Welcome back to Physics 211

Today's agenda:

- *Center of Mass*
- *Equilibrium of extended bodies*
- *Torque*

Current assignments

- Prelecture 13-1 due Tuesday 10:30am
  - Reading: 12.1-12.5
- HW#11 due this Wednesday at 5 pm.
- Exam 3 on Thursday

Exam 3

- Will be this Thursday (11/13) Stolkin auditorium (here) at usual class time
- Material:
  - Focuses on chapters 8-11
    - Circular dynamics, momentum, energy, work
  - But must be familiar with concepts from before: acceleration, forces, etc.
- Practice exam available on web
  - Go over in Wednesday recitation, too

Power in terms of force and velocity

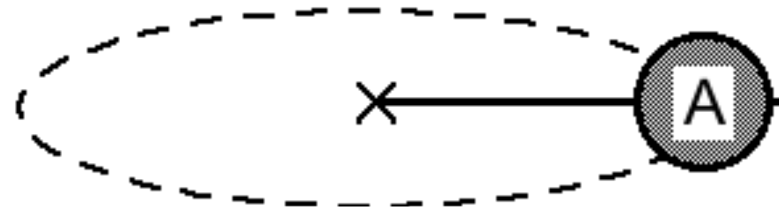
$$\text{Power} = \frac{W}{\Delta t} = \frac{\vec{F} \cdot \Delta \vec{s}}{\Delta t}$$

$$= \vec{F} \cdot \frac{\Delta \vec{s}}{\Delta t} = \vec{F} \cdot \vec{\mathbf{v}}$$

12-1.1: A locomotive accelerates a train from rest to a final speed of 40 mph by delivering constant power. If we assume that there are no losses due to air drag or friction, the acceleration of the train (while it is speeding up) is

1. decreasing
2. constant
3. increasing

12-1.2 A ball is whirled around a horizontal circle at constant speed.



If air drag forces can be neglected, the power expended by the hand is:

1. positive
2. negative
3. zero
4. “Can’t tell.”

Motion of Real Objects

- So far discussed motion of idealized point-like objects

- Saw that neglecting **internal** forces OK
  - only net **external** forces need to be considered for translational motion **of center of mass**
  - what about rotational motion?

Rigid Bodies

- Real extended objects can move in complicated ways (stretch, twist, etc.)
- Here, think of relative positions of each piece of object as fixed – idealize object as **rigid body**
- Can still undergo complicated motion (translational motion plus rotations)

Center of Mass

- Properties:
  - When a collection of particles making up an extended body is acted on by external forces, the ***center of mass*** moves as if all the mass of the body were concentrated there.
  - weight force can be considered to act vertically through ***center of mass***

Center of mass for system of (point) objects:

$$\vec{r}_{\mathrm{CM}} = \frac{(m_1\vec{r}_1 + m_2\vec{r}_2 + \ldots)}{(m_1 + m_2 + \ldots)}$$

$$= \frac{1}{M}\sum_i m_i\vec{r}_i$$

where $M = (m_1 + m_2 + \ldots)$

Points to note

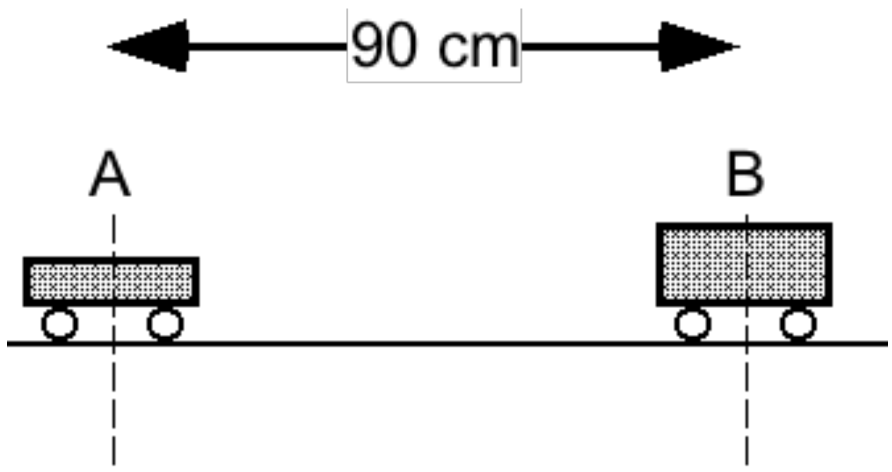
- All real bodies are just collections of point-like objects (atoms)

- It is ***not*** necessary that CM lie within volume of body

Center of mass board demos

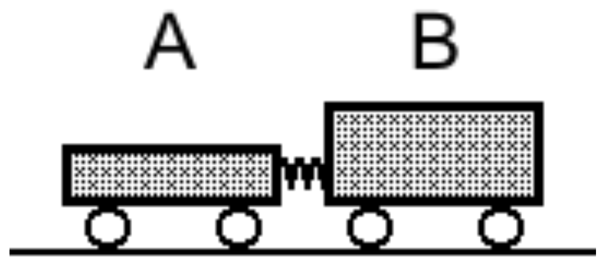
- How can we find center of mass for funny-shaped object?
- Suspend from 2 points and draw in plumb lines.
- Where lines intersect yields CM

12-1.3: Two carts, A and B, of different mass ($m_B = 2 m_A$) are placed a distance of 90 cm apart. The location of the center of mass of the two carts is



1. 30 cm to the right of cart A.
2. 45 cm to the right of cart A.
3. 60 cm to the right of cart A.
4. None of the above.

12-1.4: Two carts, A and B, of different mass ($m_B = 2 m_A$) are placed end-to-end on a low-friction track with a compressed spring between them. After the spring is released, cart A moves to the left; cart B, to the right.



Will the center of mass of the system

1. move to the right,
2. move to the left, or
3. stay at rest.
4. No clue.

Use conservation of momentum

$$m_1\Delta v_1 + m_2\Delta v_2 = 0$$ -- *no external forces*

- So, $m_1 v_1 + m_2 v_2 = \text{constant}$
- Initially at rest → constant = 0
- Thus:
  a) $m_1\Delta r_1 + m_2\Delta r_2 = 0$
  b) $\Delta(m_1 r_1 + m_2 r_2) = 0$
  c) r_{CM} does not move!

Situations with $F_{net} = 0$

- Consider 2 particles and $M = m_1 + m_2$
- CM definition → $Mr_{CM} = m_1 r_1 + m_2 r_2$

$M\Delta r_{CM}/\Delta t = m_1 \Delta r_1/\Delta t + m_2 \Delta r_2/\Delta t = m_1 v_1 + m_2 v_2$

  - RHS is total momentum

Thus, *velocity of center of mass is constant in absence of external forces!*

Center of mass for system of (point) objects:

$$\vec{r}_{\mathrm{CM}} = \frac{(m_1\vec{r}_1 + m_2\vec{r}_2 + \ldots)}{(m_1 + m_2 + \ldots)}$$

$$= \frac{1}{M}\sum_i m_i\vec{r}_i$$

where $M = (m_1 + m_2 + \ldots)$

Sample problem 1

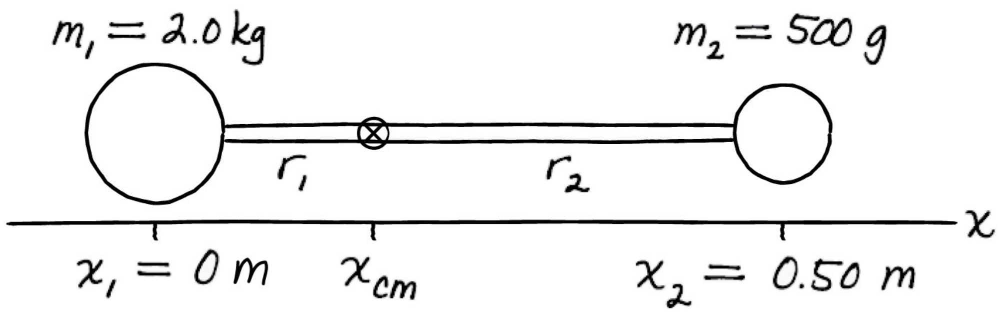
EXAMPLE 12.1 The center of mass

A 500 g ball and a 2.0 kg ball are connected by a massless 50-cm-long rod.

a. Where is the center of mass?
b. What is the speed of each ball if they rotate about the center of mass at 40 rpm?

MODEL Model each ball as a particle.

VISUALIZE The figure shows the two masses. We've chosen a coordinate system in which the masses are on the *x*-axis with the 2.0 kg mass at the origin.



Conclusions

If there is ***no net force*** on a system, the center of mass of the system:

- will stay at rest if it is initially at rest,

  or

- will continue to move with the same velocity if it is initially moving.

12-1.5: A cart (of mass *m*) moving to the right at speed *v* collides with an ***identical*** stationary cart on a low-friction track. The two carts stick together after the collision and move to the right with speed 0.5 *v.*

Is the speed of the center of mass of the system after the collision

1. less than
2. equal to, or
3. greater than

the speed of the center of mass before the collision?

4. Not sure.

What about F_{ext} not zero?

$M\, r_{CM} = m_1 r_1 + m_2 r_2$

→ $M\Delta\, r_{CM} / \Delta t = m_1 \Delta r_1 / \Delta t + m_2 \Delta r_2 / \Delta t$

→ $Mv_{CM} = m_1 v_1 + m_2 v_2$

Therefore:

$M\, \Delta v_{CM} / \Delta t = m_1\, \Delta v_1 / \Delta t + m_2\, \Delta v_2 / \Delta t$

$Ma_{CM} = F_1 + F_2 = F_{ext}$ since internal forces cancel.

Motion of center of mass of a system:

$$\sum \vec{F}_{\text{ext}} = M\vec{a}_{\text{CM}}$$

The center of mass of a system of point objects moves in the same way as a single object with the same total mass would move under the influence of the same net (external) force.

Equilibrium of extended object

- Clearly net force must be zero
- Also, if want object to behave as point at center of mass → **ALL forces acting on object must pass through CM**

A few properties of the center of mass of an extended object

- The weight of an entire object can be thought of as being exerted at a single point, the center of mass.
- One can locate the center of mass of any object by suspending it from two different points and drawing vertical lines through the support points.
- Equilibrium can be ensured if all forces pass through CM
- An object at rest on a table does not tip over if the center of mass is above the area where it is supported.

Clicker 11-1.4 What is the center of mass of New York state?

1. Binghamton
2. Syracuse
3. Utica
4. Albany

12-1.6: A meterstick is pivoted at its center of mass. It is initially balanced. A mass of 200 g is then hung 20 cm to the right of the pivot point. Is it possible to balance the meter-stick again by hanging a 100-g mass from it?

1. Yes, the 100-g mass should be 20 cm to the left of the pivot point.
2. Yes, but the lighter mass has to be farther from the pivot point (and to the left of it).
3. Yes, but the lighter mass has to be closer to the pivot point (and to the left of it).
4. No, because the mass has to be the same on both sides.

Using the center of mass as the origin for a system of objects:

$$0 = \vec{r}_{\text{CM}} = \frac{(m_1\vec{r}_1 + m_2\vec{r}_2 + \ldots)}{(m_1 + m_2 + \ldots)}$$

Therefore, the center of mass is the point which, if taken as the origin, makes:

$$(m_1\vec{r}_1 + m_2\vec{r}_2 + \ldots) = 0$$

Restatement of equilibrium conditions

$m_1 r_1 + m_2 r_2 = 0 \quad \rightarrow \quad W_1 r_1 + W_2 r_2 = 0$

- Thus, Σ force x displacement = 0
- quantity – force x displacement called ***torque (preliminary definition)***

Thus, equilibrium requires net torque to be zero

Conditions for equilibrium of an extended object

For an extended object that remains at rest and does not rotate:

- The net force on the object has to be zero.

$$\vec{F}_{\text{net}} = \sum \vec{F}_{\text{ext}} = 0$$

- The net torque on the object has to be zero.

$$\vec{\tau}_{\text{net}} = \sum \vec{\tau} = 0$$

Throwing an extended object

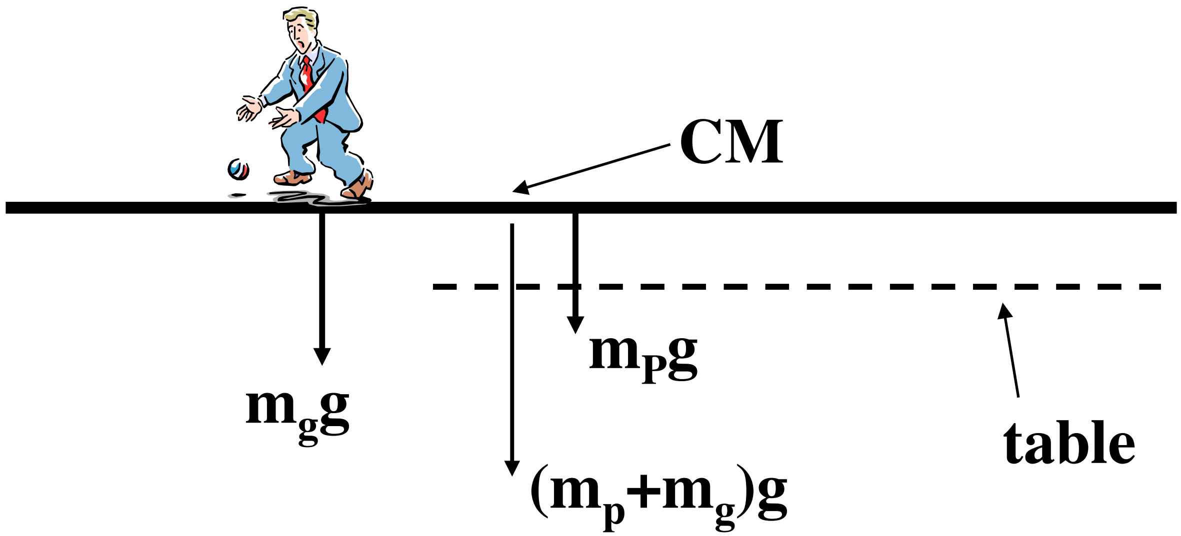
- **Demo:** odd-shaped object – 1 point has simple motion = projectile motion
  - *center of mass*

- Total external force F_{ext}

$$a_{CM} = F_{ext} /M$$

- ***Translational*** motion of system looks like all mass is concentrated at CM

Plank demo



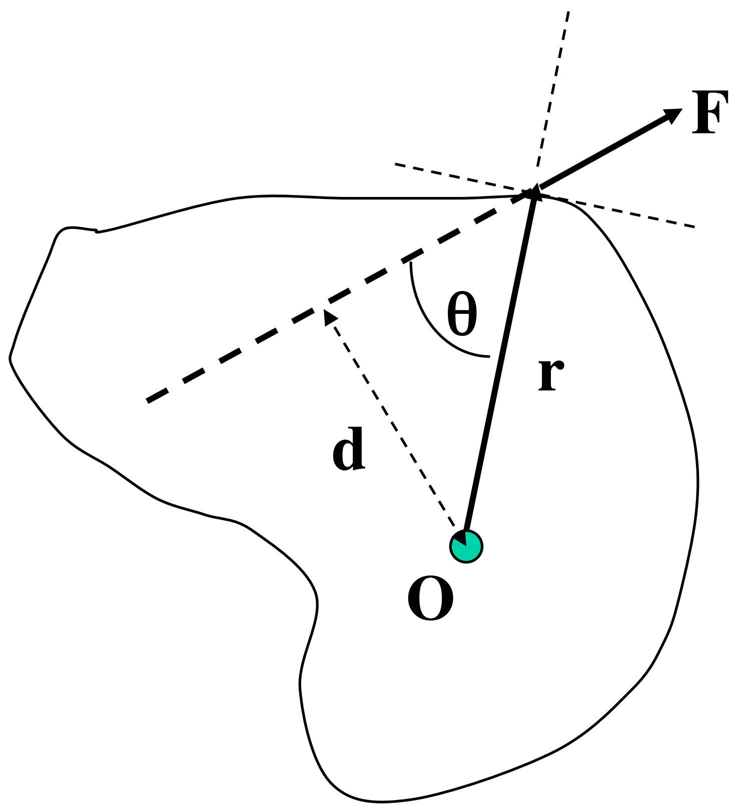
- For equilibrium of plank, center of mass of person plus plank must lie above table
- Ensures normal force can act at same point and system can be thought of as point-like
- If CM lies away from table – equilibrium not possible – rotates!

Preliminary definition of torque:

$$\tau = F\,d$$

The torque on an object with respect to a given pivot point and due to a given force is defined as the product of the force exerted on the object and the moment arm. The moment arm is the perpendicular distance from the pivot point to the line of action of the force.

Computing torque



$|\tau| = |F|d$
$= |F||r|\sin\theta$
$= (|F|\sin\theta)|r|$

component of force at 90^0 to position vector times distance

Definition of torque:

$$\vec{\tau} = \vec{r} \times \vec{F}$$

where ***r*** is the vector from the reference point (generally either the pivot point or the center of mass) to the point of application of the force ***F***.

$$|\vec{\tau}| = r\,F\sin\theta$$

where θ is the angle between the vectors ***r*** and ***F***.

Vector (or "cross") product of vectors

The vector product is a way to combine two vectors to obtain a *third vector* that has some similarities with multiplying numbers. It is indicated by a cross ($\times$) between the two vectors.

The ***magnitude*** of the vector cross product is given by:

$$\left|\vec{A}\times\vec{B}\right| = AB\sin\theta$$

The ***direction*** of the vector **A**×**B** is *perpendicular* to the plane of vectors **A** and **B** and given by the right-hand rule.

Interpretation of torque

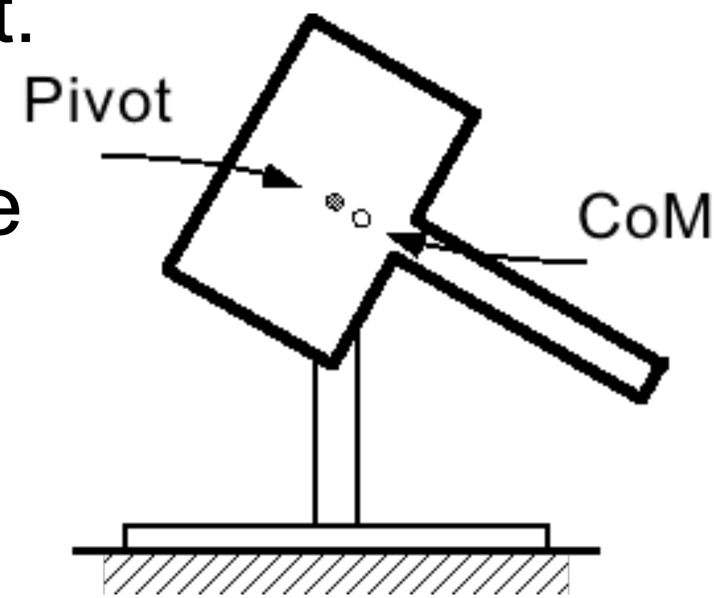
- Measures tendency of any force to cause rotation
- Torque is defined with respect to some origin – must talk about "torque of force about point X", etc.
- Torques can cause clockwise (-) or anticlockwise rotation (+) about pivot point

Extended objects need *extended free-body diagrams*

- Point free-body diagrams allow finding net force since points of application do not matter. $$\vec{F}_{\text{net}} = \sum \vec{F}_{\text{ext}}$$

- *Extended free-body diagrams* show point of application for each force and allow finding net torque. $$\vec{\tau}_{\text{net}} = \sum \vec{\tau}$$

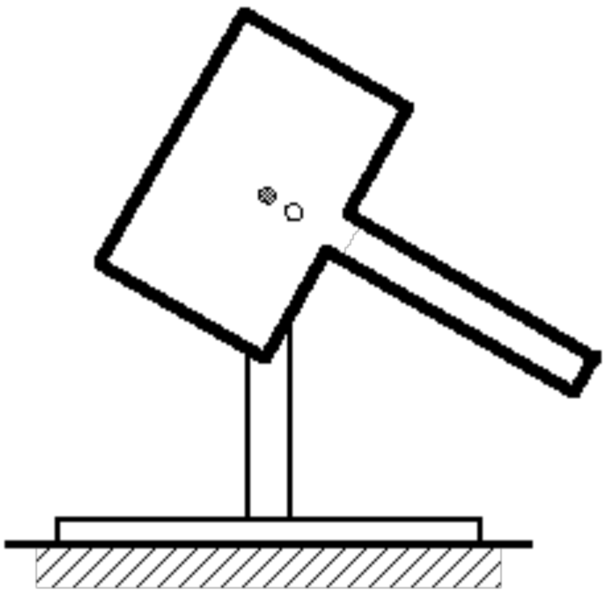
12-1.3 A T-shaped board is supported such that its center of mass is to the right of and below the pivot point.

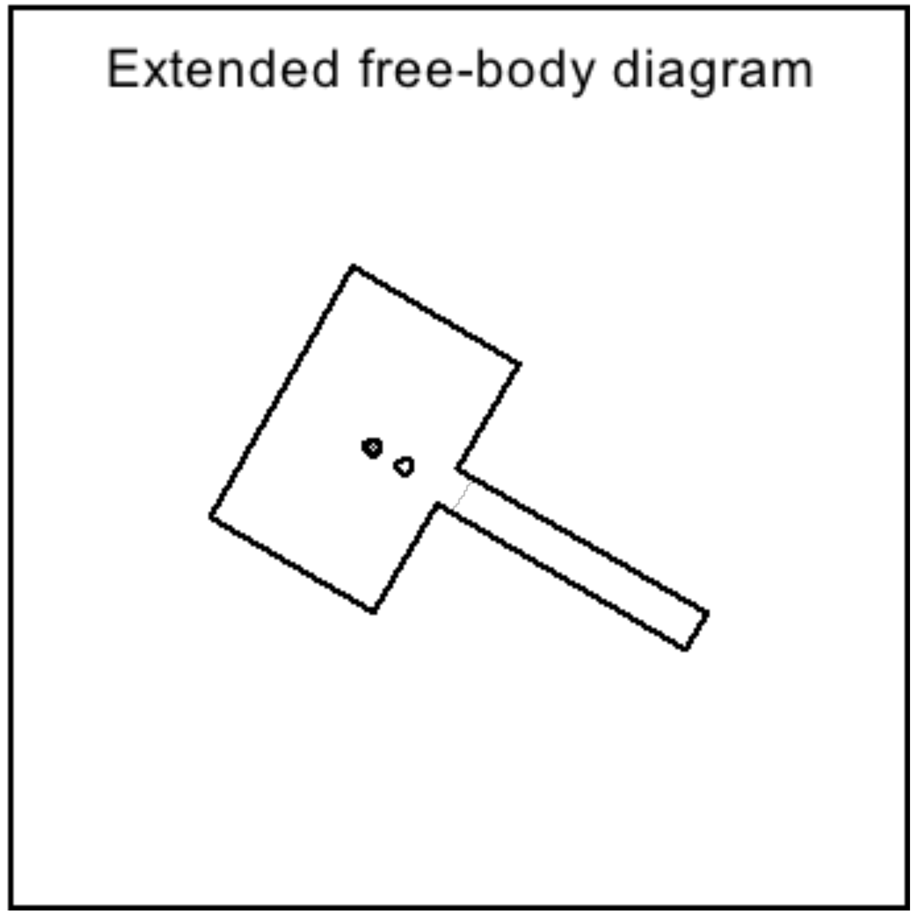


Which way will it rotate once the support is removed?

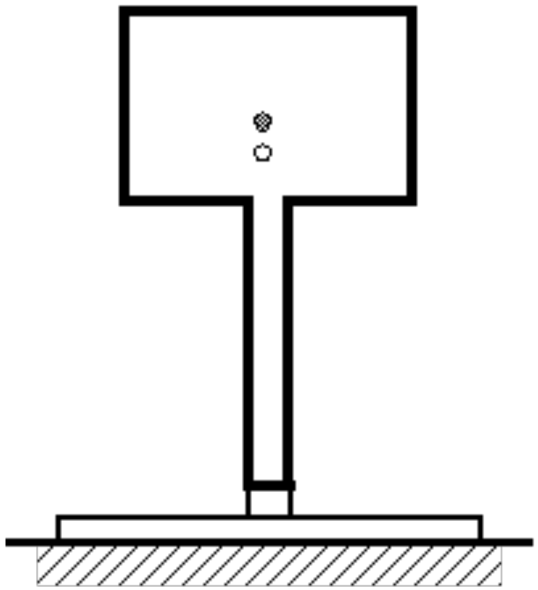
1. Clockwise.
2. Counter-clockwise.
3. Not at all.
4. Not sure what will happen.

Center of mass is *to the right of* and *below* pivot

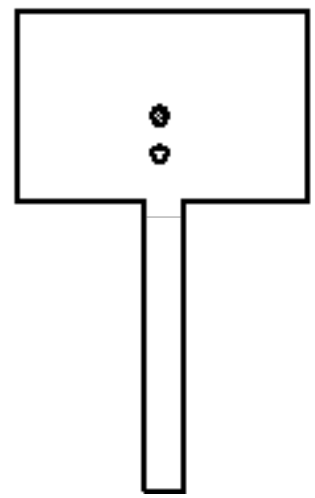




Center of mass *directly below* pivot



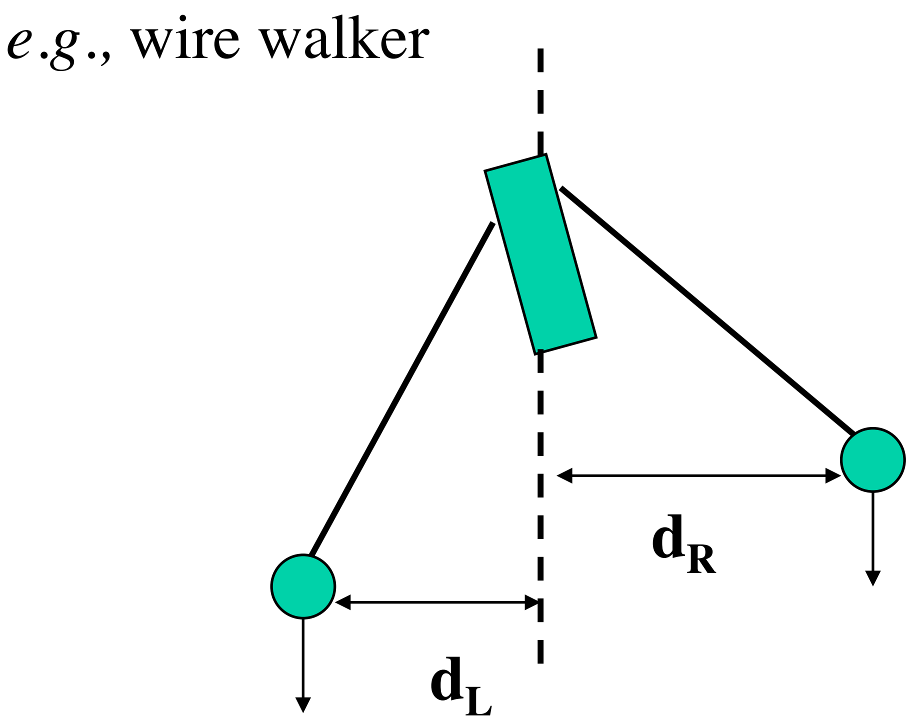
Extended free-body diagram



Examples of stable and unstable rotational equilibrium

- Wire walker – no net torque when figure vertical.
- Small deviations lead to a net ***restoring*** torque ➔ stable

Restoring torque



Consider displacing anticlockwise

- τ_R increases
- τ_L decreases

net torque causes clockwise rotation!

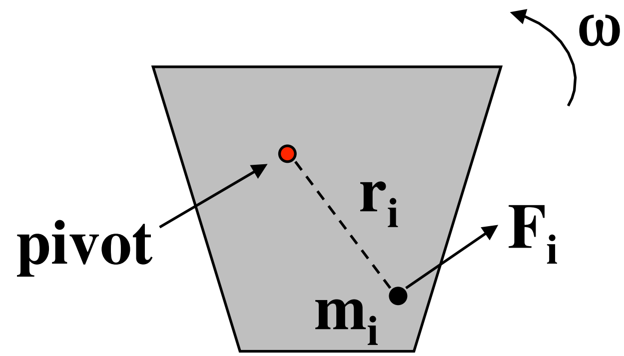
Rotations about fixed axis

- Every particle in body undergoes circular motion (not necessarily constant speed) with ***same time period***

- $v = (2\pi r)/T = \omega r$. Quantity ω is called ***angular velocity***

- Similarly can define ***angular acceleration*** $\alpha = \Delta\omega/\Delta t$

Rotational Motion



* Particle *i*:

  $|v_i| = r_i\,\omega$ at 90° to r_i

* Newton's 2nd law:

  $m_i \Delta v_i / \Delta t = F_i^T$ ← component at 90° to r_i

* Substitute for v_i and multiply by r_i :

  $m_i r_i^2 \Delta\omega / \Delta t = F_i^T\, r_i = \tau_i$

* Finally, sum over all masses:

  $(\Delta\omega/\Delta t) \sum m_i r_i^2 = \sum \tau_i = \tau_{net}$

Discussion

$$(\Delta\omega/\Delta t)\,\Sigma\, m_i r_i^2 = \tau_{net}$$

α - angular acceleration

Moment of inertia, I

$$I\alpha = \tau_{net}$$

compare this with Newton's 2nd law

$$Ma = F$$

Moment of Inertia

$$I = m_1 r_1^{\ 2} + m_2 r_2^{\ 2} + \cdots + m_N r_N^{\ 2} = \sum_{i=1}^{N} m_i r_i^{\ 2}$$

* I must be defined with respect to a particular axis