Welcome back to Physics 211

Today's agenda:

- *Moment of Inertia*
- *Angular momentum*

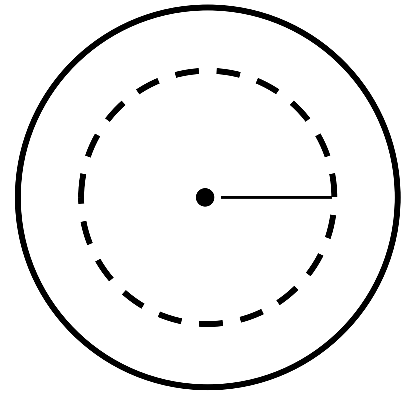
Current assignments

- Prelecture due Tuesday after Thanksgiving
- HW#13 due next Wednesday, 11/24

  Turn in written assignment to TA on 12/3

Relating linear and angular kinematics

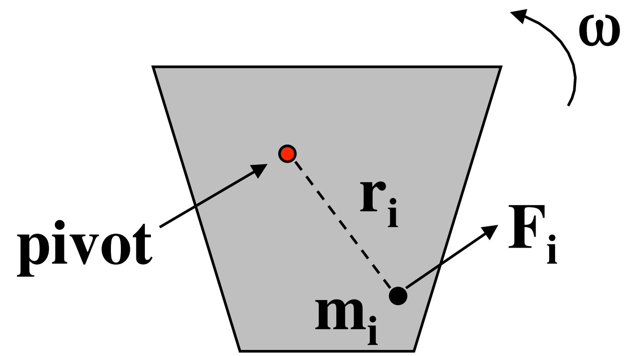
- Linear speed: $v = (2\pi r)/T = \omega r$



- Tangential acceleration: $a_{tan} = r\alpha$

- Radial acceleration: $a_{rad} = v^2/r = \omega^2 r$

Rotational Motion



* Particle *i*:

  $|v_i| = r_i\,\omega$ at 90° to r_i

* Newton's 2nd law:

  $m_i \Delta v_i / \Delta t = F_i^T$ ← component at 90° to r_i

* Substitute for v_i and multiply by r_i :

  $m_i r_i^2 \Delta\omega / \Delta t = F_i^T r_i = \tau_i$

* Finally, sum over all masses:

  $(\Delta\omega/\Delta t) \sum m_i r_i^2 = \sum \tau_i = \tau_{net}$

Discussion

$$(\Delta\omega/\Delta t) \sum m_i r_i^2 = \tau_{net}$$

α - angular acceleration

Moment of inertia, I

$$I\alpha = \tau_{net}$$

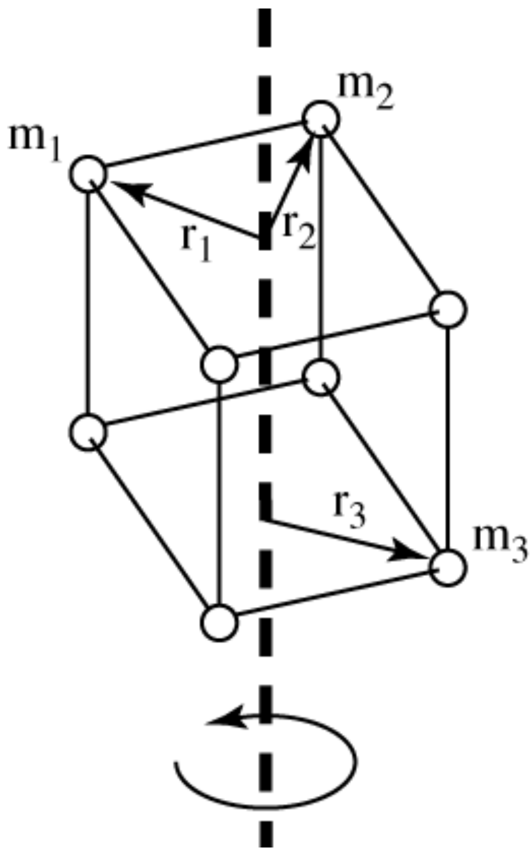
compare this with Newton's 2nd law

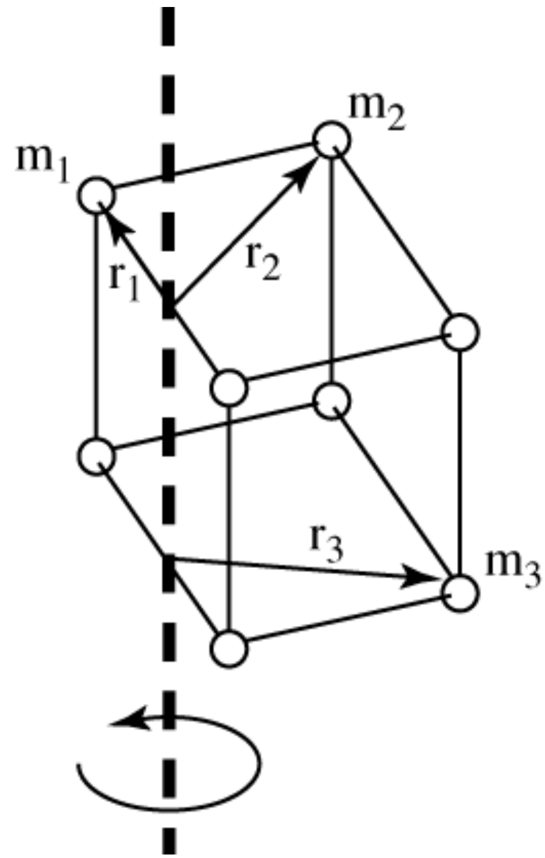
$$Ma = F$$

Moment of Inertia

$$I = m_1 r_1^2 + m_2 r_2^2 + \cdots + m_N r_N^2 = \sum_{i=1}^{N} m_i r_i^2$$

* I must be defined with respect to a particular axis

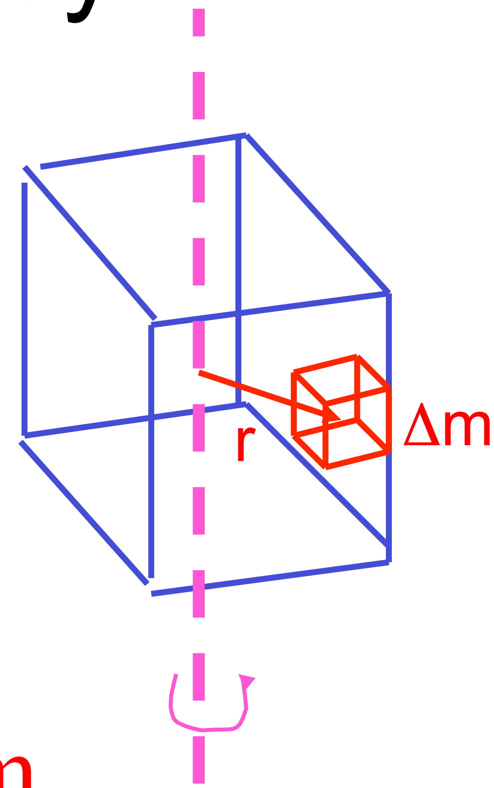




Moment of Inertia of Continuous Body

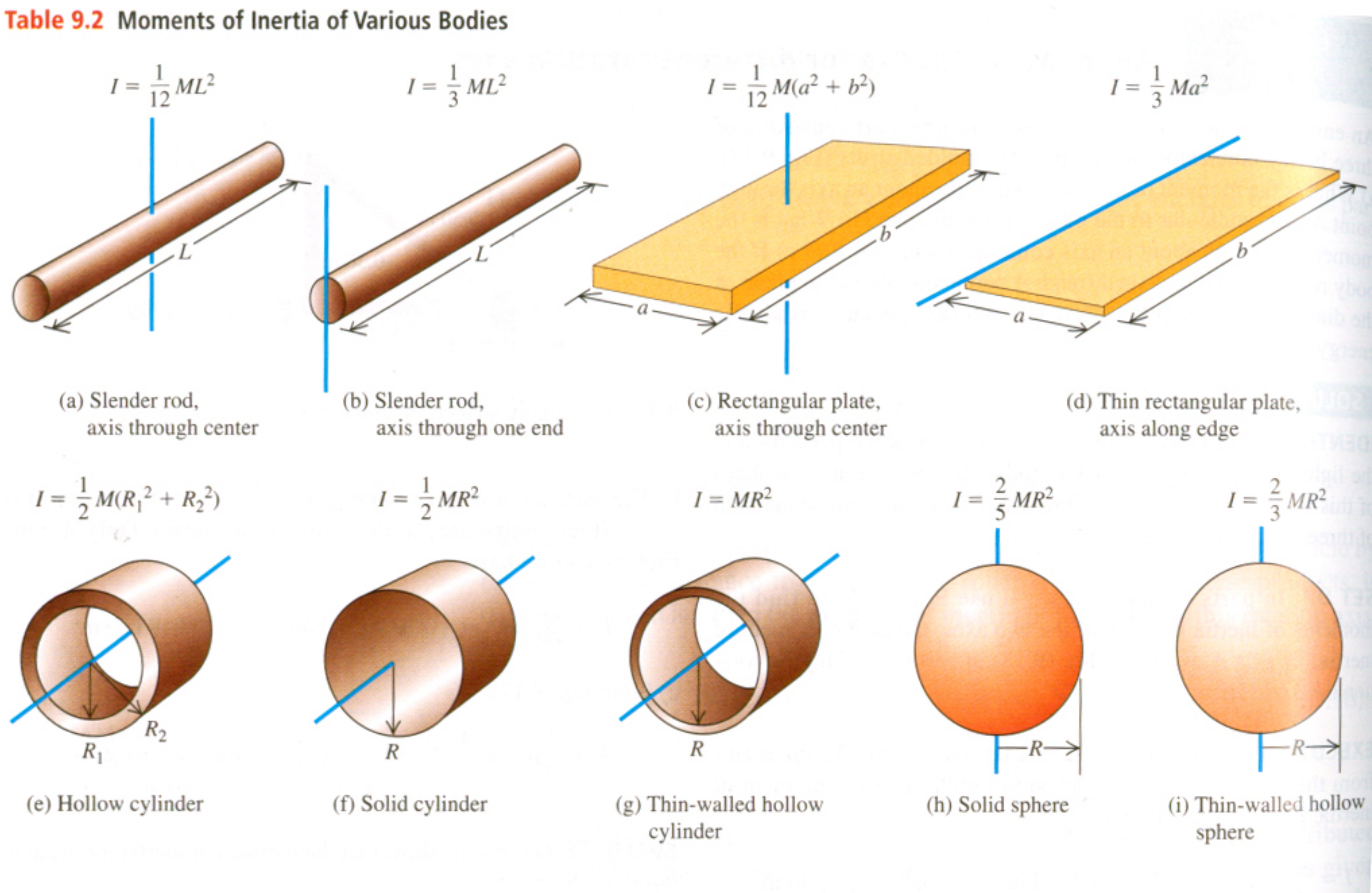
$\Delta m \mapsto 0$

$$\sum \Rightarrow \int$$



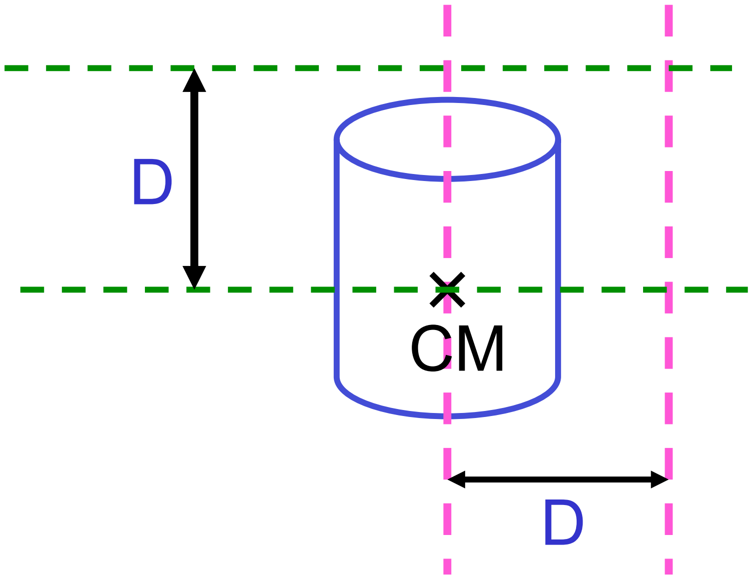
$$I = \sum_{i=1}^{N} m_i r_i^2 \Rightarrow I = \int r^2 dm$$

Tabulated Results for Moments of Inertia of some rigid, uniform objects



(from p.299 of *University Physics*, Young & Freedman)

Parallel-Axis Theorem



$$I = I_{CM} + M\,D^2$$

*Smallest *I* will always be along axis passing through CM

Practical Comments on Calculation of Moment of Inertia for Complex Object

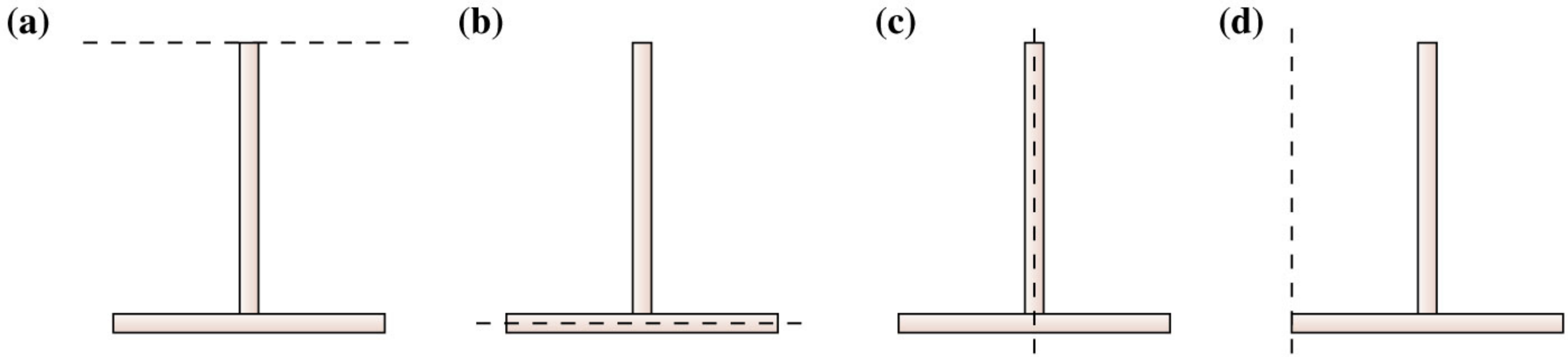
1. To find I for a ***complex*** object, **split** it into ***simple*** geometrical shapes that can be found in Table 9.2

2. Use Table 9.2 to get I_{CM} for each part about the axis ***parallel*** to the axis of rotation and ***going through the center-of-mass***

3. If needed use **parallel-axis theorem** to get I for each part about the axis of rotation

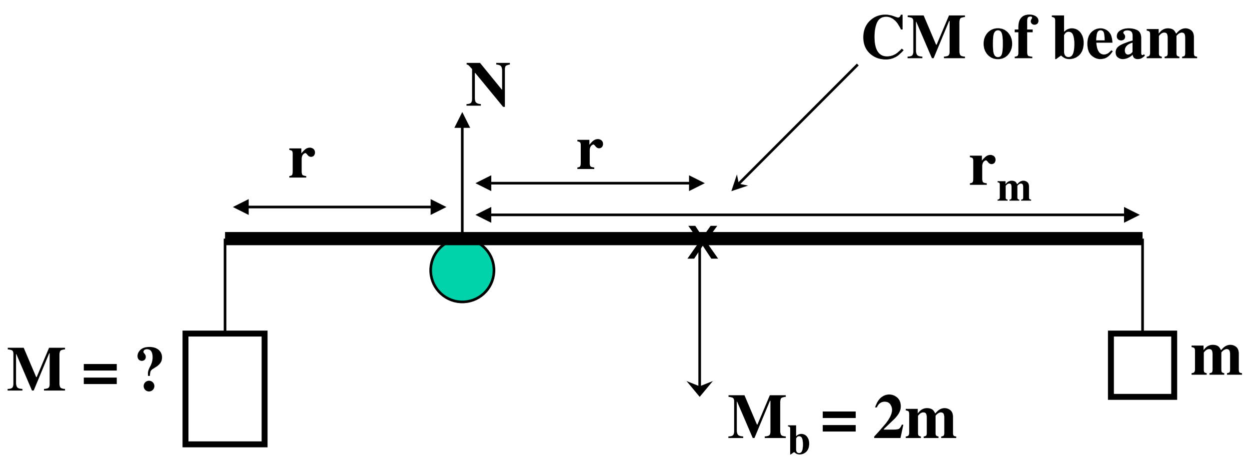
4. **Add** up moments of inertia of all parts

13-2.1: The T's are made of two identical rods of equal mass and length. Rank in order, from smallest to largest, the moments of inertia for rotation about the dashed line?



- 1. c, b, d, a
- 2. c, b, a, d
- 3. b, c, d, a
- 4. c, d, b, a

Beam resting on pivot



Vertical equilibrium? $\sum F =$

Rotational equilibrium? $\sum \tau =$

M =

N =

Suppose M replaced by M/2 ?

- vertical equilibrium? $\sum F =$
- rotational dynamics?

$\sum \tau =$

- net torque?
- which way rotates?
- initial angular acceleration?

Moment of Inertia?

$I = \Sigma m_i r_i^2$

* depends on pivot position!

$I =$

* Hence $\alpha = \tau / I =$

Rotational Kinetic Energy

$$K = \Sigma_i (1/2) m_i v_i^2 = (1/2)\omega^2 \Sigma_i m_i r_i^2$$

- Hence

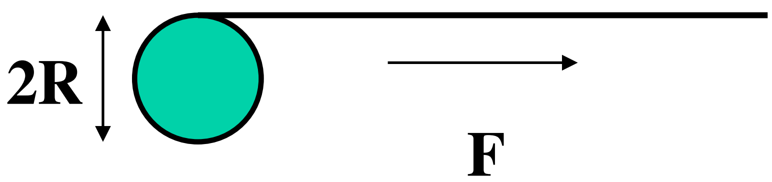
$$K = (1/2) I \omega^2$$

- This is the energy that a rigid body possesses by virtue of rotation

13-2.2: Two spheres of equal radius, one a shell of mass m_1, the other a solid sphere of mass $m_2 > m_1$, race down an incline. Which one wins?

- 1. The solid sphere
- 2. The spherical shell
- 3. They tie
- 4. need more information

Sample problem: Spinning a cylinder



Cable wrapped around cylinder. Pull off with constant force F. Suppose unwind a distance d of cable

- What is final angular speed of cylinder?

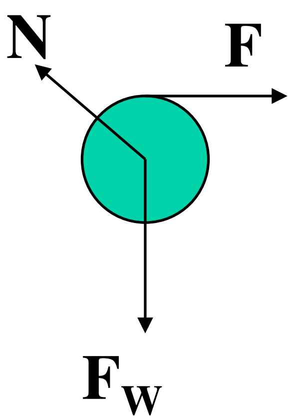
- Use work-KE theorem

$$W = Fd = K_f = (1/2)I\omega^2$$

- Mom. of inertia of cyl.? -- from table: $(1/2)mR^2$
  - from table: $(1/2)mR^2$

$$\omega = [2Fd/(mR^2/2)]^{1/2} = [4Fd/(mR^2)]^{1/2}$$

cylinder+cable problem -- constant acceleration method



radius R

extended free body diagram

* no torque due to N or F_W
* why direction of N ?
* torque due to τ = FR
* hence $\alpha = FR/[(1/2)MR^2]$
  $= 2F/(MR)$

$$\Delta\theta = (1/2)\alpha t^2 = d/R;\ t = [(MR/F)(d/R)]^{1/2}$$

$$\rightarrow \omega = \alpha t = [4Fd/(MR^2)]^{1/2}$$

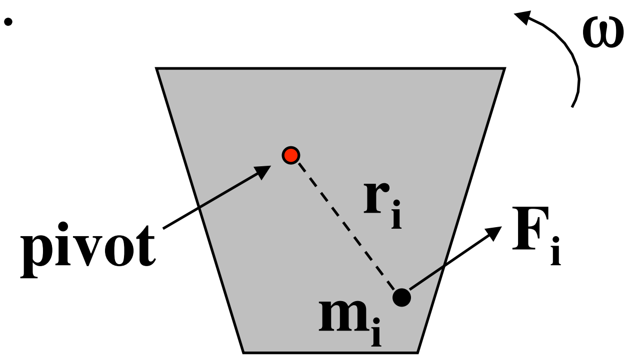
Angular Momentum

- can define rotational analog of linear momentum called ***angular momentum***
- in absence of ***external torque*** it will be conserved in time
- True even in situations where Newton's laws fail ….

Definition of Angular Momentum

* Back to slide on rotational dynamics:

$$m_i r_i^2 \Delta\omega/\Delta t = \tau_i$$



* Rewrite, using $l_i = m_i r_i^2 \omega$:

$$\Delta l_i/\Delta t = \tau_i$$

* Summing over all particles in body:

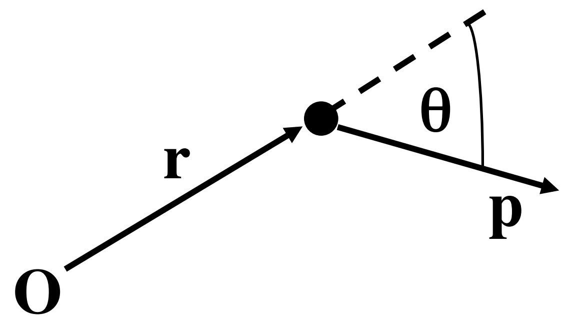
$$\Delta L/\Delta t = \tau_{ext}$$

L = angular momentum = $I\omega$

13-2.3: An ice skater spins about a vertical axis through her body with her arms held out. As she draws her arms in, her angular velocity

- 1. increases
- 2. decreases
- 3. remains the same
- 4. need more information

Angular Momentum 1.



Point particle:

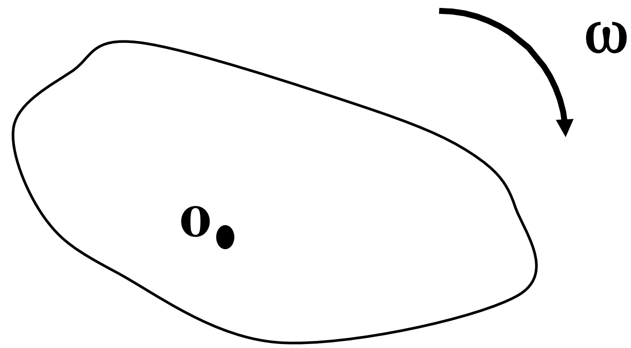
$|L| = |r||p|\sin(\theta) = m|r||v|\sin(\theta)$

vector form → **L** = **r** x **p**

– direction of L given by right hand rule (into paper here)

L = mvr if v is at 90^0 to r for single particle

Angular Momentum 2.



rigid body:

* $|L| = I\omega$ (fixed axis of rotation)
* direction – along axis – into paper here

Rotational Dynamics

$$\boldsymbol{\tau} = I\boldsymbol{\alpha}$$

$$\Delta\mathbf{L}/\Delta t = \boldsymbol{\tau}$$

- These are equivalent statements

- If no net external torque: $\tau = 0$ →
  - * **L** is constant in time
  - * *Conservation of Angular Momentum*
  - * Internal forces/torques do not contribute to external torque.

Linear and rotational motion

- Force
- Acceleration

$$\vec{F}_{\text{net}} = \sum \vec{F} = m\vec{a}$$

- Momentum

$$\vec{p} = m\vec{v}$$

- Kinetic energy

$$K = \tfrac{1}{2} m v^2$$

- Torque
- Angular acceleration

$$\vec{\tau}_{\text{net}} = \sum \vec{\tau} = I\vec{\alpha}$$

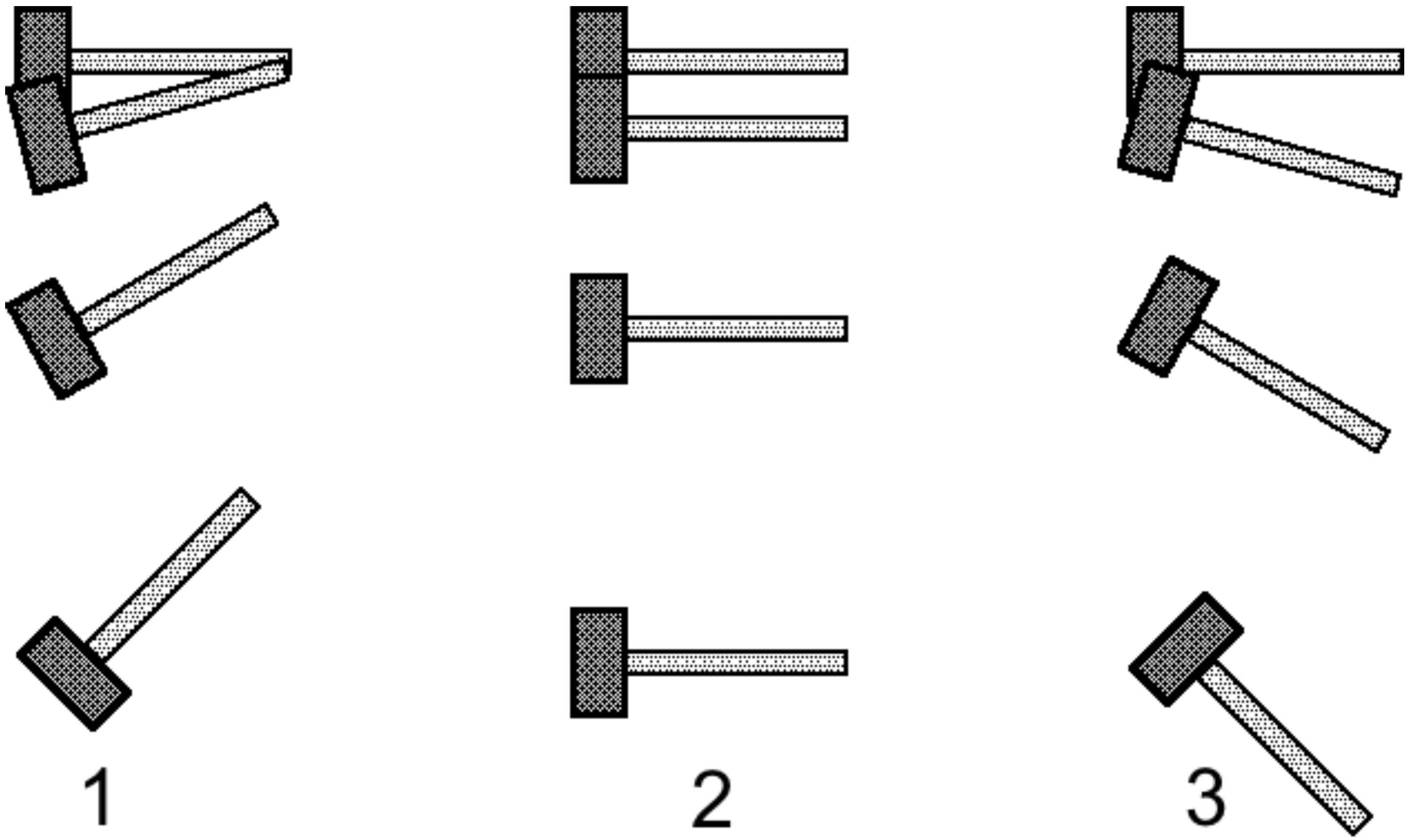
- Angular momentum**

$$\vec{L} = I\vec{\omega}$$

- Kinetic energy

$$K = \tfrac{1}{2} I\omega^2$$

A hammer is held horizontally and then released. Which way will it fall?



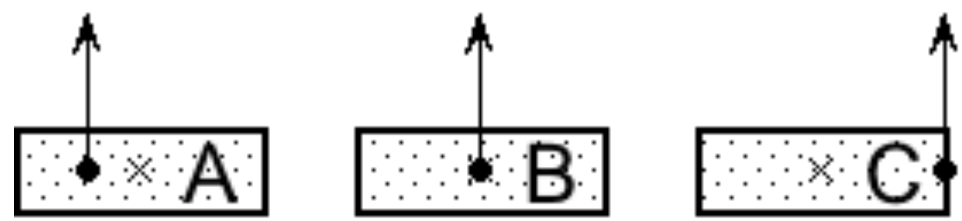
General motion of extended objects

- Net force ➔ acceleration of CM
- Net torque about CM ➔ angular acceleration (rotation) about CM
- Resultant motion is superposition of these two motions
- Total kinetic energy $K = K_{CM} + K_{rot}$

Three identical rectangular blocks are at rest on a flat, frictionless table. The same force is exerted on each of the three blocks for a very short time interval. The force is exerted at a different point on each block, as shown.

After the force has stopped acting on each block, which block will spin the fastest?

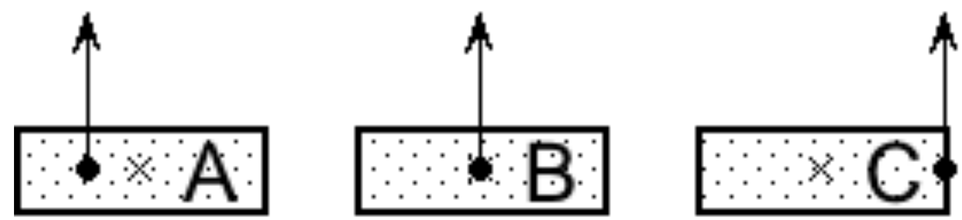
1. A.
2. B.
3. C.
4. A and C.



Top-view diagram

Three identical rectangular blocks are at rest on a flat, frictionless table. The same force is exerted on each of the three blocks for a very short time interval. The force is exerted at a different point on each block, as shown.

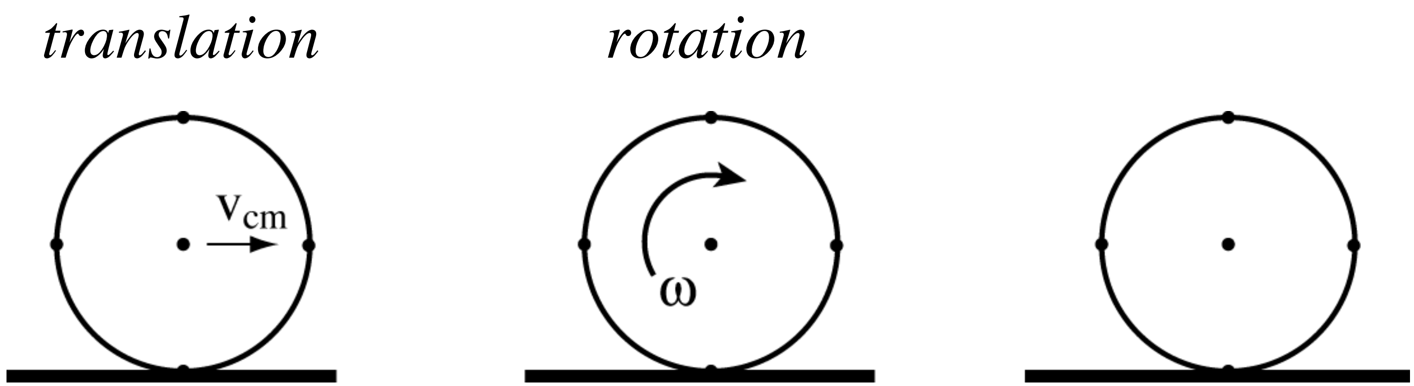
After each force has stopped acting, which block's center of mass will have the greatest speed?



Top-view diagram

1. A.
2. B.
3. C.
4. A, B, and C have the same C.O.M. speed.

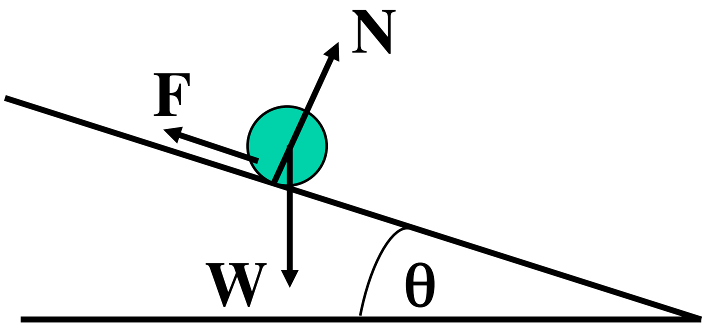
Rolling without slipping



$v_{cm} =$

$a_{cm} =$

Rolling without slipping



$\sum F = ma_{CM}$

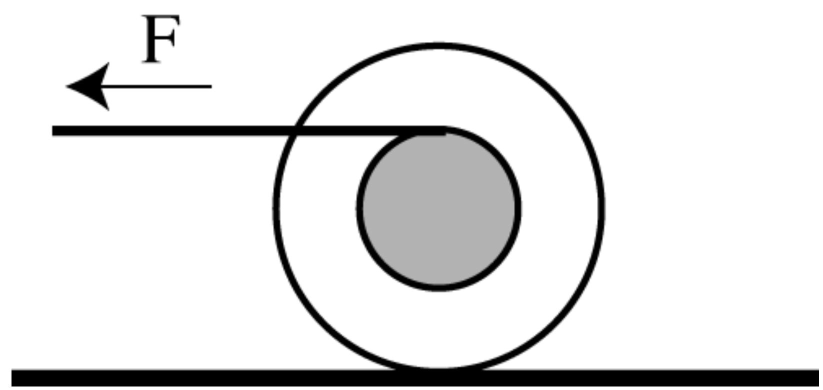
$\sum \tau = I\alpha$

Now $a_{CM} = R\alpha$ if no slipping

So, $m a_{CM}$

and F =

A ribbon is wound up on a spool. A person pulls the ribbon as shown.



Will the spool move to the left, to the right, or will it not move at all?

1. The spool will move to the left.
2. The spool will move to the right.
3. The spool will not move at all.

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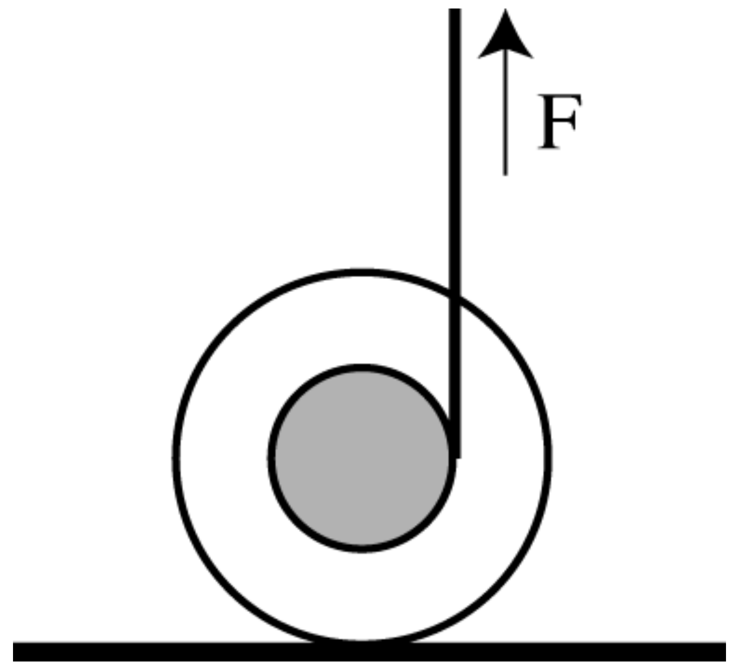

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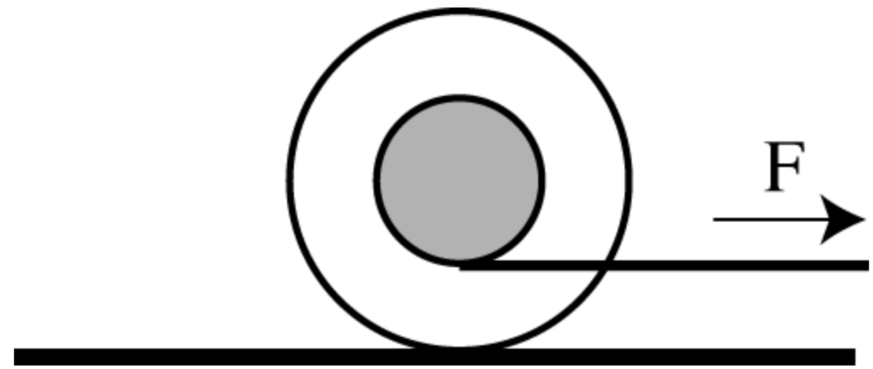

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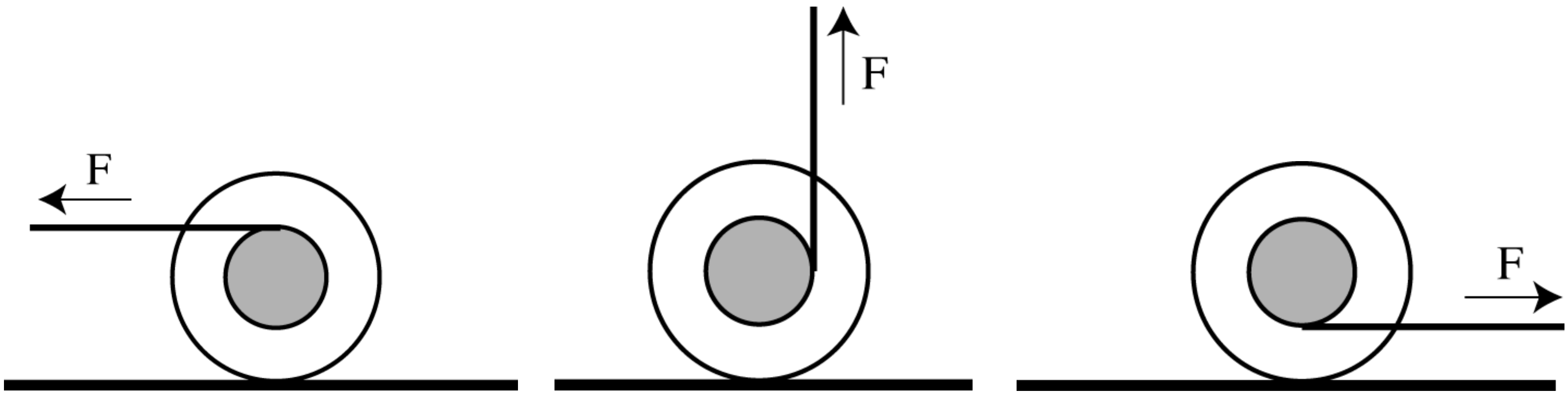
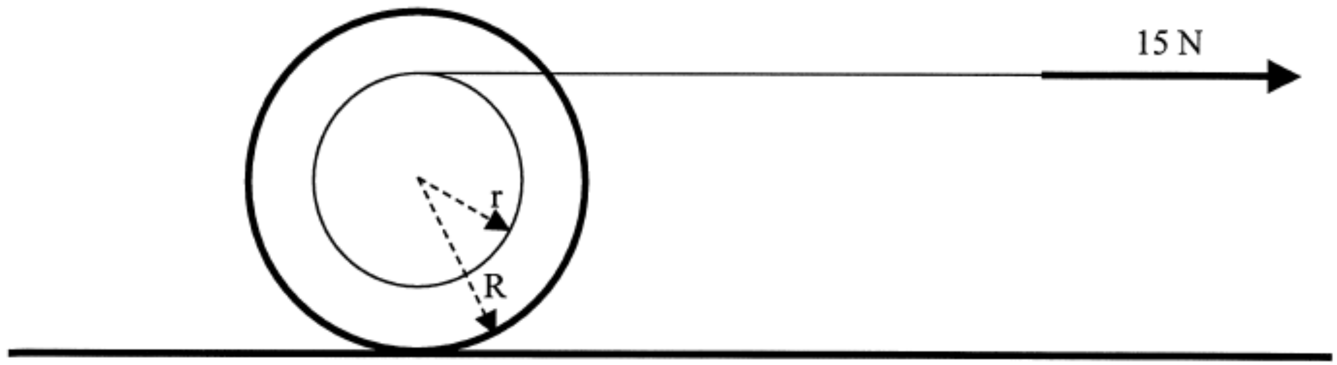

F
F
F

4. [30pts total + 5 bonus pts] A string is wound around the spool. The spool has a mass of M=7 kg, an outer radius of R=0.6 m and an inner radius of r=0.4 m. Moment of inertia with respect to the axis going through the center of mass is I_{cm}=0.8 kg m^2. Somebody is pooling on the string in horizontal direction with a force of 15 N. (Parts a,b,c of this problem are independent of each other).



4a. [20pts] Find linear acceleration of the center-of-mass of the spool (a_{cm}) rolling without slipping.

4b. [10pts] What is the kinetic energy of the spool rolling without slipping if velocity of the center-of-mass is v_{cm}=3 m/s?

4c. [bonus 5pts] Find linear acceleration of the center-of-mass of the spool (a_{cm}) if it is rolling with slipping and the coefficient of kinetic friction between the spool and the ground is μ_k=0.15 (use g=10 m/s^2).