PHY 211 – Final (Version 1)

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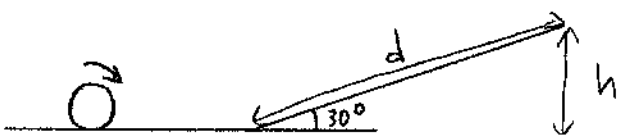
It is very important that you print your name at the top of the exam page. Please do it before you read any questions!

Document your work. Use the back of each sheet if you run out of space.

1.[25 pts total] A hollow sphere of mass 0.2 kg rolls along a horizontal floor without slipping and then up a 30° incline. It is rolling along the horizontal surface with speed 3 m/s. (The moment of inertia of a hollow sphere about an axis through its center is I = 2/3 m r^2.)



a. [5 pts] What is the total kinetic energy of the sphere when it is rolling along the horizontal surface?

$K = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$, for rolling without slipping, $v = \omega r$

Thus, $K = \frac{1}{2}mv^2 + \frac{1}{2}I\frac{v^2}{r^2}$, $I = \frac{2}{3}mr^2$

so $K = \frac{1}{2}mv^2 + \frac{1}{\cancel{2}} \times \frac{\cancel{2}}{3} m \cancel{r^2} \frac{v^2}{\cancel{r^2}} = \frac{1}{2}mv^2 + \frac{1}{3}mv^2 = \frac{5}{6}mv^2$

$\Rightarrow K = \frac{5}{6} 0.2 \times 3^2 = \boxed{1.5\ J}$

b. [6 pts] How far up the ramp does the sphere go before starting to roll back down?

$E_i = E_f$

$K = mgh$ $\quad h = \frac{K}{mg}$, $d \sin\theta = h \Rightarrow d = \frac{h}{\sin\theta}$

$\Rightarrow d = \frac{K}{mg\sin\theta} = \frac{1.5}{0.2 \times 9.8 \times \sin 30°} = \boxed{1.53\ m}$

c. [4 pts] If we doubled the radius of the ball, how would the distance that the sphere goes up the ramp change? Explain.

$$d = \frac{K}{mg\sin\theta} = \frac{\frac{5}{6}\cancel{m}v^2}{\cancel{m}g\sin\theta}$$ is independent of r, so d is unchanged.

d. [3 pts] Now the ramp and floor are greased so that we can ignore friction. A block of mass 0.2 kg slides along the horizontal surface with the same initial speed as the sphere, 3 m/s. What is the total kinetic energy of the block as it slides along the horizontal surface?

$$K = \frac{1}{2}mv^2 = \frac{1}{2}0.2 \times 3^2 = \boxed{0.9\ \text{J}}$$

e. [4 pts] How far up the ramp does the block go?

$$E_i = E_f$$

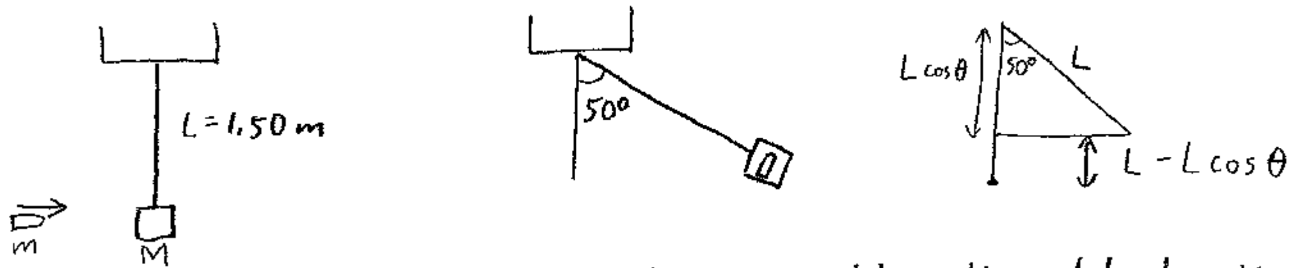
$$K = mgh \Rightarrow h = \frac{K}{mg},\quad d = \frac{K}{mg\sin\theta}$$

$$= \frac{\frac{1}{2}\cancel{m}v^2}{\cancel{m}g\sin\theta} = \frac{\frac{1}{2}\times 3^2}{9.8\times\sin 30°} = \boxed{0.918\ \text{m}}$$

f. [3 pts] Which goes higher up the ramp, the sphere or the block? Explain.

The sphere goes higher up the ramp. In both cases all of the kinetic energy is transformed into gravitational potential energy and the sphere starts out with more kinetic energy since, although they have the same speed and mass, the sphere is also rotating.

2. [25 pts total] A 12 g bullet is fired into a 1400 g wood block that is hanging from a string of length 1.50 m. The bullet embeds itself into the block, and the block swings out to an angle of 50°. What was the speed of the bullet?



During the impact of the bullet with the block, the momentum of this subsystem is conserved. Thus, $p_i = p_f$

$$m v_0 = (m + M) v_1$$

Mechanical energy is not conserved during this collision, since it is perfectly inelastic. However, mechanical energy is conserved as the pendulum rises. Thus,

$$E_i = E_f$$

$$\frac{1}{2}\cancel{(m+M)} v_1^2 = \cancel{(m+M)} g L (1 - \cos\theta)$$

$$v_1 = \sqrt{2gL(1-\cos\theta)}$$

$$\Rightarrow v_0 = \frac{m+M}{m} v_1 = \frac{m+M}{m}\sqrt{2gL(1-\cos\theta)}$$

$$\Rightarrow v_0 = \frac{0.012 + 1.4}{0.012}\sqrt{2 \times 9.8 \times 1.50 \times (1 - \cos 50°)}$$

$$= \boxed{381 \ \frac{m}{s}}$$

e. [3pts] What is the total instantaneous acceleration of the end of the rod where the rope was attached at the instant after the rope breaks?

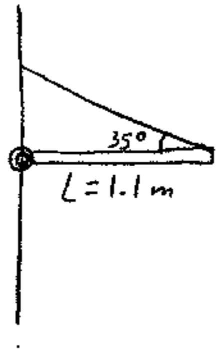
$$a = \alpha r \Rightarrow a = \alpha L = -13.4 \tfrac{rad}{s^2} \times 1.1\ m = \boxed{-14.7 \tfrac{m}{s^2}}$$

$\boxed{\text{straight down}}$

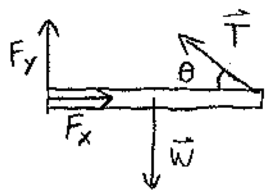
f. [3pts] What is the total instantaneous acceleration of the end of the rod at the pivot at the instant after the rope breaks?

$$a = \alpha r, \text{ but here, } r = 0, \text{ so } \boxed{a = 0}$$

3. [25 pts total] A 3.0 kg rod of length 1.1 m is attached to a pivot and held in place by a rope attached to one of its ends as shown in the diagram. The angle between the rod and the rope is 35°.



a. [3pts] Draw the extended free-body diagram for the rod.



b. [4pts] What is the tension in the rope?

$$\Sigma \vec{F} = 0 \; , \quad \Sigma \vec{\tau} = 0 \Rightarrow -mg\frac{L}{2} + TL\sin\theta = 0$$

$$T\sin\theta + F_y - mg = 0 \qquad T\cancel{L}\sin\theta = mg\frac{\cancel{L}}{2}$$

$$-T\cos\theta + F_x = 0 \qquad T = \frac{mg}{2\sin\theta} = \frac{3 \times 9.8}{2\sin 35^\circ} = 25.6\,N$$

c. [6pts] What are the x and y components of the force acting at the hinge?

$$F_x = T\cos\theta = 25.6\cos 35^\circ = \boxed{21.0\ N}$$

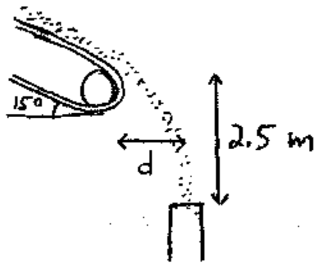
$$F_y = mg - T\sin\theta = 3 \times 9.8 - 25.6\sin 35^\circ = \boxed{14.7\ N}$$

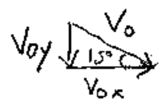
d. [6pts] The rope breaks. At that instant, what is the instantaneous angular acceleration of the rod about the pivot? (The moment of inertia of a thin rod about its center is 1/12 m L^2. The moment of inertia of a thin rod about its end is 1/3 m L^2.)

$$\Sigma \tau = I\alpha \; , \quad I = \frac{1}{3}mL^2 \; , \quad \tau = -mg\frac{L}{2}$$

$$\alpha = \frac{\tau}{I} = \frac{-3\cancel{m}g\cancel{L}}{2\cancel{m}L^{\cancel{2}}} = -\frac{3g}{2L} = \frac{-3 \times 9.8\frac{m}{s^2}}{2 \times 1.1\,m} = \boxed{-13.4\ \frac{rad}{s^2}}$$

4. [25 pts total] Sand moves without slipping at 5.0 m/s down a conveyer that is tilted at 15°. The sand enters a pipe that is 2.5 m below the end of the conveyer belt, as shown in the figure.





a. [5 pts] What is the speed of the sand as it enters the pipe?

$$E_i = E_f$$

$$\frac{1}{2}\cancel{m}V_0^2 + \cancel{m}gh = \frac{1}{2}\cancel{m}V_f^2$$

$$V_f^2 = V_0^2 + 2gh \Rightarrow V_f = \sqrt{V_0^2 + 2gh}$$

$$V_f = \sqrt{5.0^2 + 2 \times 9.8 \times 2.5} = \boxed{8.60\ \frac{m}{s}}$$

b. [10 pts] What is the horizontal distance between the conveyer belt and the pipe?

$$|V_{ox}| = V_0 \cos\theta \quad , \quad |V_{oy}| = V_0 \sin\theta$$

$$= 5\tfrac{m}{s}\cos 15° = 4.83\ \tfrac{m}{s} \qquad = 5\tfrac{m}{s}\sin 15° = 1.29\ \tfrac{m}{s}$$

$$y - y_0 = -h = -V_{oy}t - \frac{1}{2}gt^2 \Rightarrow -\frac{1}{2}gt^2 - V_{oy}t + h = 0$$

$$\Rightarrow t = \frac{V_{oy} \pm \sqrt{V_{oy}^2 + 2gh}}{-g} = \frac{-1.29 \mp \sqrt{1.29^2 + 2\cdot 9.8\cdot 2.5}}{9.8}$$

Take positive time

$$\Rightarrow t = -0.858\,s,\ 0.595\,s \qquad d = x = V_{ox}t = 4.83 \times 0.595 = \boxed{2.87\ m}$$

c. [10 pts] What is the horizontal distance if we assume that the conveyer belt is not on Earth but on Mars? (Mass of Mars=6.42 x 10^{23} kg, mean radius of Mars=3.37 x 10^6 m, Mean distance between Mars and sun=2.28 x 10^{11} m.)

$$G = 6.67 \times 10^{-11}\ N\,m^2/kg^2$$

On mars, $$\cancel{m}g_m = G\frac{M_m\cancel{m}}{r_m^2} \Rightarrow g_m = 6.67\times10^{-11}\,\frac{6.42\times10^{23}}{(3.37\times10^6)^2}$$

$$= 3.77\ \frac{m}{s^2}$$

Thus

$$t = \frac{V_{oy} \pm \sqrt{V_{oy}^2 + 2g_m h}}{-g_m} = \frac{-1.29 \mp \sqrt{1.29^2 + 2\cdot 3.77\cdot 2.5}}{3.77}$$

$$\Rightarrow t = 0.859\,s \qquad \Rightarrow d = x = V_{ox}t = 4.83 \times 0.859 = \boxed{4.15\ m}$$