Welcome back to Physics 211

Today's agenda:

- *Torque*
- *Rotational Dynamics*

Current assignments

- Prelecture Thursday, Nov 20th at 10:30am
- HW#13 due this Friday at 5 pm.

Clicker 13-1.1 What is the center of mass of New York state?

1. Binghamton
2. Syracuse
3. Utica
4. Albany

Motion of center of mass of a system:

$$\sum \vec{F}_{\text{ext}} = M\vec{a}_{\text{CM}}$$

The center of mass of a system of point objects moves in the same way as a single object with the same total mass would move under the influence of the same net (external) force.

Throwing an extended object

- **Demo:** odd-shaped object – 1 point has simple motion = projectile motion
  - *center of mass*

- Total external force F_{ext}

$$a_{CM} = F_{ext} /M$$

- ***Translational*** motion of system looks like all mass is concentrated at CM

Equilibrium of extended object

- Clearly net force must be zero

- Also, if want object to behave as point at center of mass → **ALL forces acting on object must pass through CM**

13-1.2: A meterstick is pivoted at its center of mass. It is initially balanced. A mass of 200 g is then hung 20 cm to the right of the pivot point. Is it possible to balance the meter-stick again by hanging a 100-g mass from it?

1. Yes, the 100-g mass should be 20 cm to the left of the pivot point.
2. Yes, but the lighter mass has to be farther from the pivot point (and to the left of it).
3. Yes, but the lighter mass has to be closer to the pivot point (and to the left of it).
4. No, because the mass has to be the same on both sides.

Using the center of mass as the origin for a system of objects:

$$0 = \vec{r}_{\mathrm{CM}} = \frac{(m_1\vec{r}_1 + m_2\vec{r}_2 + \ldots)}{(m_1 + m_2 + \ldots)}$$

Therefore, the center of mass is the point which, if taken as the origin, makes:

$$(m_1\vec{r}_1 + m_2\vec{r}_2 + \ldots) = 0$$

Restatement of equilibrium conditions

$m_1 r_1 + m_2 r_2 = 0 \quad \rightarrow \quad W_1 r_1 + W_2 r_2 = 0$

- Thus, Σ force x displacement = 0
- quantity – force x displacement called ***torque (preliminary definition)***

Thus, equilibrium requires net torque to be zero

Conditions for equilibrium of an extended object

For an extended object that remains at rest and does not rotate:

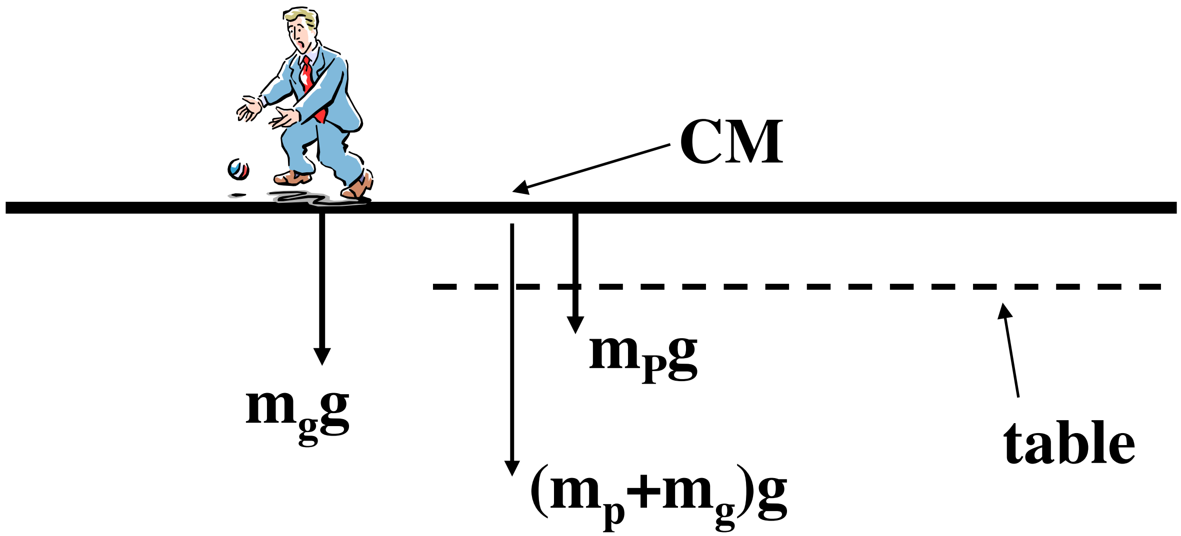
- The net force on the object has to be zero.

$$\vec{F}_{\text{net}} = \sum \vec{F}_{\text{ext}} = 0$$

- The net torque on the object has to be zero.

$$\vec{\tau}_{\text{net}} = \sum \vec{\tau} = 0$$

Plank demo



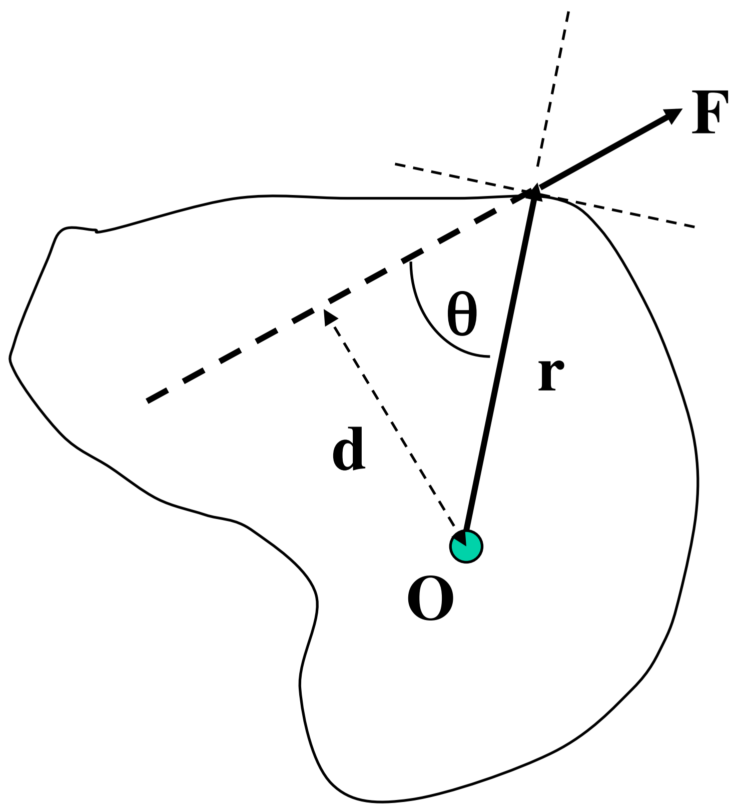
- For equilibrium of plank, center of mass of person plus plank must lie above table
- Ensures normal force can act at same point and system can be thought of as point-like
- If CM lies away from table – equilibrium not possible – rotates!

Preliminary definition of torque:

$$\tau = F\, d$$

The torque on an object with respect to a given pivot point and due to a given force is defined as the product of the force exerted on the object and the moment arm. The moment arm is the perpendicular distance from the pivot point to the line of action of the force.

Computing torque



$$|\tau| = |F|d$$
$$= |F||r|\sin\theta$$
$$= (|F|\,\sin\theta)|r|$$

component of force at 90^0 to position vector times distance

Definition of torque:

$$\vec{\tau} = \vec{r} \times \vec{F}$$

where ***r*** is the vector from the reference point (generally either the pivot point or the center of mass) to the point of application of the force ***F***.

$$|\vec{\tau}| = r\,F\sin\theta$$

where θ is the angle between the vectors ***r*** and ***F***.

Vector (or “cross”) product of vectors

The vector product is a way to combine two vectors to obtain a *third vector* that has some similarities with multiplying numbers. It is indicated by a cross ($\times$) between the two vectors.

The ***magnitude*** of the vector cross product is given by:

$$\left|\vec{A}\times\vec{B}\right| = AB\sin\theta$$

The ***direction*** of the vector **A**×**B** is *perpendicular* to the plane of vectors **A** and **B** and given by the right-hand rule.

Interpretation of torque

- Measures tendency of any force to cause rotation
- Torque is defined with respect to some origin – must talk about "torque of force about point X", etc.
- Torques can cause clockwise (-) or anticlockwise rotation (+) about pivot point

Extended objects
need *extended free-body diagrams*

- Point free-body diagrams allow finding net force since points of application do not matter.

$$\vec{F}_{\text{net}} = \sum \vec{F}_{\text{ext}}$$

- *Extended free-body diagrams* show point of application for each force and allow finding net torque.

$$\vec{\tau}_{\text{net}} = \sum \vec{\tau}$$

Recall: Definition of torque:

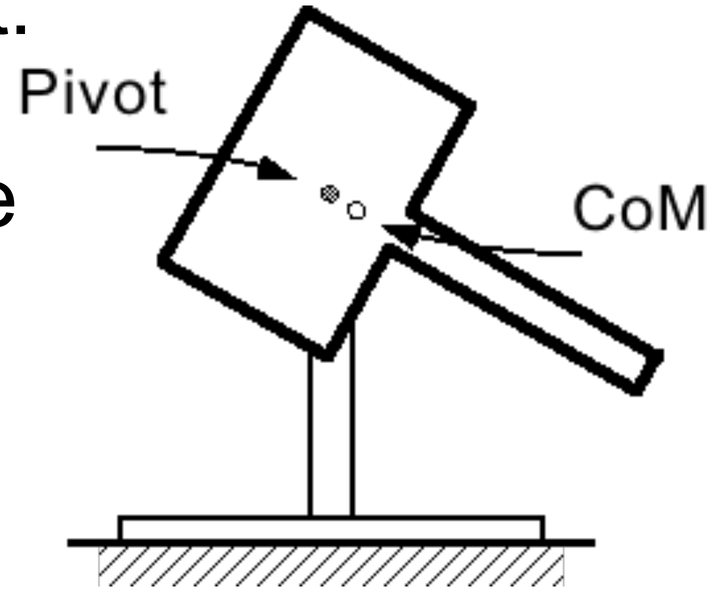
$$\vec{\tau} = \vec{r} \times \vec{F}$$

where ***r*** is the vector from the reference point (generally either the pivot point or the center of mass) to the point of application of the force ***F***.

$$|\vec{\tau}| = r\,F\sin\theta$$

where θ is the angle between the vectors ***r*** and ***F***.

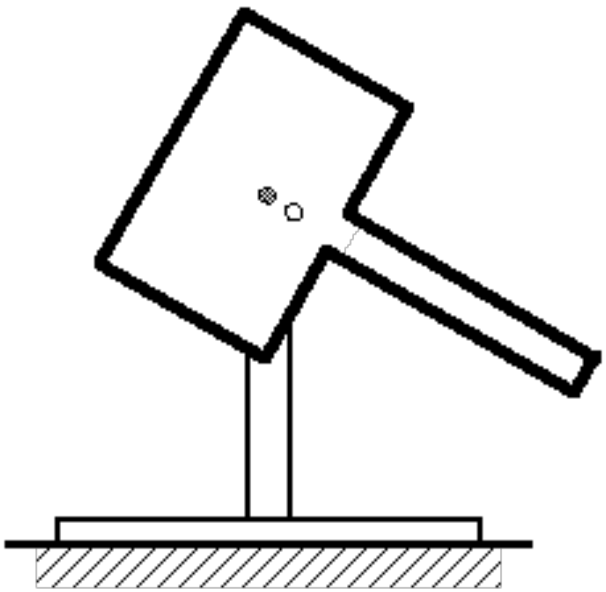
13-1.3 A T-shaped board is supported such that its center of mass is to the right of and below the pivot point.

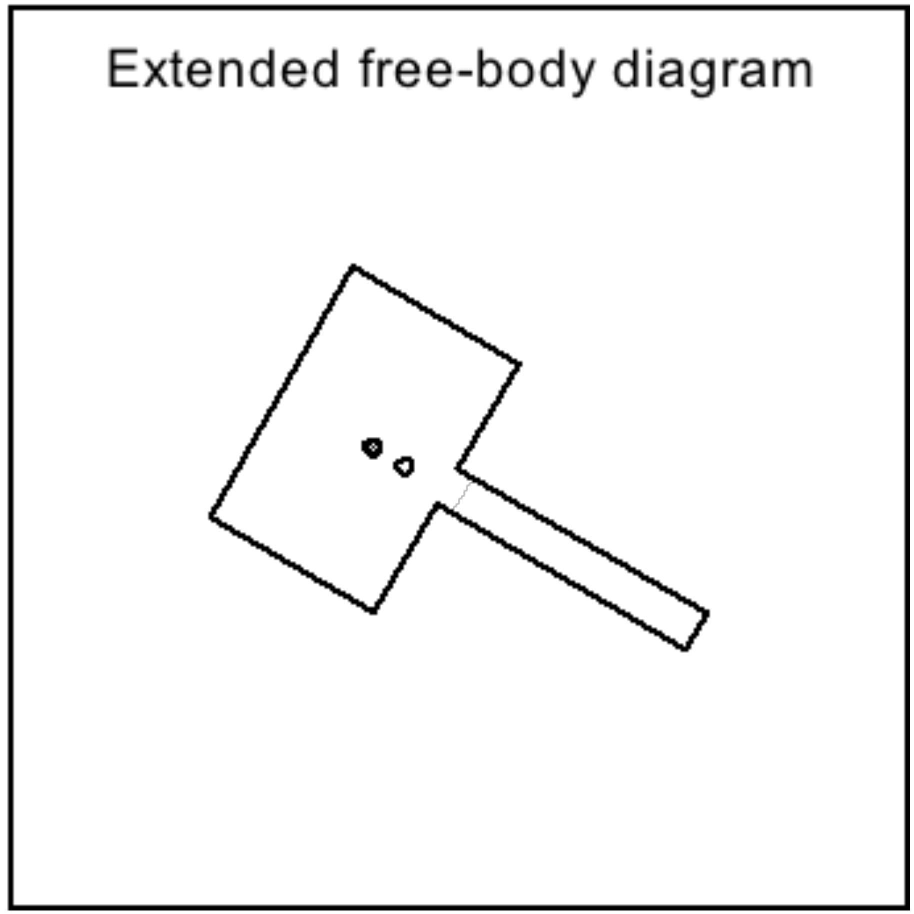


Which way will it rotate once the support is removed?

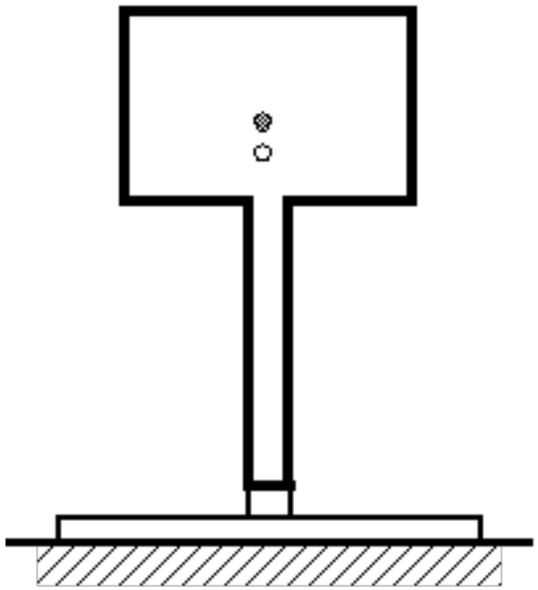
1. Clockwise.
2. Counter-clockwise.
3. Not at all.
4. Not sure what will happen.

Center of mass is *to the right of* and *below* pivot

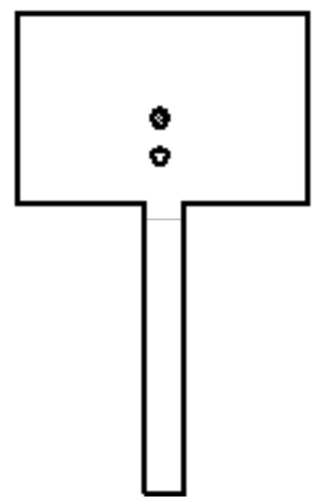




Center of mass *directly below* pivot



Extended free-body diagram



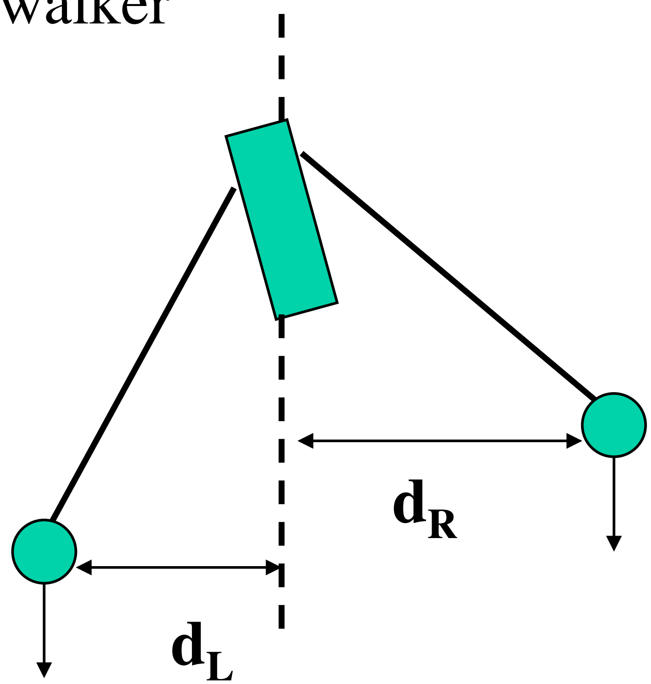
Examples of stable and unstable rotational equilibrium

- Wire walker – no net torque when figure vertical.

- Small deviations lead to a net ***restoring*** torque ➔ stable

Restoring torque

e.g., wire walker



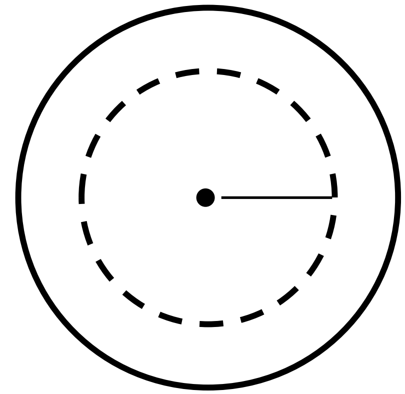
Consider displacing counterclockwise

- τ_R increases
- τ_L decreases

net torque causes clockwise rotation!

Rotations about fixed axis

- Linear speed: $v = (2\pi r)/T = \omega\ r$. Quantity ω is called ***angular velocity***

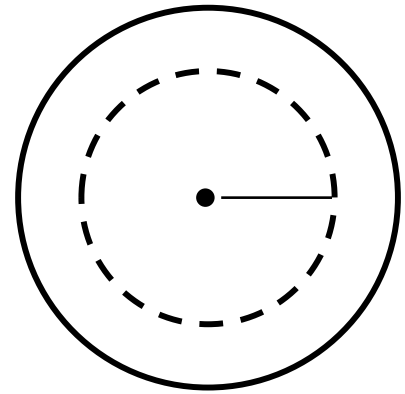


- ω is a vector! Use right hand rule to find direction of ω.

- Angular acceleration $\alpha = \Delta\omega/\Delta t$ is also a vector!
  - ω and α *parallel* $\rightarrow$ angular speed increasing
  - ω and α *antiparallel* $\rightarrow$ angular speed decreasing

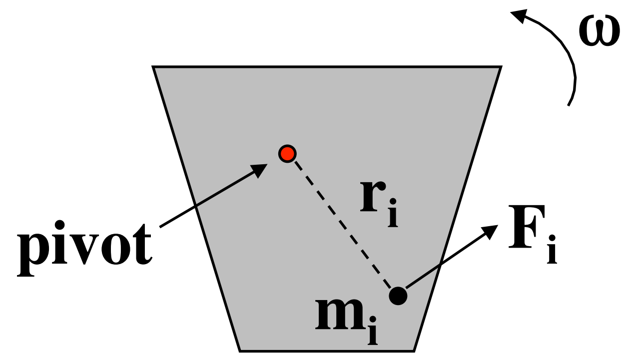
Relating linear and angular kinematics

- Linear speed: $v = (2\pi r)/T = \omega\, r$



- Tangential acceleration: $a_{tan} = r\alpha$
- Radial acceleration: $a_{rad} = v^2/r = \omega^2 r$

Rotational Motion



* Particle *i*:

  $|v_i| = r_i\,\omega$ at 90° to r_i

* Newton's 2nd law:

  $m_i \Delta v_i / \Delta t = F_i^T$ ← component at 90° to r_i

* Substitute for v_i and multiply by r_i :

  $m_i r_i^2 \Delta\omega / \Delta t = F_i^T r_i = \tau_i$

* Finally, sum over all masses:

  $$(\Delta\omega/\Delta t) \sum m_i r_i^2 = \sum \tau_i = \tau_{net}$$

Discussion

$$(\Delta\omega/\Delta t) \sum m_i r_i^2 = \tau_{net}$$

α - angular acceleration

Moment of inertia, I

$$I\alpha = \tau_{net}$$

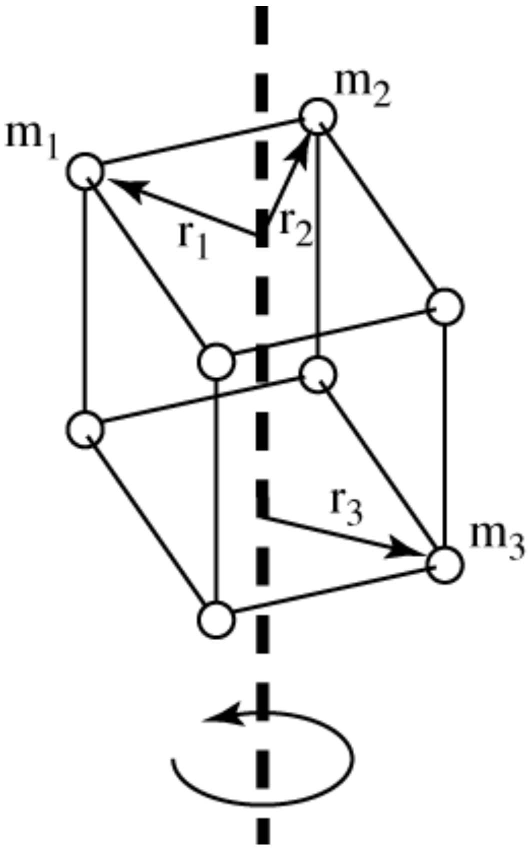
compare this with Newton's 2nd law

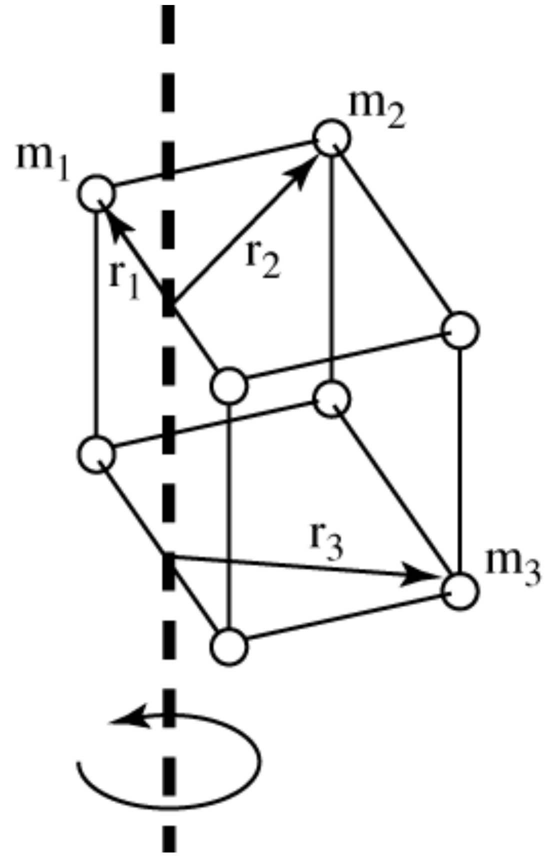
$$Ma = F$$

Moment of Inertia

$$\boxed{I = m_1 r_1^2 + m_2 r_2^2 + \cdots + m_N r_N^2 = \sum_{i=1}^{N} m_i r_i^2}$$

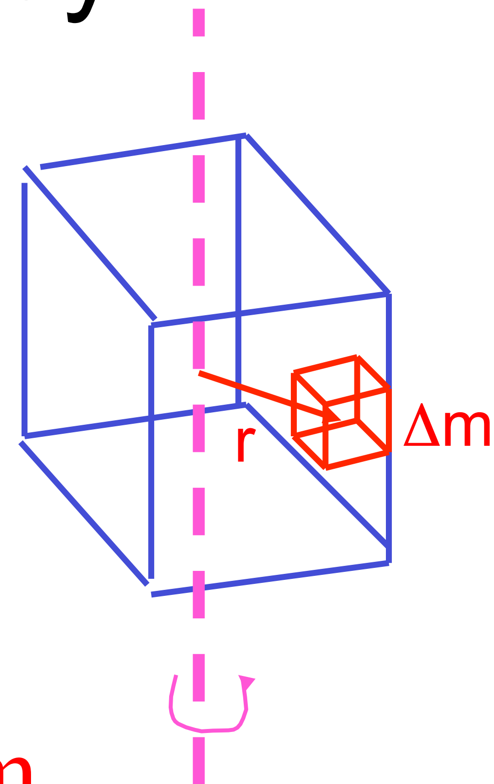
* I must be defined with respect to a particular axis





Moment of Inertia of Continuous Body

$\Delta m \mapsto 0$

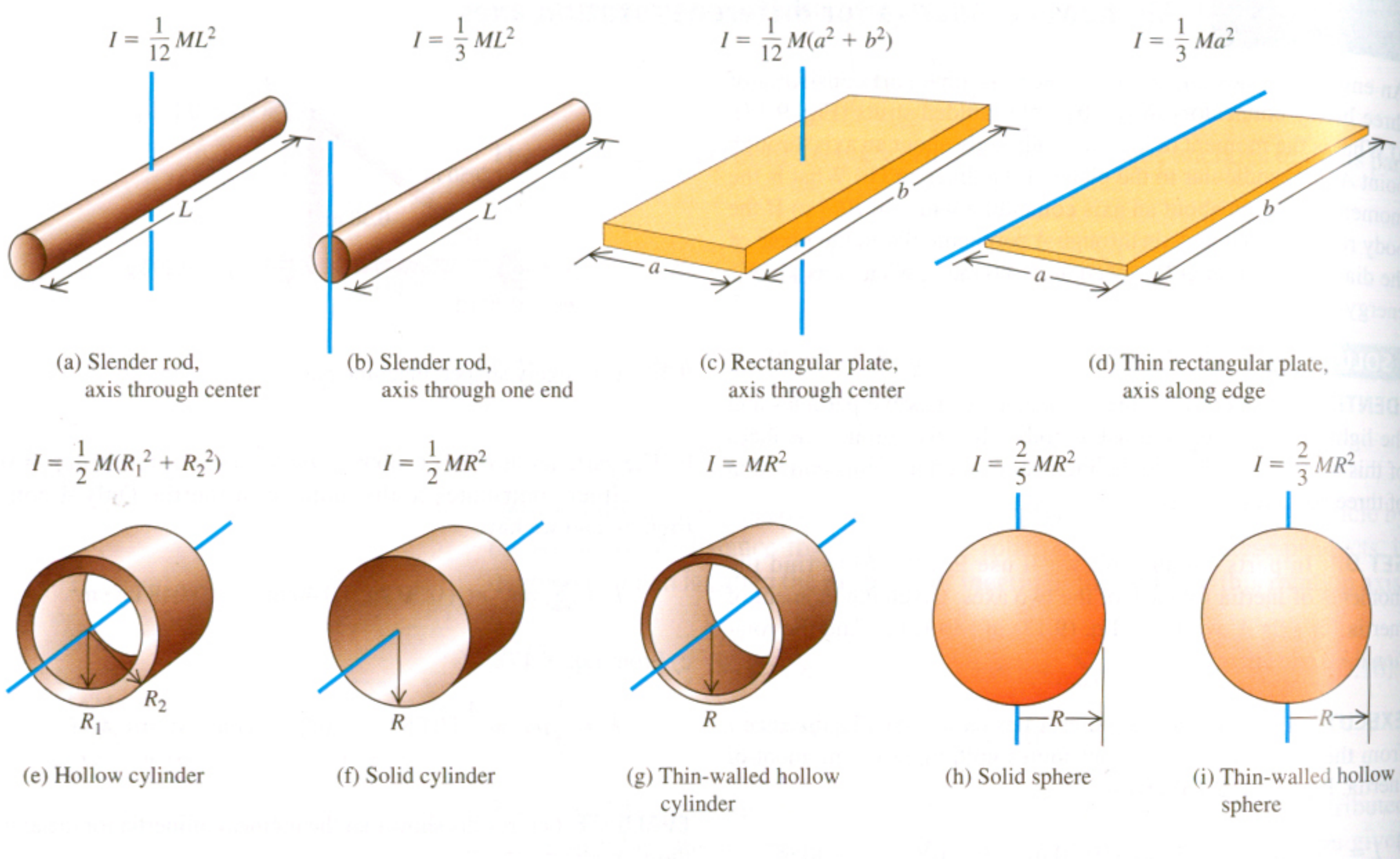


$$\sum \Rightarrow \int$$

$$I = \sum_{i=1}^{N} m_i r_i^2 \quad \Rightarrow \quad I = \int r^2 dm$$

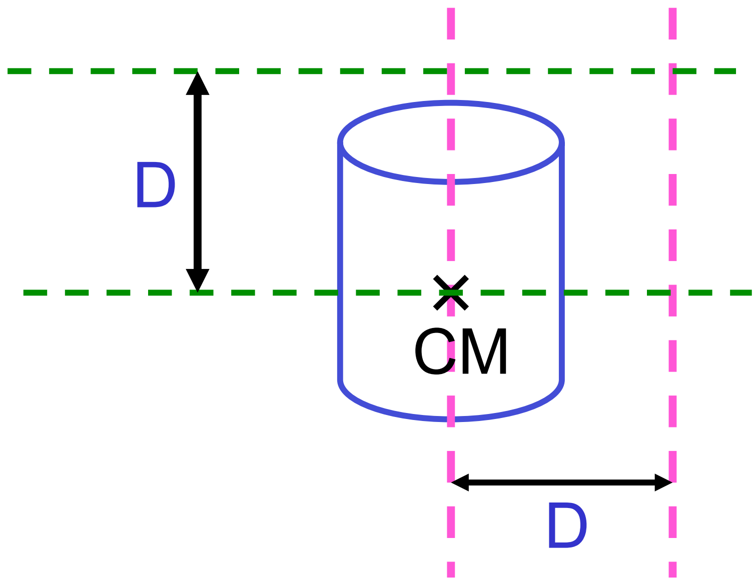
Tabulated Results for Moments of Inertia of some rigid, uniform objects

Table 9.2 Moments of Inertia of Various Bodies



(from p.299 of *University Physics*, Young & Freedman)

Parallel-Axis Theorem



$$I = I_{CM} + MD^2$$

*Smallest I will always be along axis passing through CM

Practical Comments on Calculation of Moment of Inertia for Complex Object

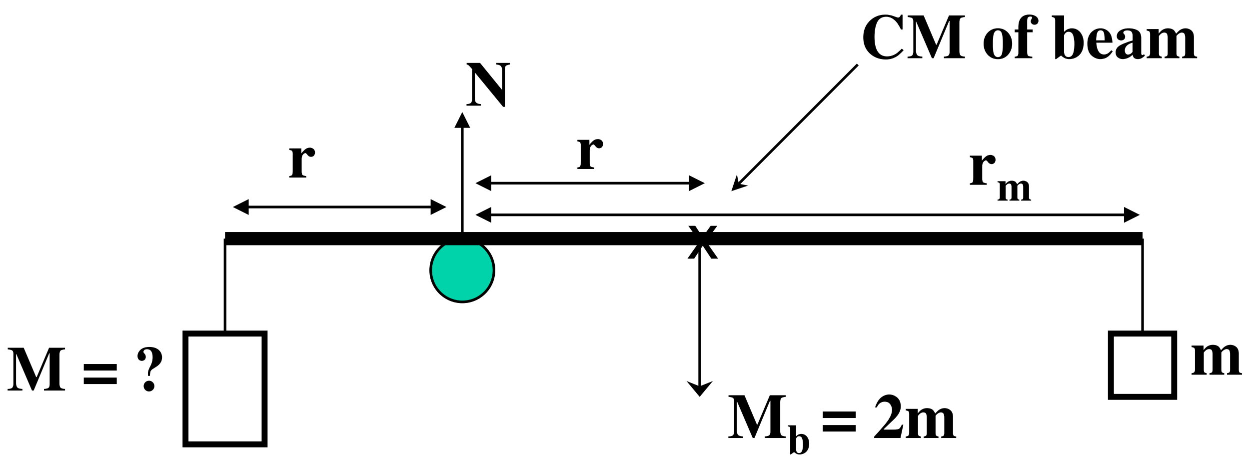
1. To find I for a ***complex*** object, **split** it into ***simple*** geometrical shapes that can be found in Table 9.2

2. Use Table 9.2 to get I_{CM} for each part about the axis ***parallel*** to the axis of rotation and ***going through the center-of-mass***

3. If needed use **parallel-axis theorem** to get I for each part about the axis of rotation

4. **Add** up moments of inertia of all parts

Beam resting on pivot



Vertical equilibrium? $\sum F =$

Rotational equilibrium? $\sum \tau =$

$M =$

$N =$

Suppose M replaced by M/2 ?

- vertical equilibrium? $\sum F =$
- rotational dynamics?

$\sum \tau =$

- net torque?
- which way rotates?
- initial angular acceleration?

Moment of Inertia?

$\mathbf{I = \Sigma m_i r_i^2}$

* depends on pivot position!

$I =$

* Hence $\alpha = \tau / I =$

Rotational Kinetic Energy

$$K = \Sigma_i (1/2) m_i v_i^2 = (1/2)\omega^2 \Sigma_i m_i r_i^2$$

- Hence

$$K = (1/2) I \omega^2$$

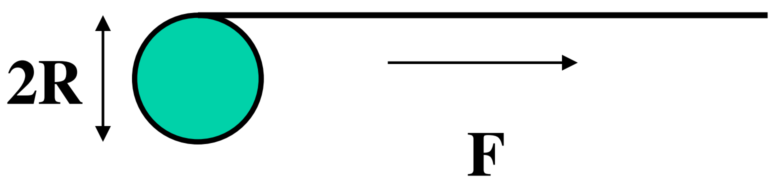
- This is the energy that a rigid body possesses by virtue of rotation

13-1.4: Two spheres of equal radius, one a shell of mass m_1, the other a solid sphere of mass $m_2 > m_1$, race down an incline. Which one wins?

- 1. The solid sphere
- 2. The spherical shell
- 3. They tie
- 4. need more information

The great race

Spinning a cylinder



Cable wrapped around cylinder. Pull off with constant force F. Suppose unwind a distance d of cable

- What is final angular speed of cylinder?

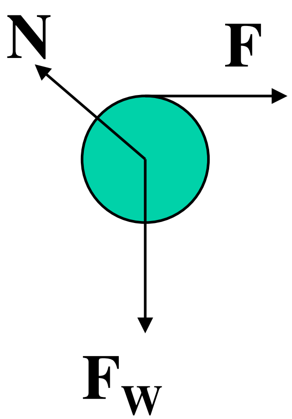
- Use work-KE theorem

$$W = Fd = K_f = (1/2)I\omega^2$$

- Mom. of inertia of cyl.? -- from table: $(1/2)mR^2$
  - from table: $(1/2)mR^2$

$$\omega = [2Fd/(mR^2/2)]^{1/2} = [4Fd/(mR^2)]^{1/2}$$

cylinder+cable problem -- constant acceleration method



radius R

extended free body diagram

* no torque due to N or F_W

* why direction of N ?

* torque due to $\tau = FR$

* hence $\alpha = FR/[(1/2)MR^2]$
$= 2F/(MR)$

$\Delta\theta = (1/2)\alpha t^2 = d/R$; $t = [(MR/F)(d/R)]^{1/2}$

$\rightarrow \omega = \alpha t = [4Fd/(MR^2)]^{1/2}$

Reading assignment

- Rotational dynamics, Angular momentum
- Remainder of Ch. 12 in textbook