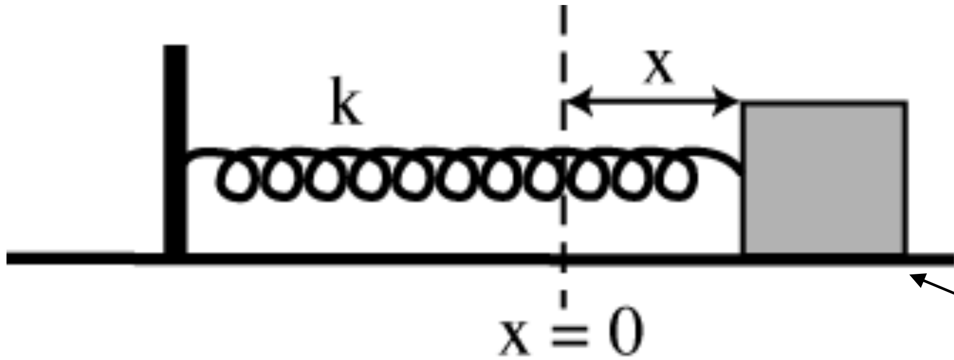
- *Work*

Current assignments

- Prelecture 11-2 due Thursday 10:30
  - Reading: Chapter 11
- HW#11 due date will be extended to next Wednesday, and a few problems on Chapter 11 will be added.
- Exam 3 will be next Thursday, 11/13

It will cover Chapters 8-11

(Horizontal) Spring



frictionless table

- x = displacement from relaxed state of spring

- Elastic potential energy stored in spring: $\mathbf{U_s = (1/2)kx^2}$

$$\mathbf{(1/2)kx^2 + (1/2)mv^2 = constant}$$

Work, Energy

- Newton's Laws are ***vector*** equations

- Sometimes easier to relate speed of a particle to how far it moves under a force – a single equation can be used – need to introduce concept of ***work***

What is work?

- Assume ***constant*** force in 1D
- Consider:

$$v_F^2 = v_I^2 + 2a\ \Delta x$$

- Multiply by m/2 →

$$(1/2)mv_F^2 - (1/2)mv_I^2 = ma\Delta x$$

- But: F = ma

$$\rightarrow (1/2)mv_F^2 - (1/2)mv_I^2 = F\Delta x$$

Work-Kinetic Energy theorem (1D)

$$(1/2)mv_F^2 - (1/2)mv_I^2 = Fs$$

- *Define s* = Δx = displacement
- W = Fs → defines **work done** on particle

  = force times displacement

- K = (1/2)mv^2 → defines **kinetic energy**

  =1/2 mass times square of v

Improved definition of work (many forces)

- For forces, write F → F_{AB}
- Thus W = Fs → $W_{AB} = F_{AB}\,\Delta s_A$ is **work done on A by B as A undergoes displacement Δs_A**
- Work-kinetic energy theorem:

$$W_{net,A} = \Sigma_B W_{AB} = \Delta K$$

The Work - Kinetic Energy Theorem

$$W_{net} = \Delta K = K_f - K_i$$

The *net work* done on an object is equal to the *change in kinetic energy* of the object.

11.1-1 Suppose a tennis ball and a bowling ball are rolling toward you. The tennis ball is moving much faster, but both have the *same momentum (mv),* and you exert the same force to stop each.

Which of the following statements is correct?

1. It takes equal distances to stop each ball.
2. It takes equal time intervals to stop each ball.
3. Both of the above.
4. Neither of the above.

11-1.2 Suppose a tennis ball and a bowling ball are rolling toward you. Both have the *same momentum (mv),* and you exert the same force to stop each.

It takes equal time intervals to stop each ball.

The distance taken for the bowling ball to stop is

1. less than.
2. equal to
3. greater than

the distance taken for the tennis ball to stop.

Work and Kinetic Energy in 2D

Work done on object 1 by object 2:

$$W_{(\text{on 1 by 2})} = \vec{F}_{1,2} \bullet \Delta\vec{s}_{\text{of 1}}$$

Kinetic energy of an object:

$$K = \tfrac{1}{2} m \mathbf{v}^2 \quad [\text{or: } \tfrac{1}{2} m(\vec{\mathbf{v}} \bullet \vec{\mathbf{v}})]$$

Scalar (or "dot") product of vectors

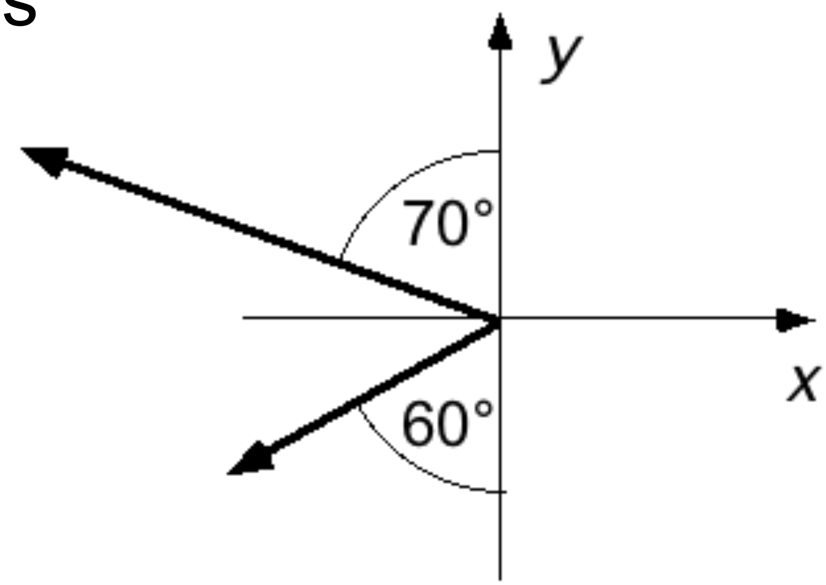
The scalar product is a way to combine two vectors to obtain a number (or *scalar*). It is indicated by a dot (•) between the two vectors.

(Note: component of A in direction n is just A•n)

$$\vec{A} \bullet \vec{B} = AB\cos\theta$$

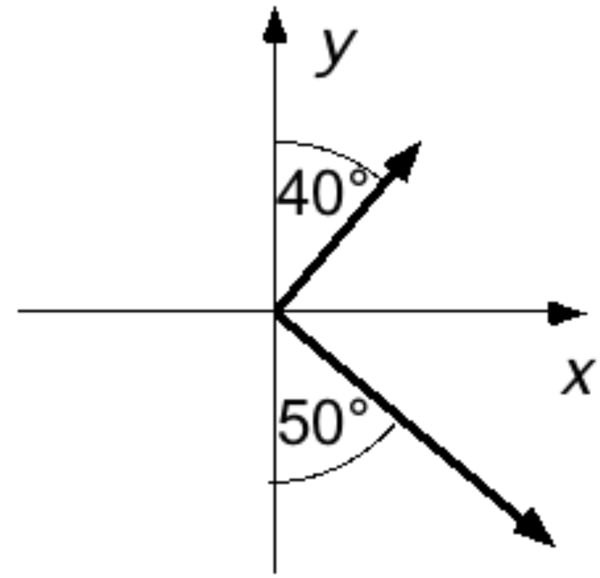
$$= A_x B_x + A_y B_y + A_z B_z$$

11-1.3 Is the scalar ("dot") product of the two vectors



1. positive
2. negative
3. equal to zero
4. "Can't tell."

11-1.4 Is the scalar (“dot”) product of the two vectors

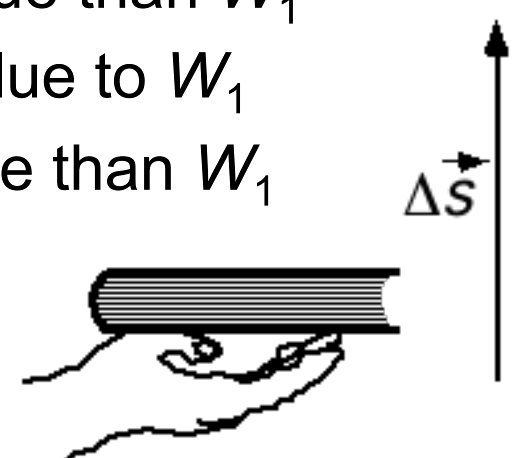


1. positive
2. negative
3. equal to zero
4. “Can’t tell.”

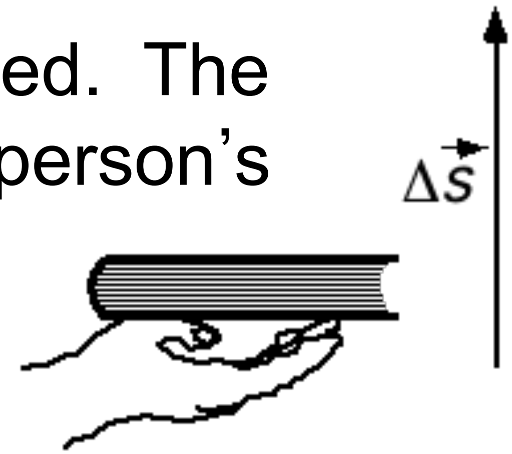
11-1.5 A person lifts a book at constant speed. Since the force exerted on the book by the person's hand is in the same direction as the displacement of the book, the work (W_1) done on the book by the person's hand is positive.

The work done on the book by the earth is:

1. negative and equal in absolute value to W_1
2. negative and less in absolute value than W_1
3. positive and equal in absolute value to W_1
4. positive and less in absolute value than W_1



A person lifts a book at constant speed. The work (W_1) done on the book by the person's hand is positive.



Work done on the book by the earth:

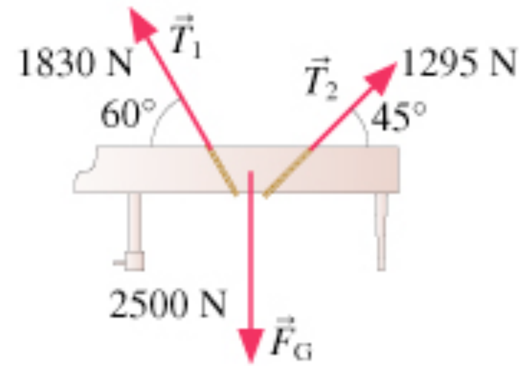
Net work done on the book:

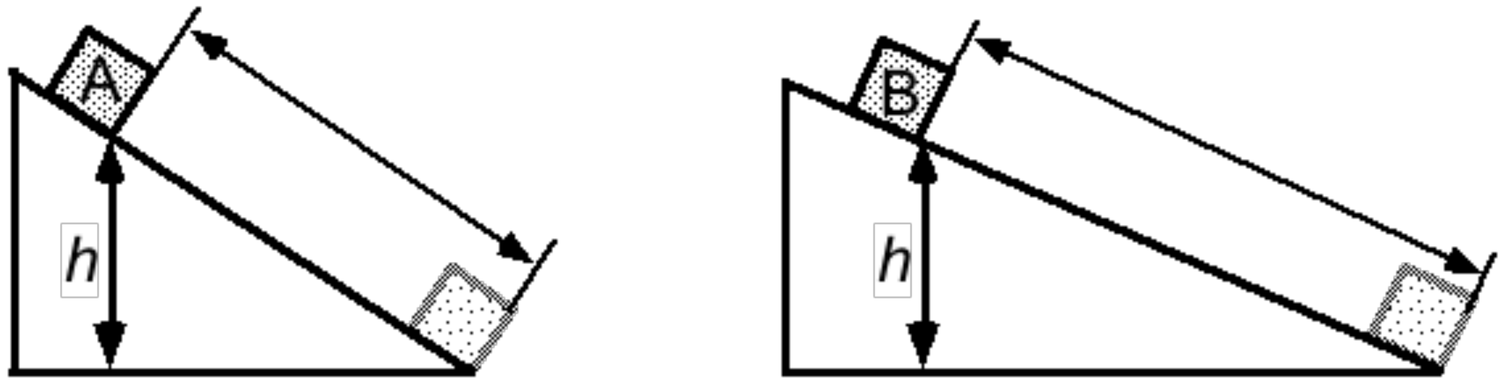
Change in kinetic energy of the book:

11-1.6 A person carries a book horizontally at constant speed. The work done on the book by the person's hand is

1. positive
2. negative
3. equal to zero
4. "Can't tell."

Sample problem: The two ropes seen in the figure are used to lower a 255 kg piano 5.1 m from a second-story window to the ground. How much work is done by each of the three forces?





11-1.7 Two identical blocks slide down two frictionless ramps. Both blocks start from the same height, but block A is on a steeper incline than block B.

The speed of block A at the bottom of its ramp is

1. less than the speed of block B.
2. equal to the speed of block B.
3. greater than the speed of block B.
4. "Can't tell."

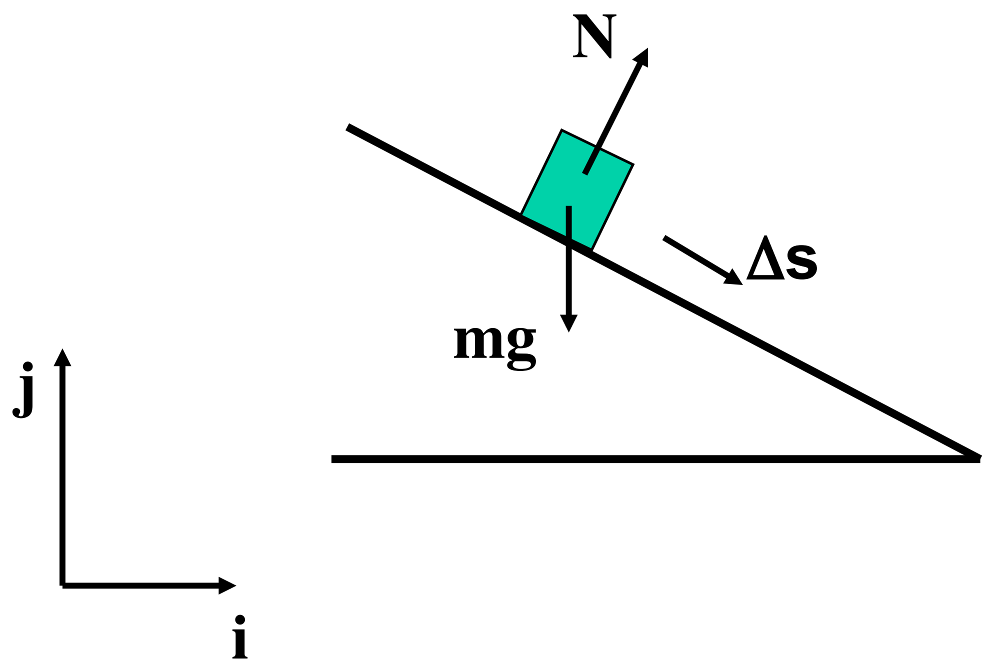
Solution

- Which forces do work on block?

- Which, if any, are constant?

- What is $\mathbf{F} \bullet \Delta \mathbf{s}$ for motion?

Work done by gravity



Work $W = -mg\, \mathbf{j} \cdot \Delta \mathbf{s}$

Therefore,

$W = -mg\Delta h$

N does no work!

Work done on an object by gravity

$$W_{(\textit{on}\text{ object }\textit{by}\text{ earth})} = -\,m\,g\,\Delta h,$$

where $\Delta h = h_{\text{final}} - h_{\text{initial}}$ is the change in height.

Object moves	Change in height Δh	Work done on object by earth
upward	positive	negative
downward	negative	positive

Defining gravitational potential energy

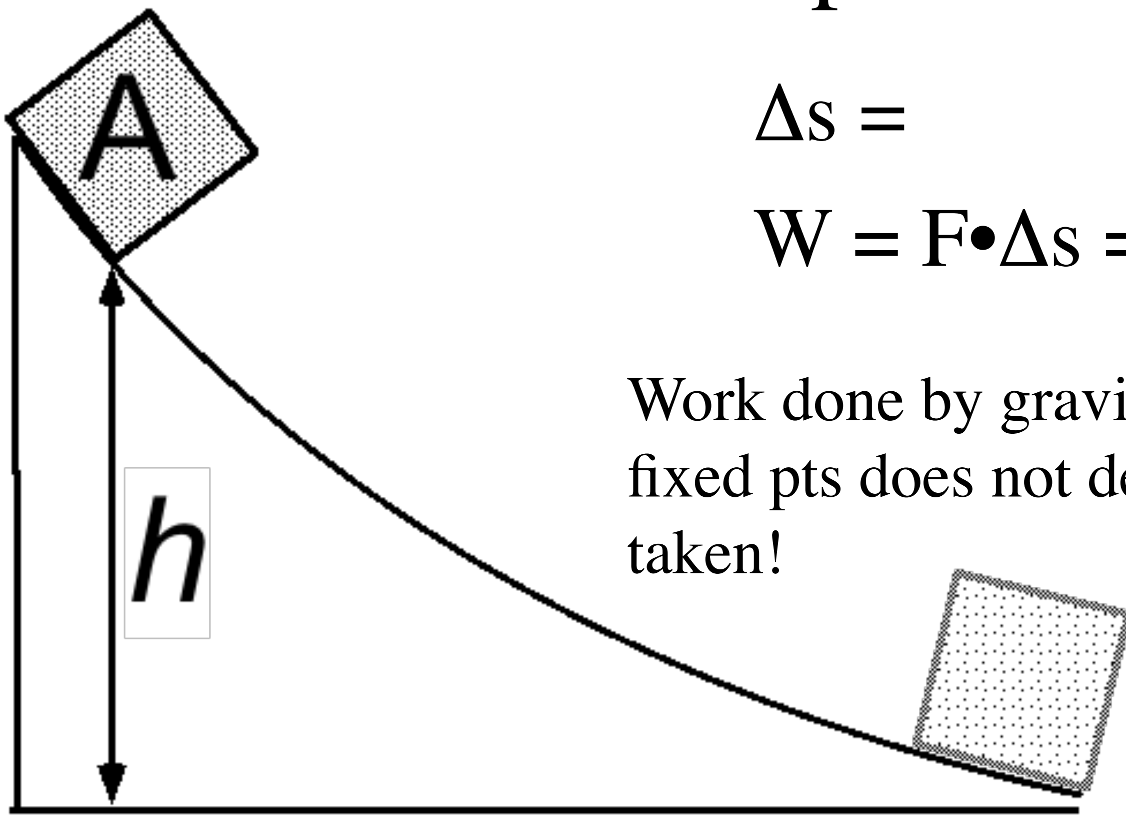
$$W_{\text{(on obj. by earth)}} = \Delta K$$

$$0 = \Delta K - W_{\text{(on obj. by earth)}}$$

$$0 = \Delta K + \Delta U_{\text{g}}$$

The *change* in gravitational potential energy of the object -earth system is just another name for the negative value of the work done on an object by the earth.

Curved ramp



$\Delta s =$

$W = F \bullet \Delta s =$

Work done by gravity between 2 fixed pts does not depend on path taken!

Work done by gravitational force in moving some object along any path is *independent* of the path depending *only* on the change in vertical height

Conservative forces

- If the work done by some force (e.g. gravity) does **not depend on path** the force is called **conservative**.
- Then gravitational potential energy U_g only depends on (vertical) position of object $U_g = U_g(h)$
- Elastic forces also conservative – elastic potential energy $U = (1/2)kx^2$...

Many forces

- For a particle which is subject to several (conservative) forces F_1, F_2 …

  $\mathbf{E = (1/2)mv^2 + U_1 + U_2 + \ldots}$ is constant

- Principle called

  Conservation of total mechanical energy

Nonconservative forces

- friction, air resistance,...

- Potential energies can only be defined for conservative forces

Nonconservative forces

- Can do work, but cannot be represented by a potential energy function
- Total mechanical energy can now change due to work done by nonconservative force
- For example, frictional force leads to decrease of total mechanical energy -- energy converted to heat, or internal energy
- Total energy = total mech. energy + internal energy -- *is conserved*

Conservation of ***total*** energy

The total energy of an object or system is said to be conserved if the sum of **all** energies (including those outside of mechanics that have not yet been discussed) never changes.

This is believed always to be true.

Demo: Infrared camera

11-1. A compressed spring fires a ping pong ball vertically upward. If the spring is compressed by 1 cm initially the ball reaches a height of 2 m above the spring. What height would the ball reach if the spring were compressed by just 0.5 cm? (neglect air resistance)

1. 2 m
2. 1 m
3. 0.5 m
4. we do not have sufficient information to calculate the new height

A 5.00-kg package slides 1.50 m down a long ramp that is inclined at 12.0° below the horizontal. The coefficient of kinetic friction between the package and the ramp is $\mu_k = 0.310$. Calculate: (a) the work done on the package by friction; (b) the work done on the package by gravity; (c) the work done on the package by the normal force; (d) the total work done on the package. (e) If the package has a speed of 2.20 m/s at the top of the ramp, what is its speed after sliding 1.50 m down the ramp?

Summary

- **Work** is defined as dot product of force with displacement vector
- Each individual force on an object gives rise to work done
- The **kinetic energy** only changes if **net work** is done on the object, which requires a **net force**

Power

Power = Rate at which work is done

$$\text{Average power} = \frac{W}{\Delta t}$$

$$\text{Inst. power} = \lim_{\Delta t \to 0} \frac{W}{\Delta t}$$

Units of power:

11-2. A sports car accelerates from zero to 30 mph in 1.5 s. How long does it take to accelerate from zero to 60 mph, assuming the power (=ΔW / Δt) of the engine to be constant?

(Neglect losses due to friction and air drag.)

1. 2.25 s
2. 3.0 s
3. 4.5 s
4. 6.0 s

Power in terms of force and velocity

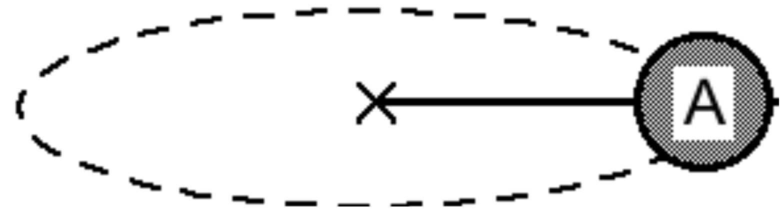
$$\text{Power} = \frac{W}{\Delta t} = \frac{\vec{F}\cdot\Delta\vec{s}}{\Delta t}$$

$$= \vec{F}\cdot\frac{\Delta\vec{s}}{\Delta t} = \vec{F}\cdot\vec{\mathbf{v}}$$

11-3. A locomotive accelerates a train from rest to a final speed of 40 mph by delivering constant power. If we assume that there are no losses due to air drag or friction, the acceleration of the train (while it is speeding up) is

1. decreasing
2. constant
3. increasing

11-4. A ball is whirled around a horizontal circle at constant speed.



If air drag forces can be neglected, the power expended by the hand is:

1. positive
2. negative
3. zero
4. “Can’t tell.”

Motion of Real Objects

- So far discussed motion of idealized point-like objects
- Saw that neglecting **internal** forces OK
  - only net **external** forces need to be considered for translational motion **of center of mass**
  - what about rotational motion?

Rigid Bodies

- Real extended objects can move in complicated ways (stretch, twist, etc.)
- Here, think of relative positions of each piece of object as fixed – idealize object as **rigid body**
- Can still undergo complicated motion (translational motion plus rotations)

Center of Mass

- Properties:
  - When a collection of particles making up an extended body is acted on by external forces, the ***center of mass*** moves as if all the mass of the body were concentrated there.

  - weight force can be considered to act vertically through ***center of mass***

Center of mass for system of (point) objects:

$$\vec{r}_{\mathrm{CM}} = \frac{(m_1\vec{r}_1 + m_2\vec{r}_2 + \ldots)}{(m_1 + m_2 + \ldots)}$$

$$= \frac{1}{M}\sum_i m_i\vec{r}_i$$

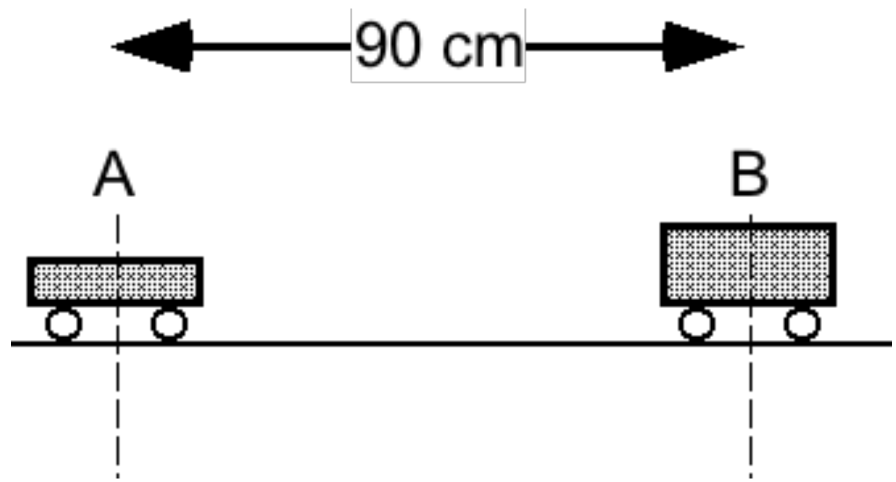
where $M = (m_1 + m_2 + \ldots)$

Points to note

- All real bodies are just collections of point-like objects (atoms)

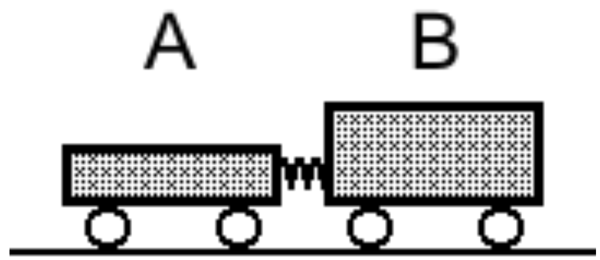
- It is ***not*** necessary that CM lie within volume of body

11.5. Two carts, A and B, of different mass ($m_B = 2\ m_A$) are placed a distance of 90 cm apart. The location of the center of mass of the two carts is



1. 30 cm to the right of cart A.
2. 45 cm to the right of cart A.
3. 60 cm to the right of cart A.
4. None of the above.

11.6. Two carts, A and B, of different mass ($m_B = 2\ m_A$) are placed end-to-end on a low-friction track with a compressed spring between them. After the spring is released, cart A moves to the left; cart B, to the right.



Will the center of mass of the system

1. move to the right,
2. move to the left, or
3. stay at rest.
4. No clue.

Use conservation of momentum

$$m_1 \Delta v_1 + m_2 \Delta v_2 = 0$$ -- *no external forces*

- So, $m_1 v_1 + m_2 v_2 = \text{constant}$
- Initially at rest → constant = 0
- Thus:
  a) $m_1 \Delta r_1 + m_2 \Delta r_2 = 0$
  b) $\Delta(m_1 r_1 + m_2 r_2) = 0$
  c) r_{CM} does not move!

11.7. A cart (of mass *m*) moving to the right at speed *v* collides with an ***identical*** stationary cart on a low-friction track. The two carts stick together after the collision and move to the right with speed 0.5 *v.*

Is the speed of the center of mass of the system after the collision

1. less than
2. equal to, or
3. greater than

the speed of the center of mass before the collision?

4. Not sure.

Situations with $F_{net} = 0$

- Consider 2 particles and $M = m_1 + m_2$
- CM definition → $Mr_{CM} = m_1r_1 + m_2r_2$

$M\Delta r_{CM}/\Delta t = m_1\Delta r_1/\Delta t + m_2\Delta r_2/\Delta t = m_1v_1 + m_2v_2$

  - RHS is total momentum

Thus, *velocity of center of mass is constant in absence of external forces!*

Conclusions

If there is ***no net force*** on a system, the center of mass of the system:

- will stay at rest if it is initially at rest, or

- will continue to move with the same velocity if it is initially moving.