Welcome back to Physics 211

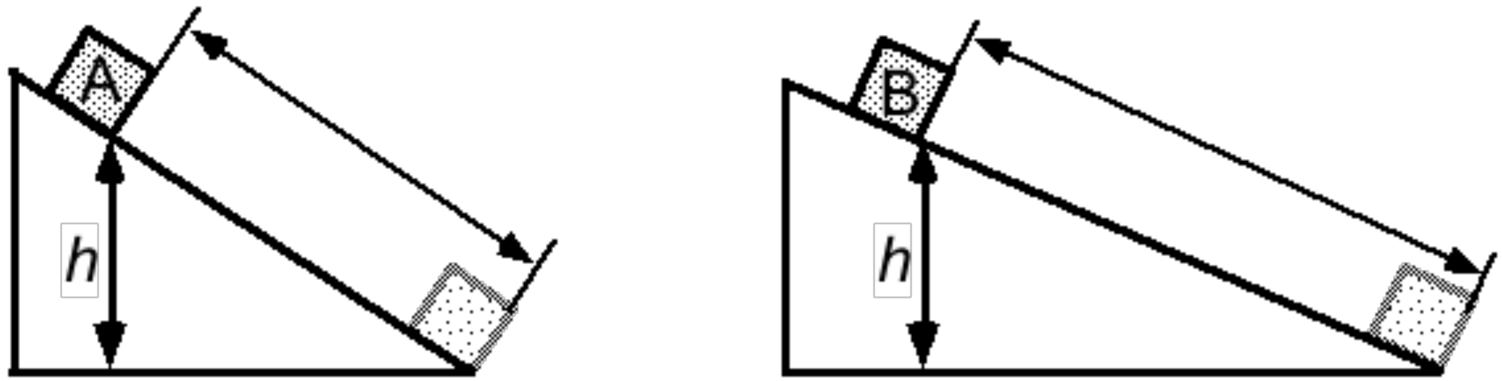
Today's agenda:

- *Work and Energy*
- *Power*

Current assignments

- No Prelecture next Tuesday
- HW#11 due next Wednesday at 5 pm.
- Exam 3 next Thursday.

  Exam 3 covers Chapters 8-11 of Knight.



11-2.1 Two identical blocks slide down two frictionless ramps. Both blocks start from the same height, but block A is on a steeper incline than block B.

The speed of block A at the bottom of its ramp is

1. less than the speed of block B.
2. equal to the speed of block B.
3. greater than the speed of block B.
4. “Can’t tell.”

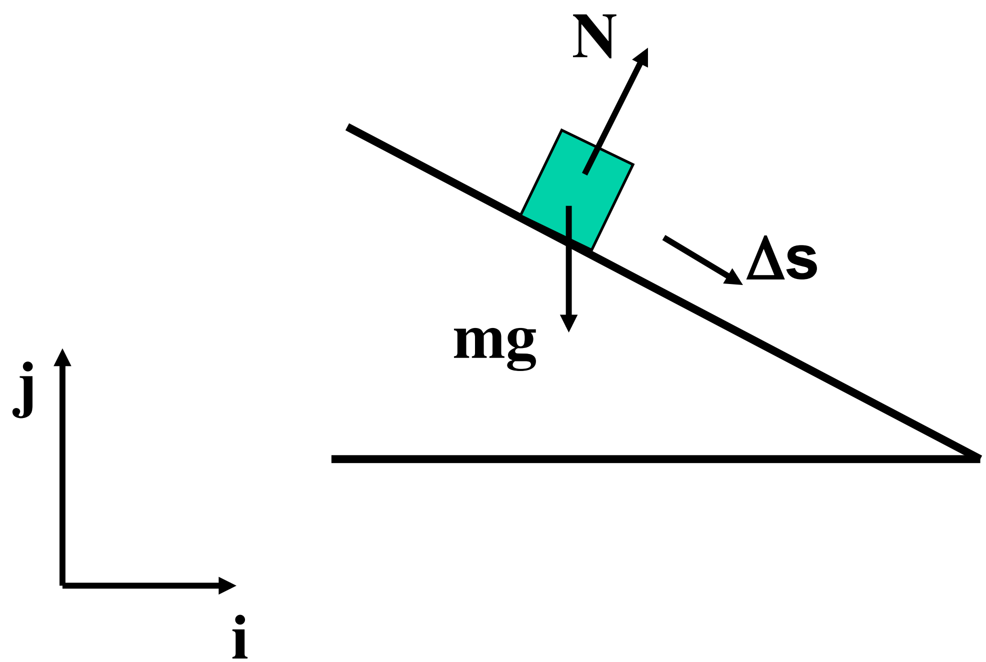
Solution

- Which forces do work on block?

- Which, if any, are constant?

- What is $\mathbf{F} \cdot \Delta \mathbf{s}$ for motion?

Work done by gravity



Work $W = -mg\,\mathbf{j}\cdot\Delta\mathbf{s}$

Therefore,

$$W = -mg\Delta h$$

N does no work!

Work done on an object by gravity

$$W_{\text{(}\textit{on}\text{ object }\textit{by}\text{ earth)}} = -\, m\, g\, \Delta h,$$

where $\Delta h = h_{\text{final}} - h_{\text{initial}}$ is the change in height.

Object moves	Change in height Δh	Work done on object by earth
upward	positive	negative
downward	negative	positive

Defining gravitational potential energy

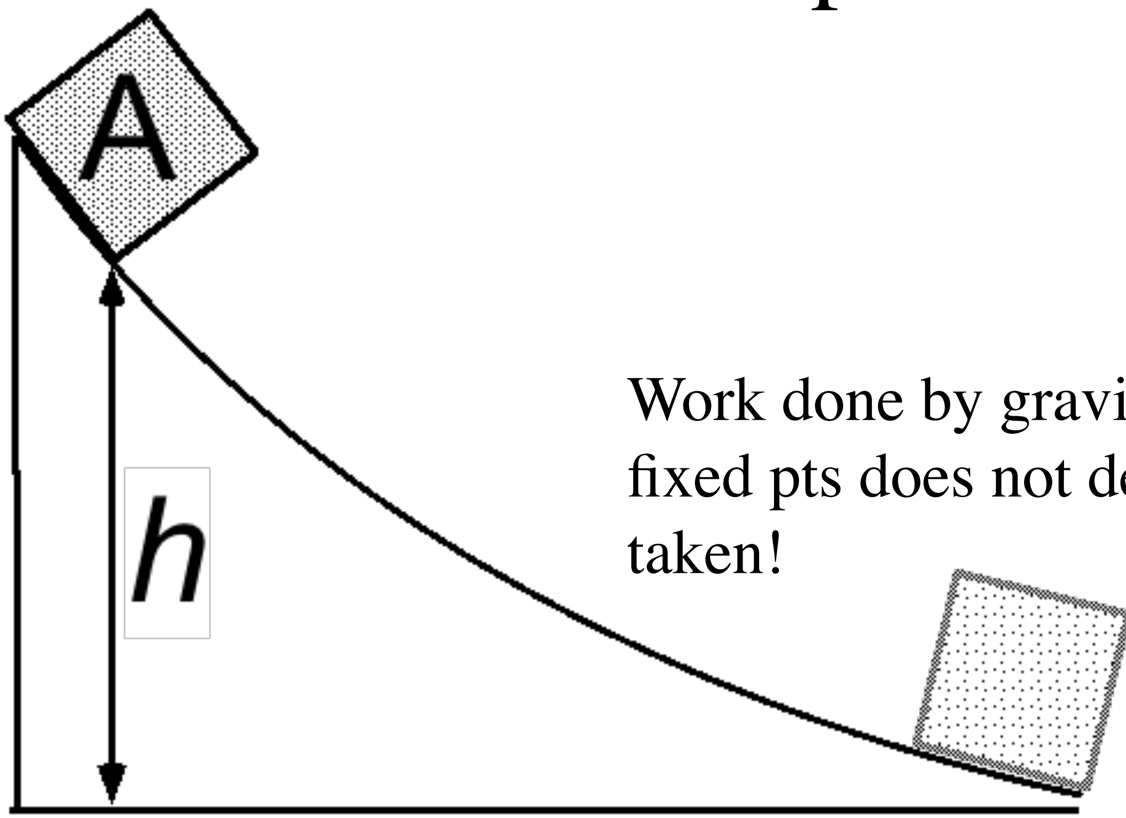
$$W_{\text{(on obj. by earth)}} = \Delta K$$

$$0 = \Delta K - W_{\text{(on obj. by earth)}}$$

$$0 = \Delta K + \Delta U_{\text{g}}$$

The *change* in gravitational potential energy of the object -earth system is just another name for the negative value of the work done on an object by the earth.

Curved ramp



Work done by gravity between 2 fixed pts does not depend on path taken!

Work done by gravitational force in moving some object along any path is *independent* of the path depending *only* on the change in vertical height

Conservative forces

- If the work done by some force (e.g. gravity) does **not depend on path** the force is called **conservative**.
- Then gravitational potential energy U_g only depends on (vertical) position of object $U_g = U_g(h)$
- Elastic forces also conservative – elastic potential energy $U = (1/2)kx^2$...

Many forces

- For a particle which is subject to several (conservative) forces F_1, F_2 …

  $\mathbf{E = (1/2)mv^2 + U_1 + U_2 + \ldots}$ is constant

- Principle called

  Conservation of total mechanical energy

Nonconservative forces

- friction, air resistance,...

- Potential energies can only be defined for conservative forces

Nonconservative forces

- Can do work, but cannot be represented by a potential energy function
- Total mechanical energy can now change due to work done by nonconservative force
- For example, frictional force leads to decrease of total mechanical energy -- energy converted to heat, or internal energy
- Total energy = total mech. energy + internal energy *-- is conserved*

Conservation of ***total*** energy

The total energy of an object or system is said to be conserved if the sum of **all** energies (including those outside of mechanics that have not yet been discussed) never changes.

This is believed always to be true.

Demo: Infrared camera

11-2.2 A compressed spring fires a ping pong ball vertically upward. If the spring is compressed by 1 cm initially the ball reaches a height of 2 m above the spring. What height would the ball reach if the spring were compressed by just 0.5 cm? (neglect air resistance)

1. 2 m
2. 1 m
3. 0.5 m
4. we do not have sufficient information to calculate the new height

A 5.00-kg package slides 1.50 m down a long ramp that is inclined at 12.0° below the horizontal. The coefficient of kinetic friction between the package and the ramp is $\mu_k = 0.310$. Calculate: (a) the work done on the package by friction; (b) the work done on the package by gravity; (c) the work done on the package by the normal force; (d) the total work done on the package. (e) If the package has a speed of 2.20 m/s at the top of the ramp, what is its speed after sliding 1.50 m down the ramp?

Summary

- **Work** is defined as dot product of force with displacement vector
- Each individual force on an object gives rise to work done
- The **kinetic energy** only changes if **net work** is done on the object, which requires a **net force**

Power

Power = Rate at which work is done

$$\text{Average power} = \frac{\Delta W}{\Delta t}$$

$$\text{Inst. power} = \lim_{\Delta t \to 0} \frac{\Delta W}{\Delta t}$$

Units of power:

11-2.3 A sports car accelerates from zero to 30 mph in 1.5 s. How long does it take to accelerate from zero to 60 mph, assuming the power ($=\Delta W / \Delta t$) of the engine to be constant?

(Neglect losses due to friction and air drag.)

1. 2.25 s
2. 3.0 s
3. 4.5 s
4. 6.0 s

Power in terms of force and velocity

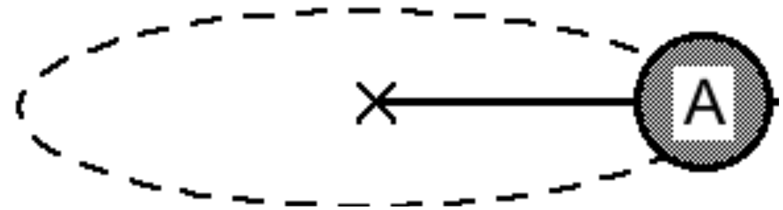
$$\text{Power} = \frac{W}{\Delta t} = \frac{\vec{F} \cdot \Delta \vec{s}}{\Delta t}$$

$$= \vec{F} \cdot \frac{\Delta \vec{s}}{\Delta t} = \vec{F} \cdot \vec{\mathbf{v}}$$

11-2.4: A locomotive accelerates a train from rest to a final speed of 40 mph by delivering constant power. If we assume that there are no losses due to air drag or friction, the acceleration of the train (while it is speeding up) is

1. decreasing
2. constant
3. increasing

11-2.5 A ball is whirled around a horizontal circle at constant speed.



If air drag forces can be neglected, the power expended by the hand is:

1. positive
2. negative
3. zero
4. "Can't tell."

Motion of Real Objects

- So far discussed motion of idealized point-like objects

- Saw that neglecting **internal** forces OK
  - only net **external** forces need to be considered for translational motion **of center of mass**
  - what about rotational motion?

Rigid Bodies

- Real extended objects can move in complicated ways (stretch, twist, etc.)
- Here, think of relative positions of each piece of object as fixed – idealize object as **rigid body**
- Can still undergo complicated motion (translational motion plus rotations)

Center of Mass

- Properties:
  - When a collection of particles making up an extended body is acted on by external forces, the ***center of mass*** moves as if all the mass of the body were concentrated there.
  - weight force can be considered to act vertically through ***center of mass***

Center of mass for system of (point) objects:

$$\vec{r}_{\mathrm{CM}} = \frac{(m_1\vec{r}_1 + m_2\vec{r}_2 + \ldots)}{(m_1 + m_2 + \ldots)}$$

$$= \frac{1}{M}\sum_i m_i\vec{r}_i$$

where $M = (m_1 + m_2 + \ldots)$

Points to note

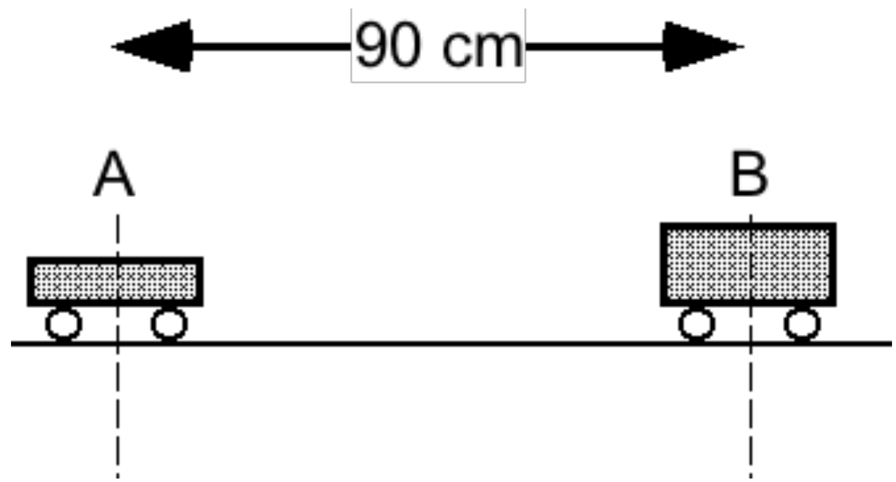
- All real bodies are just collections of point-like objects (atoms)

- It is ***not*** necessary that CM lie within volume of body

Center of mass board demos

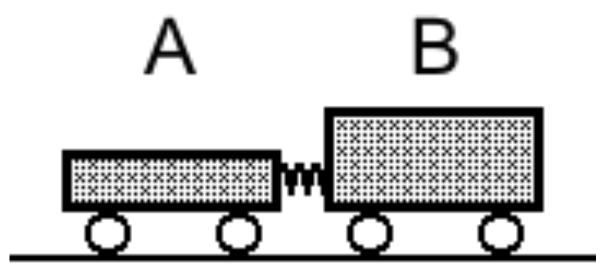
- How can we find center of mass for funny-shaped object?
- Suspend from 2 points and draw in plumb lines.
- Where lines intersect yields CM

11-2.6: Two carts, A and B, of different mass ($m_B = 2 m_A$) are placed a distance of 90 cm apart. The location of the center of mass of the two carts is



1. 30 cm to the right of cart A.
2. 45 cm to the right of cart A.
3. 60 cm to the right of cart A.
4. None of the above.

11-2.7: Two carts, A and B, of different mass ($m_B = 2 m_A$) are placed end-to-end on a low-friction track with a compressed spring between them. After the spring is released, cart A moves to the left; cart B, to the right.



Will the center of mass of the system

1. move to the right,
2. move to the left, or
3. stay at rest.
4. No clue.

Use conservation of momentum

$$m_1 \Delta v_1 + m_2 \Delta v_2 = 0$$ -- *no external forces*

- So, $m_1 v_1 + m_2 v_2 = \text{constant}$
- Initially at rest → constant = 0
- Thus:
  a) $m_1 \Delta r_1 + m_2 \Delta r_2 = 0$
  b) $\Delta(m_1 r_1 + m_2 r_2) = 0$
  c) r_{CM} does not move!

Situations with $F_{net} = 0$

- Consider 2 particles and $M = m_1+m_2$
- CM definition → $Mr_{CM} = m_1r_1 + m_2r_2$

$M\Delta r_{CM}/\Delta t = m_1\Delta r_1/\Delta t + m_2\Delta r_2/\Delta t = m_1v_1 + m_2v_2$

  - RHS is total momentum

Thus, *velocity of center of mass is constant in absence of external forces!*

Conclusions

If there is ***no net force*** on a system, the center of mass of the system:

- will stay at rest if it is initially at rest,

  or

- will continue to move with the same velocity if it is initially moving.

A cart (of mass *m*) moving to the right at speed *v* collides with an ***identical*** stationary cart on a low-friction track. The two carts stick together after the collision and move to the right with speed 0.5 *v.*

Is the speed of the center of mass of the system after the collision

1. less than
2. equal to, or
3. greater than

the speed of the center of mass before the collision?

4. Not sure.

What about F_{ext} not zero?

$M\, r_{CM} = m_1 r_1 + m_2 r_2$

→ $M\Delta\, r_{CM} / \Delta t = m_1 \Delta r_1 / \Delta t + m_2 \Delta r_2 / \Delta t$

→ $M v_{CM} = m_1 v_1 + m_2 v_2$

Therefore:

$M\, \Delta v_{CM} / \Delta t = m_1\, \Delta v_1 / \Delta t + m_2\, \Delta v_2 / \Delta t$

$M a_{CM} = F_1 + F_2 = F_{ext}$ since internal forces cancel.

Motion of center of mass of a system:

$$\sum \vec{F}_{\text{ext}} = M\vec{a}_{\text{CM}}$$

The center of mass of a system of point objects moves in the same way as a single object with the same total mass would move under the influence of the same net (external) force.

Equilibrium of extended object

- Clearly net force must be zero
- Also, if want object to behave as point at center of mass → **ALL forces acting on object must pass through CM**

A few properties of the center of mass of an extended object

- The weight of an entire object can be thought of as being exerted at a single point, the center of mass.
- One can locate the center of mass of any object by suspending it from two different points and drawing vertical lines through the support points.
- Equilibrium can be ensured if all forces pass through CM
- An object at rest on a table does not tip over if the center of mass is above the area where it is supported.

A meterstick is pivoted at its center of mass. It is initially balanced. A mass of 200 g is then hung 20 cm to the right of the pivot point. Is it possible to balance the meter-stick again by hanging a 100-g mass from it?

1. Yes, the 100-g mass should be 20 cm to the left of the pivot point.
2. Yes, but the lighter mass has to be farther from the pivot point (and to the left of it).
3. Yes, but the lighter mass has to be closer to the pivot point (and to the left of it).
4. No, because the mass has to be the same on both sides.

Using the center of mass as the origin for a system of objects:

$$0 = \vec{r}_{\mathrm{CM}} = \frac{(m_1\vec{r}_1 + m_2\vec{r}_2 + \ldots)}{(m_1 + m_2 + \ldots)}$$

Therefore, the center of mass is the point which, if taken as the origin, makes:

$$(m_1\vec{r}_1 + m_2\vec{r}_2 + \ldots) = 0$$

Restatement of equilibrium conditions

$m_1r_1 + m_2r_2 = 0 \quad \rightarrow \quad W_1r_1 + W_2r_2 = 0$

- Thus, Σ force x displacement = 0
- quantity – force x displacement called ***torque (preliminary definition)***

Thus, equilibrium requires net torque to be zero

Conditions for equilibrium of an extended object

For an extended object that remains at rest and does not rotate:

- The net force on the object has to be zero.

$$\vec{F}_{\text{net}} = \sum \vec{F}_{\text{ext}} = 0$$

- The net torque on the object has to be zero.

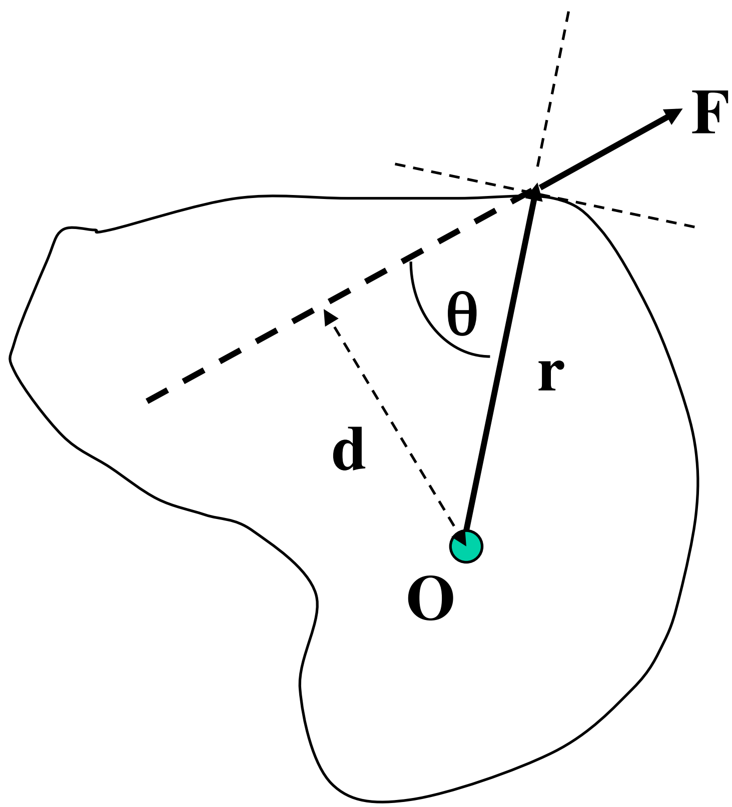
$$\vec{\tau}_{\text{net}} = \sum \vec{\tau} = 0$$

Preliminary definition of torque:

$$\tau = F\,d$$

The torque on an object with respect to a given pivot point and due to a given force is defined as the product of the force exerted on the object and the moment arm. The moment arm is the perpendicular distance from the pivot point to the line of action of the force.

Computing torque



$|\tau| = |F|d$
$= |F||r|\sin\theta$
$= (|F| \sin\theta)|r|$

component of force at 90^0 to position vector times distance

Definition of torque:

$$\vec{\tau} = \vec{r} \times \vec{F}$$

where ***r*** is the vector from the reference point (generally either the pivot point or the center of mass) to the point of application of the force ***F***.

$$|\vec{\tau}| = r\,F\sin\theta$$

where θ is the angle between the vectors ***r*** and ***F***.

Vector (or "cross") product of vectors

The vector product is a way to combine two vectors to obtain a *third vector* that has some similarities with multiplying numbers. It is indicated by a cross (×) between the two vectors.

The ***magnitude*** of the vector cross product is given by:

$$\left|\vec{A}\times\vec{B}\right| = AB\sin\theta$$

The ***direction*** of the vector **A**×**B** is *perpendicular* to the plane of vectors **A** and **B** and given by the right-hand rule.

Interpretation of torque

- Measures tendency of any force to cause rotation
- Torque is defined with respect to some origin – must talk about "torque of force about point X", etc.
- Torques can cause clockwise (+) or anticlockwise rotation (-) about pivot point

Extended objects need *extended free-body diagrams*

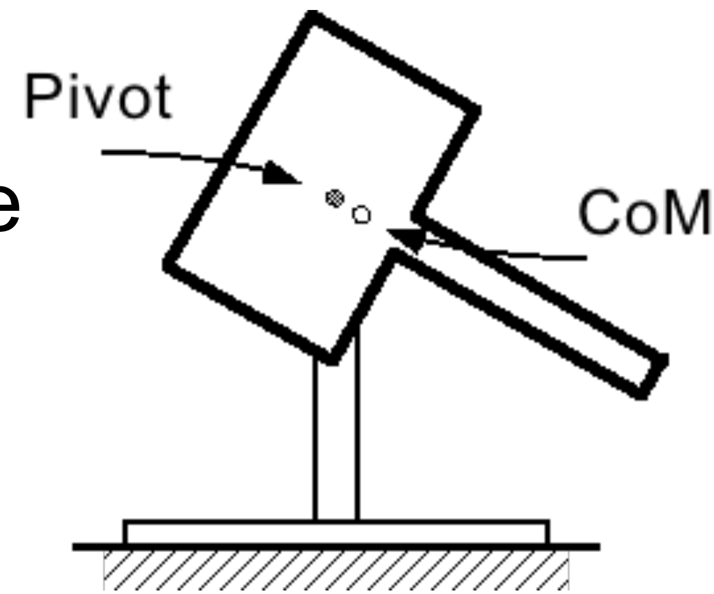
- Point free-body diagrams allow finding net force since points of application do not matter.

$$\vec{F}_{\text{net}} = \sum \vec{F}_{\text{ext}}$$

- *Extended free-body diagrams* show point of application for each force and allow finding net torque.

$$\vec{\tau}_{\text{net}} = \sum \vec{\tau}$$

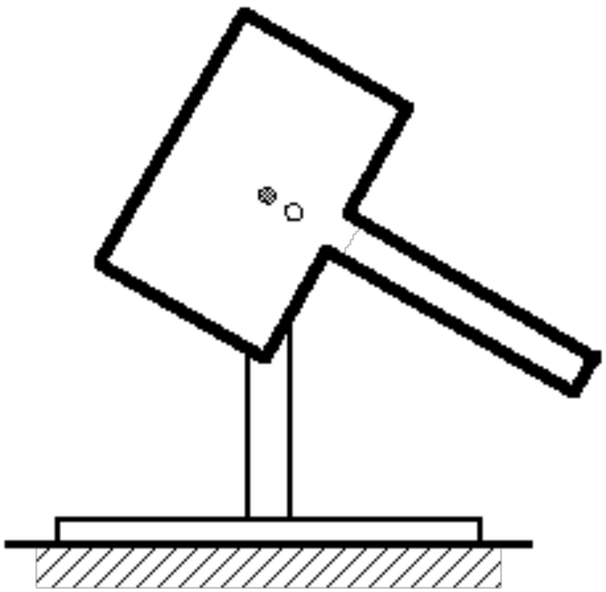
A T-shaped board is supported such that its center of mass is to the right of and below the pivot point.

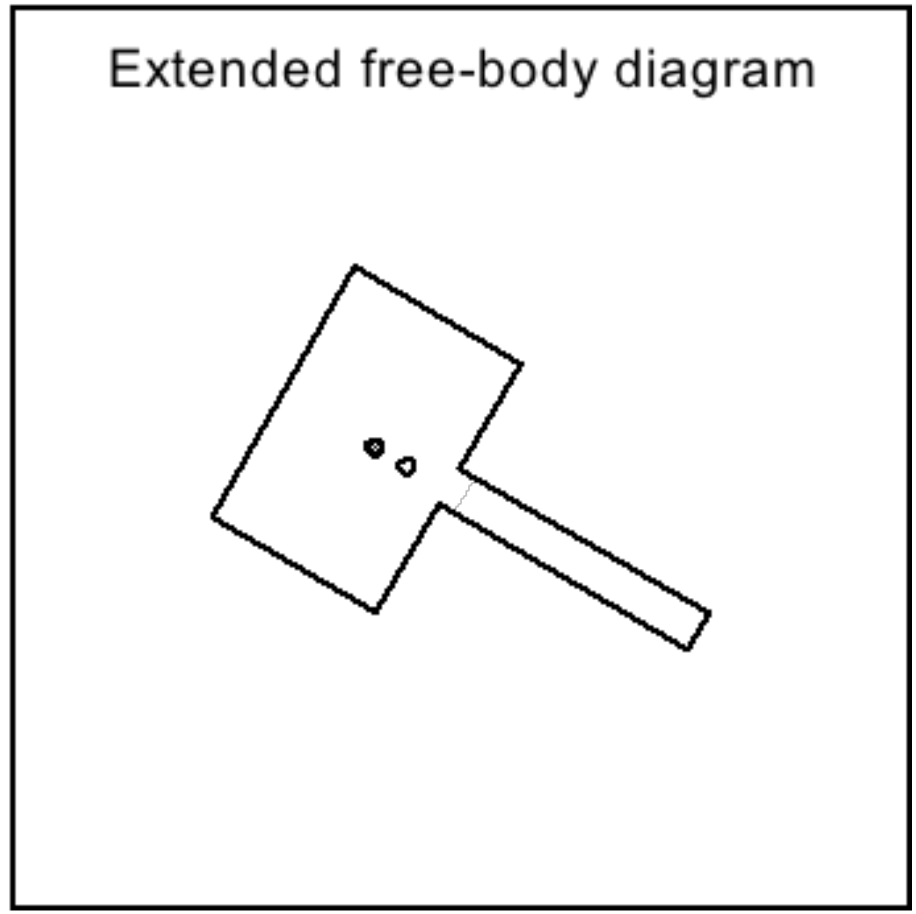


Which way will it rotate once the support is removed?

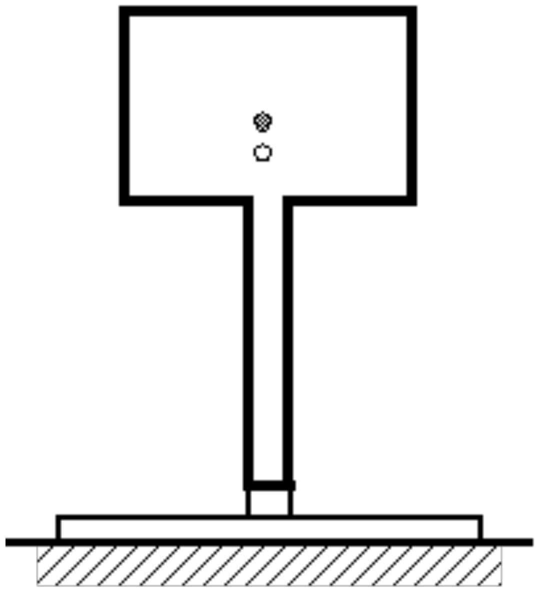
1. Clockwise.
2. Counter-clockwise.
3. Not at all.
4. Not sure what will happen.

Center of mass is *to the right of* and *below* pivot

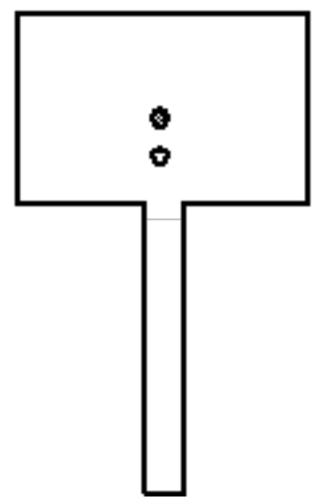




Center of mass *directly below* pivot



Extended free-body diagram

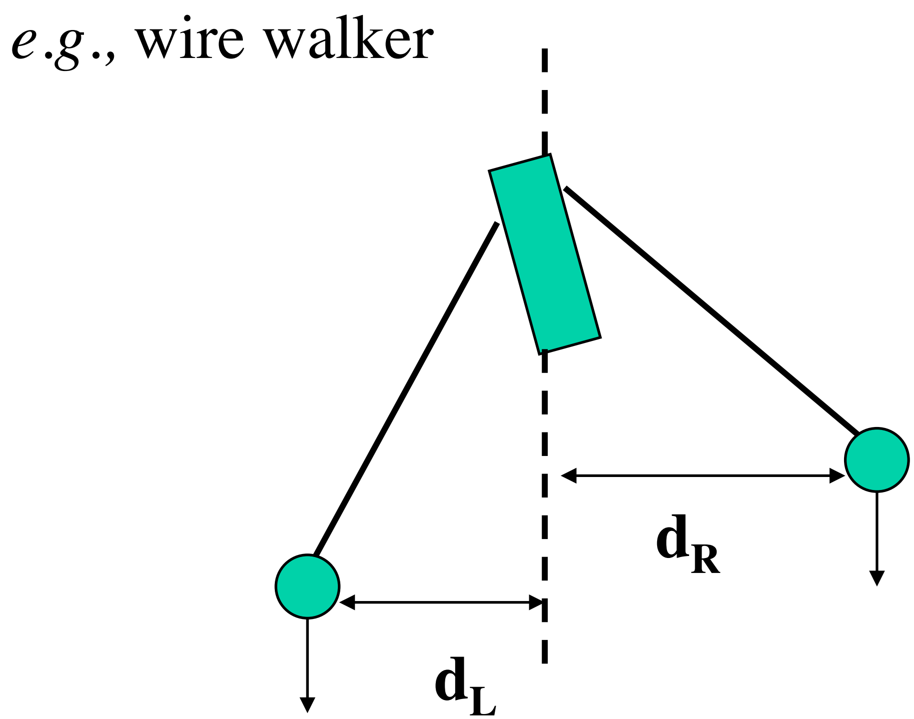


Examples of stable and unstable rotational equilibrium

- Wire walker – no net torque when figure vertical.

- Small deviations lead to a net ***restoring*** torque ➔ stable

Restoring torque



Consider displacing anticlockwise

- τ_R increases
- τ_L decreases

net torque causes clockwise rotation!

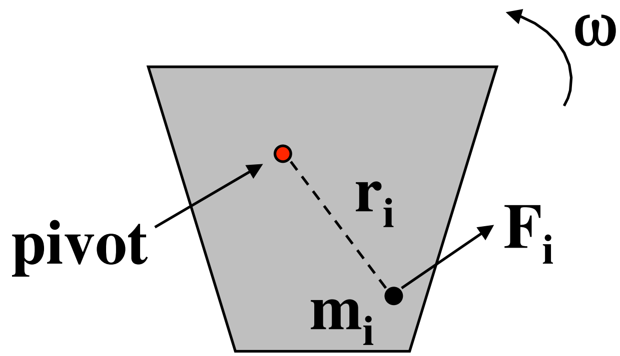
Rotations about fixed axis

- Every particle in body undergoes circular motion (not necessarily constant speed) with ***same time period***

- $v = (2\pi r)/T = \omega r$. Quantity ω is called ***angular velocity***

- Similarly can define ***angular acceleration*** $\alpha = \Delta\omega/\Delta t$

Rotational Motion



* Particle *i*:

$|v_i| = r_i\,\omega$ at 90° to r_i

* Newton's 2[nd] law:

$m_i \Delta v_i/\Delta t = F_i^T$ ← component at 90° to r_i

* Substitute for v_i and multiply by r_i :

$m_i r_i^2 \Delta\omega/\Delta t = F_i^T r_i = \tau_i$

* Finally, sum over all masses:

$$(\Delta\omega/\Delta t) \sum m_i r_i^2 = \sum \tau_i = \tau_{net}$$

Discussion

$$(\Delta\omega/\Delta t) \sum m_i r_i^2 = \tau_{net}$$

α - angular acceleration

Moment of inertia, I

$$I\alpha = \tau_{net}$$

compare this with Newton's 2nd law

$$Ma = F$$

Moment of Inertia

$$I = m_1 r_1^2 + m_2 r_2^2 + \cdots + m_N r_N^2 = \sum_{i=1}^{N} m_i r_i^2$$

* I must be defined with respect to a particular axis