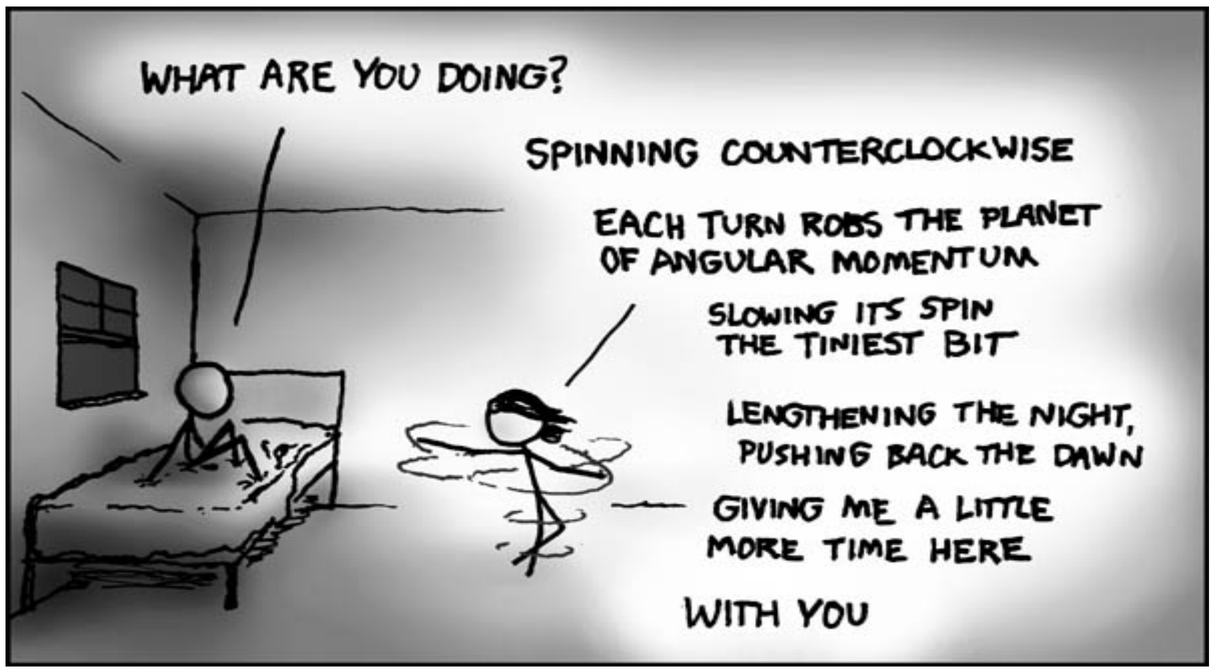
Welcome back to Physics 211

Today's agenda:

- *Rolling without slipping*
- *Angular momentum*

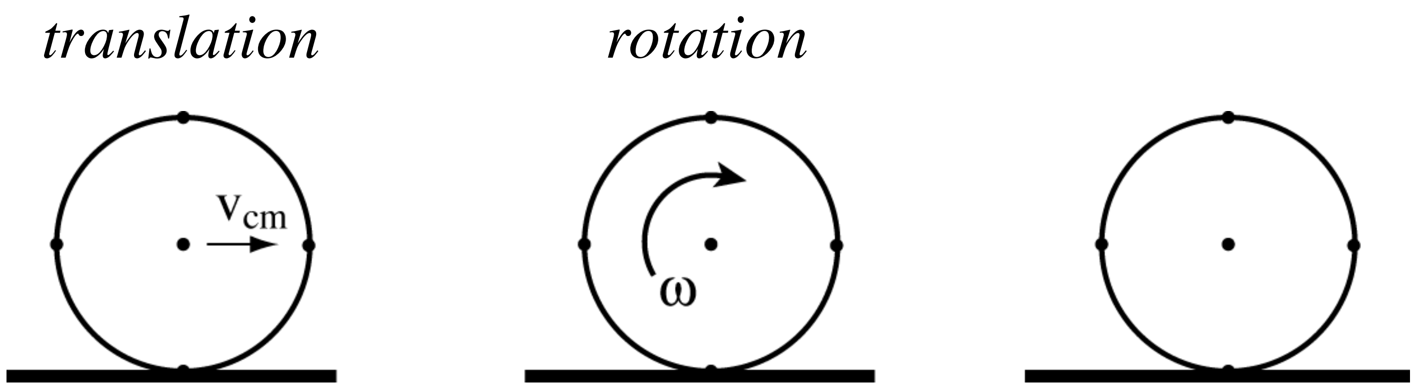


Current assignments

- No Prelecture Thursday
- HW#15 due Friday, 12/5
- Final Exam, Wednesday, Dec. 10th, 3-5pm in Stolkin. One sheet of handwritten notes and a calculator are allowed (as usual).

The great race

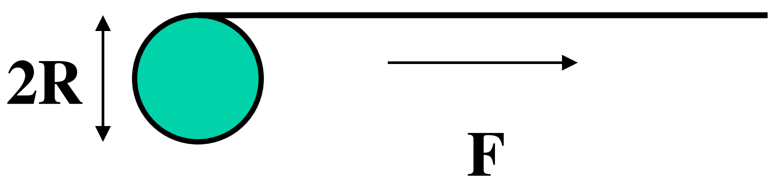
Rolling without slipping



$v_{cm} =$

$a_{cm} =$

Sample problem: Spinning a cylinder



Cable wrapped around cylinder. Pull off with constant force F. Suppose unwind a distance d of cable

- What is final angular speed of cylinder?

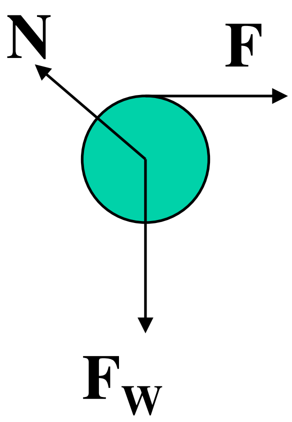
- Use work-KE theorem

$$W = Fd = K_f = (1/2)I\omega^2$$

- Mom. of inertia of cyl.? -- from table: $(1/2)mR^2$
  - from table: $(1/2)mR^2$

$$\omega = [2Fd/(mR^2/2)]^{1/2} = [4Fd/(mR^2)]^{1/2}$$

cylinder+cable problem -- constant acceleration method



radius R

extended free body diagram

* no torque due to N or F_W

* why direction of N ?

* torque due to $\tau = FR$

* hence $\alpha = FR/[(1/2)MR^2]$
$= 2F/(MR)$

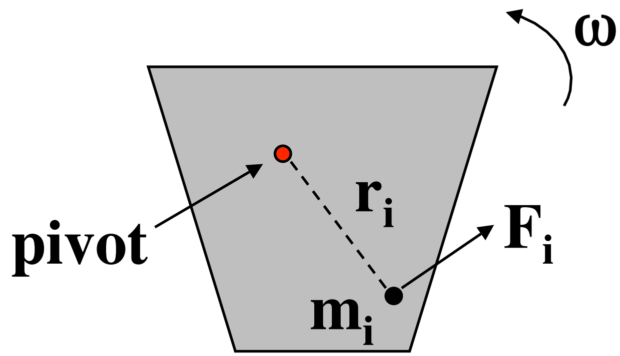
$\Delta\theta = (1/2)\alpha t^2 = d/R;\ t = [(MR/F)(d/R)]^{1/2}$

$\rightarrow \omega = \alpha t = [4Fd/(MR^2)]^{1/2}$

Angular Momentum

- can define rotational analog of linear momentum called ***angular momentum***
- in absence of ***external torque*** it will be conserved in time
- True even in situations where Newton's laws fail ….

Rotational Motion



* Particle *i*:

  $|v_i| = r_i\,\omega$ at 90° to r_i

* Newton's 2nd law:

  $m_i \Delta v_i/\Delta t = F_i^T$ ← component at 90° to r_i

* Substitute for v_i and multiply by r_i :

  $m_i r_i^2 \Delta\omega/\Delta t = F_i^T r_i = \tau_i$

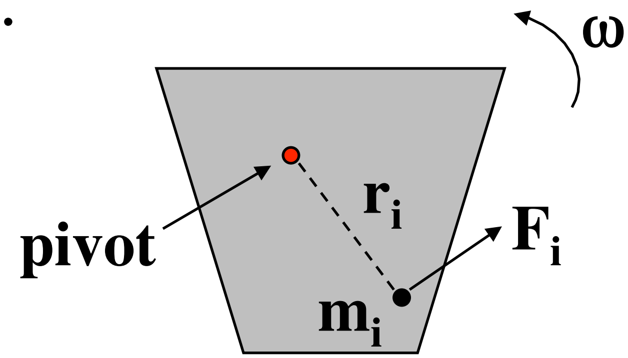
* Finally, sum over all masses:

  $(\Delta\omega/\Delta t) \sum m_i r_i^2 = \sum \tau_i = \tau_{net}$

Definition of Angular Momentum

* Back to slide on rotational dynamics:

$$m_i r_i^2 \Delta\omega/\Delta t = \tau_i$$



* Rewrite, using $l_i = m_i r_i^2 \omega$:

$$\Delta l_i/\Delta t = \tau_i$$

* Summing over all particles in body:

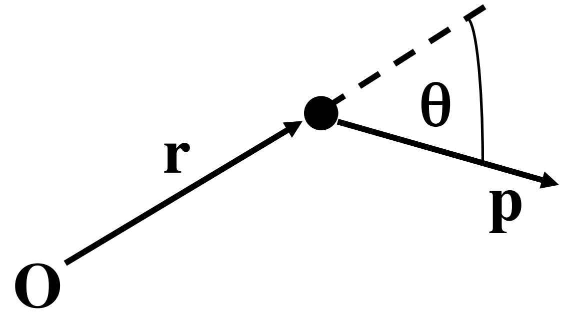
$$\Delta L/\Delta t = \tau_{ext}$$

L = angular momentum = $I\omega$

15-1.1: An ice skater spins about a vertical axis through her body with her arms held out. As she draws her arms in, her angular velocity

- 1. increases
- 2. decreases
- 3. remains the same
- 4. need more information

Angular Momentum 1.



Point particle:

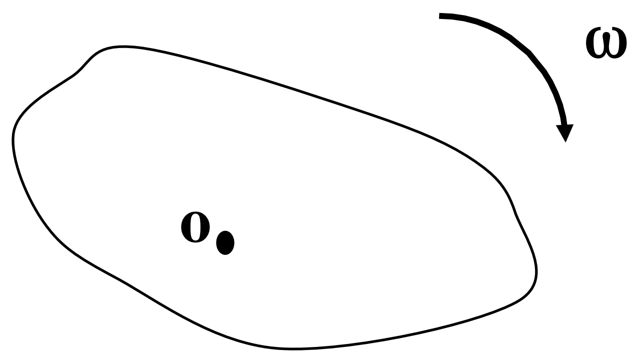
$|L| = |r||p|\sin(\theta) = m|r||v| \sin(\theta)$

vector form → $\mathbf{L} = \mathbf{r} \times \mathbf{p}$

– direction of L given by right hand rule (into paper here)

L = mvr if v is at 90^0 to r for single particle

Angular Momentum 2.



rigid body:

* $|L| = I\omega$ (fixed axis of rotation)
* direction – along axis – into paper here

Rotational Dynamics

$$\boldsymbol{\tau} = I\boldsymbol{\alpha}$$

$$\Delta \mathbf{L}/\Delta t = \boldsymbol{\tau}$$

- These are equivalent statements
- If no net external torque: $\tau = 0 \rightarrow$
  - * **L** is constant in time
  - * *Conservation of Angular Momentum*
  - * Internal forces/torques do not contribute to external torque.

Linear and rotational motion

- Force
- Acceleration

$$\vec{F}_{\text{net}} = \sum \vec{F} = m\vec{a}$$

- Momentum

$$\vec{p} = m\vec{v}$$

- Kinetic energy

$$K = \tfrac{1}{2} m v^2$$

- Torque
- Angular acceleration

$$\vec{\tau}_{\text{net}} = \sum \vec{\tau} = I\vec{\alpha}$$

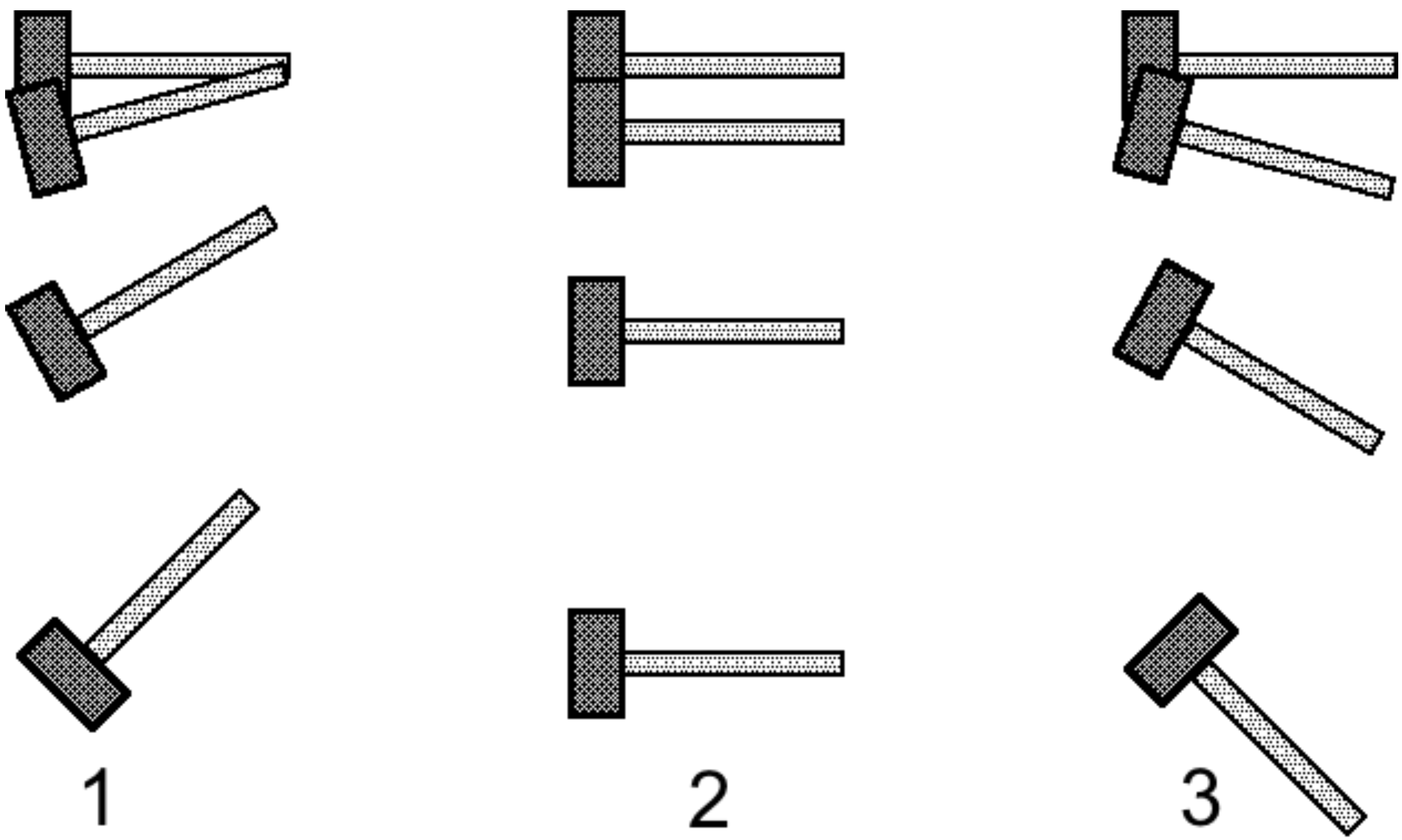
- Angular momentum**

$$\vec{L} = I\vec{\omega}$$

- Kinetic energy

$$K = \tfrac{1}{2} I\omega^2$$

15-1.2: A hammer is held horizontally and then released. Which way will it fall?



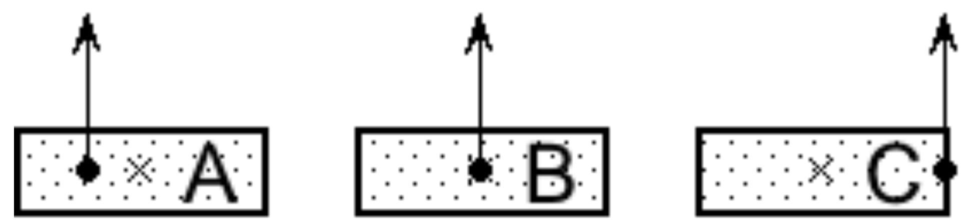
General motion of extended objects

- Net force ➔ acceleration of CM
- Net torque about CM ➔ angular acceleration (rotation) about CM
- Resultant motion is superposition of these two motions
- Total kinetic energy $K = K_{CM} + K_{rot}$

15-1.3: Three identical rectangular blocks are at rest on a flat, frictionless table. The same force is exerted on each of the three blocks for a very short time interval. The force is exerted at a different point on each block, as shown.

After the force has stopped acting on each block, which block will spin the fastest?

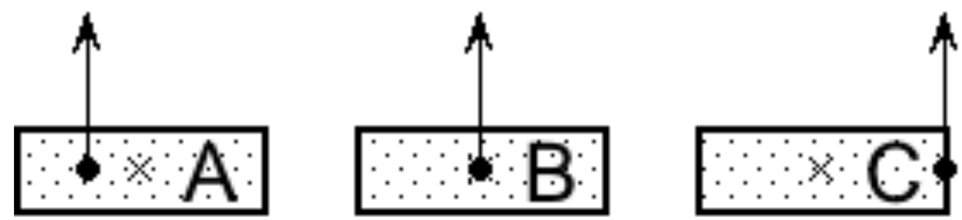
1. A.
2. B.
3. C.
4. A and C.



Top-view diagram

15-1.4: Three identical rectangular blocks are at rest on a flat, frictionless table. The same force is exerted on each of the three blocks for a very short time interval. The force is exerted at a different point on each block, as shown.

After each force has stopped acting, which block's center of mass will have the greatest speed?

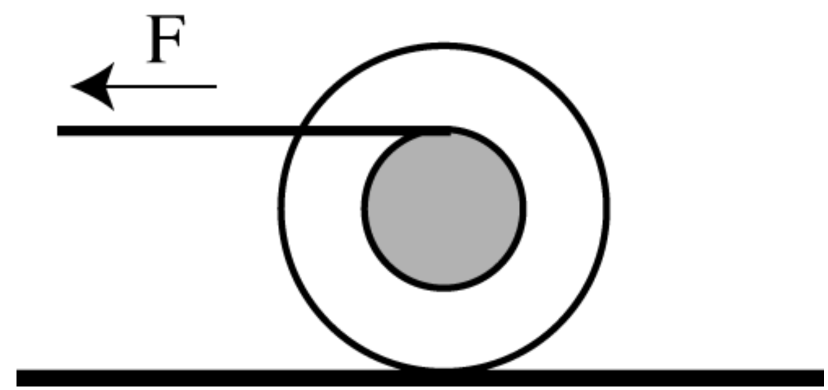


Top-view diagram

1. A.
2. B.
3. C.
4. A, B, and C have the same C.O.M. speed.

15-1.5: A ribbon is wound up on a spool. A person pulls the ribbon as shown.

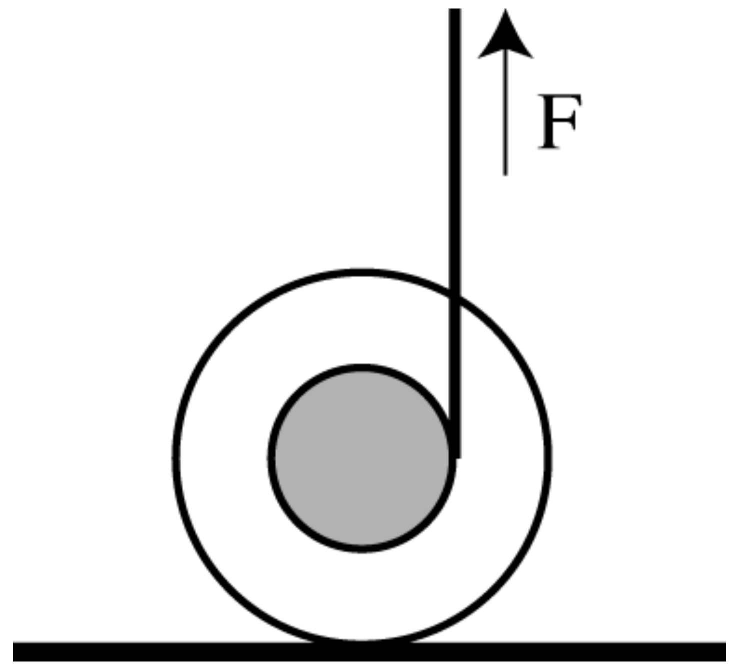
Will the spool move to the left, to the right, or will it not move at all?



1. The spool will move to the left.
2. The spool will move to the right.
3. The spool will not move at all.

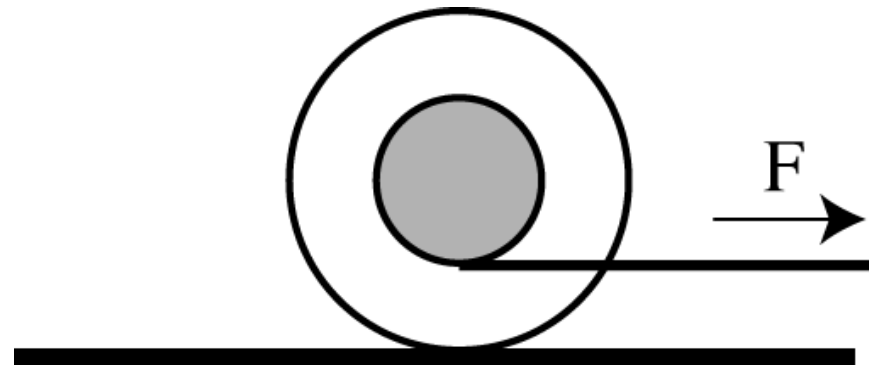
15-1.6: A ribbon is wound up on a spool. A person pulls the ribbon as shown.

Will the spool move to the left, to the right, or will it not move at all?



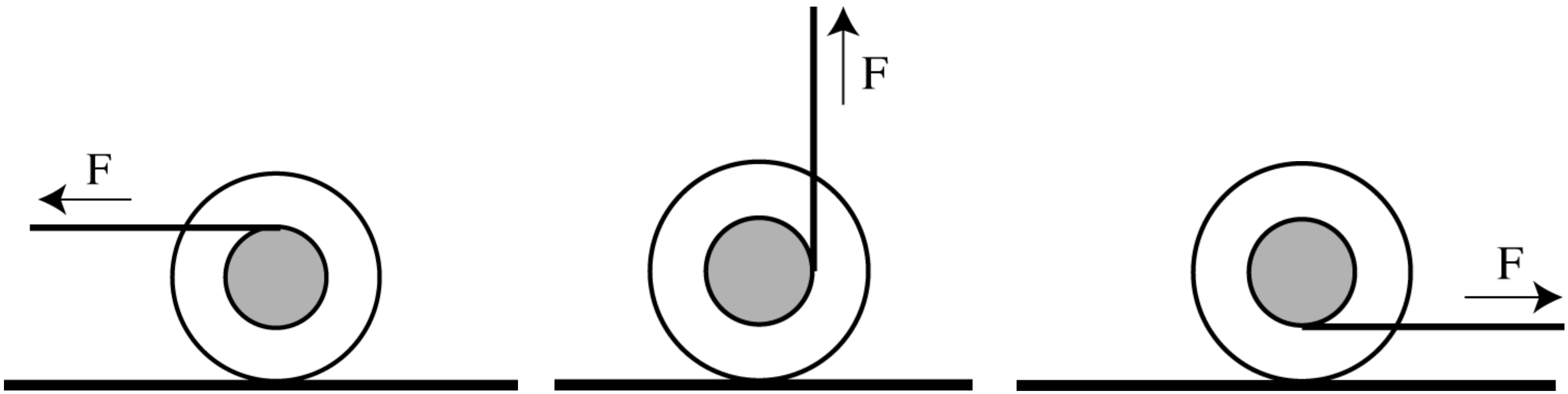
1. The spool will move to the left.
2. The spool will move to the right.
3. The spool will not move at all.

15-1.7: A ribbon is wound up on a spool. A person pulls the ribbon as shown.

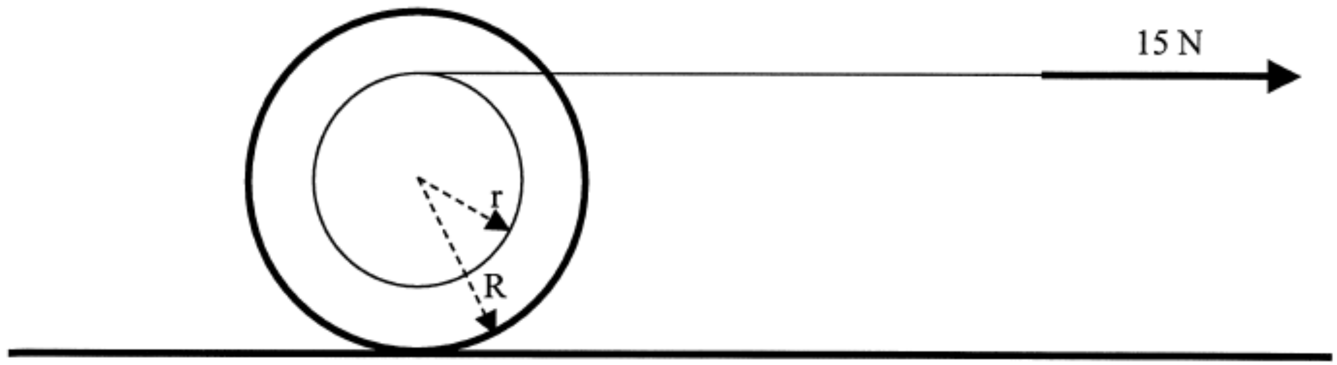


Will the spool move to the left, to the right, or will it not move at all?

1. The spool will move to the left.
2. The spool will move to the right.
3. The spool will not move at all.


F
F
F

4. [30pts total + 5 bonus pts] A string is wound around the spool. The spool has a mass of M=7 kg, an outer radius of R=0.6 m and an inner radius of r=0.4 m. Moment of inertia with respect to the axis going through the center of mass is I_{cm}=0.8 kg m^2. Somebody is pooling on the string in horizontal direction with a force of 15 N. (Parts a,b,c of this problem are independent of each other).



4a. [20pts] Find linear acceleration of the center-of-mass of the spool (a_{cm}) rolling without slipping.

4b. [10pts] What is the kinetic energy of the spool rolling without slipping if velocity of the center-of-mass is v_{cm}=3 m/s?

4c. [bonus 5pts] Find linear acceleration of the center-of-mass of the spool (a_{cm}) if it is rolling with slipping and the coefficient of kinetic friction between the spool and the ground is μ_k=0.15 (use g=10 m/s^2).

Rotational Dynamics

$$\boldsymbol{\tau} = I\boldsymbol{\alpha}$$

$$\Delta \mathbf{L}/\Delta t = \boldsymbol{\tau}$$

- These are equivalent statements

- If no net external torque: $\tau = 0 \rightarrow$
  * **L** is constant in time
  * *Conservation of Angular Momentum*
  * Internal forces/torques do not contribute to external torque.

Rotating Star Demo

Gravity

- Before 1687, large amount of data collected on motion of planets and Moon (Copernicus, Galileo, Brahe, Kepler)

- Newton showed that this could all be understood with a new ***Law of Universal Gravitation***

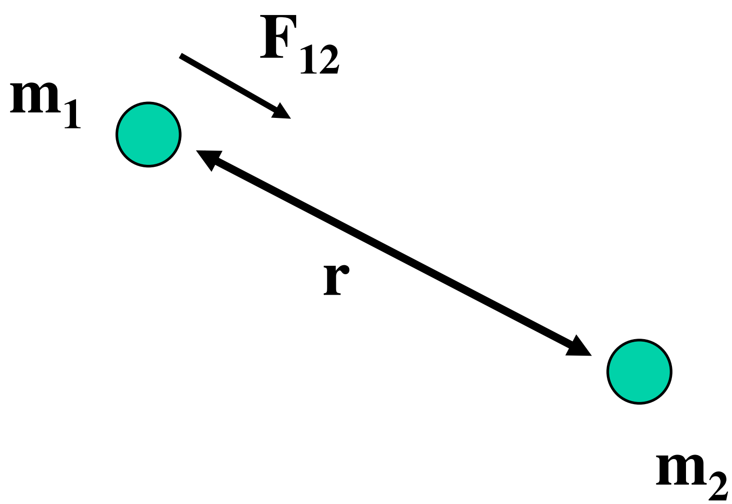
Universal Gravity

- Mathematical Principles of Natural Philosophy:

Every particle in the Universe attracts every other with a force that is directly proportional to their masses and inversely proportional to the square of the distance between them.

Inverse square law

$F = Gm_1m_2/r^2$



Interpretation

- F acts along line between bodies
- $F_{12} = -F_{21}$ in accord with Newton's Third Law
- ***Acts at a distance*** (even through a vacuum) …
- G is a universal constant = $6.7 \times 10^{-11}\ \mathrm{N{\cdot}m^2/kg^2}$

The Earth exerts a gravitational force of 800 N on a physics professor. What is the magnitude of the gravitational force (in Newtons) exerted by the professor on the Earth ?

1. 800 divided by mass of Earth
2. 800
3. zero
4. depends on how fast the Earth is spinning

Motivations for law of gravity

- Newton reasoned that Moon was accelerating – so a force must act
- Assumed that force was same as that which caused 'apple to fall'
- Assume this varies like r^{-p}
- Compare acceleration with known acceleration of Moon → find p

Apple and Moon calculation

$a_M = kr_M^{-p}$

$a_{apple} = kr_E^{-p}$

$a_M/a_{apple} = (r_M/r_E)^{-p}$

But: $a_{rad} = v^2/r$; $v = 2\pi r/T$

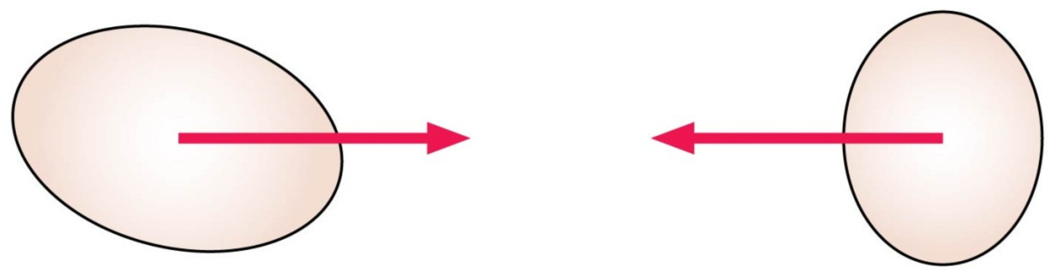
$a_M = (2\pi r_M/T)^2/r_M = 4\pi^2 r_M/T^2 = 2.7 \times 10^{-3}\ \mathrm{m/s^2}$

$a_M/a_{apple} = 2.7 \times 10^{-4}$

$r_M/r_E = 3.8 \times 10^8/6.4 \times 10^6 = 59.0 \rightarrow p = 2!$

The gravitational force between two asteroids is 1,000,000 N. What will the force be if the distance between the asteroids is doubled?

1. 250,000 N.
2. 500,000 N.
3. 1,000,000 N.
4. 2,000,000 N.
5. 4,000,000 N.



What is g ?

- Force on body close to r_E =
  $GM_Em/r_E^2 = mg \rightarrow g = GM_E/r_E^2 = 9.81\ m/s^2$
- Constant for bodies near surface
- Assumed gravitational effect of Earth can be thought of as acting at center (ultimately justified for p = 2)

Planet X has free-fall acceleration 8 m/s^2 at the surface. Planet Y has twice the mass and twice the radius of planet X. On Planet Y

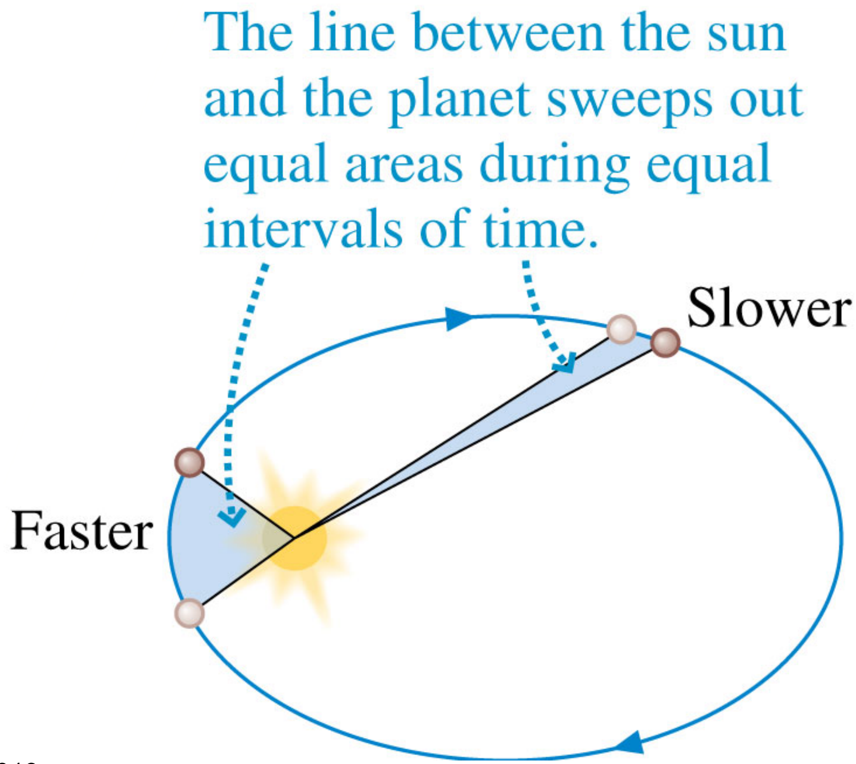
1. $g = 2 \text{ m/s}^2$.
2. $g = 4 \text{ m/s}^2$.
3. $g = 8 \text{ m/s}^2$.
4. $g = 16 \text{ m/s}^2$.
5. $g = 32 \text{ m/s}^2$.

Kepler's Laws experimental observations

1. *Planets move on ellipses with the sun at one focus of the ellipse (actually, CM of sun + planet at focus).*

Kepler's Laws experimental observations

2. A line from the sun to a given planet sweeps out equal areas in equal times.

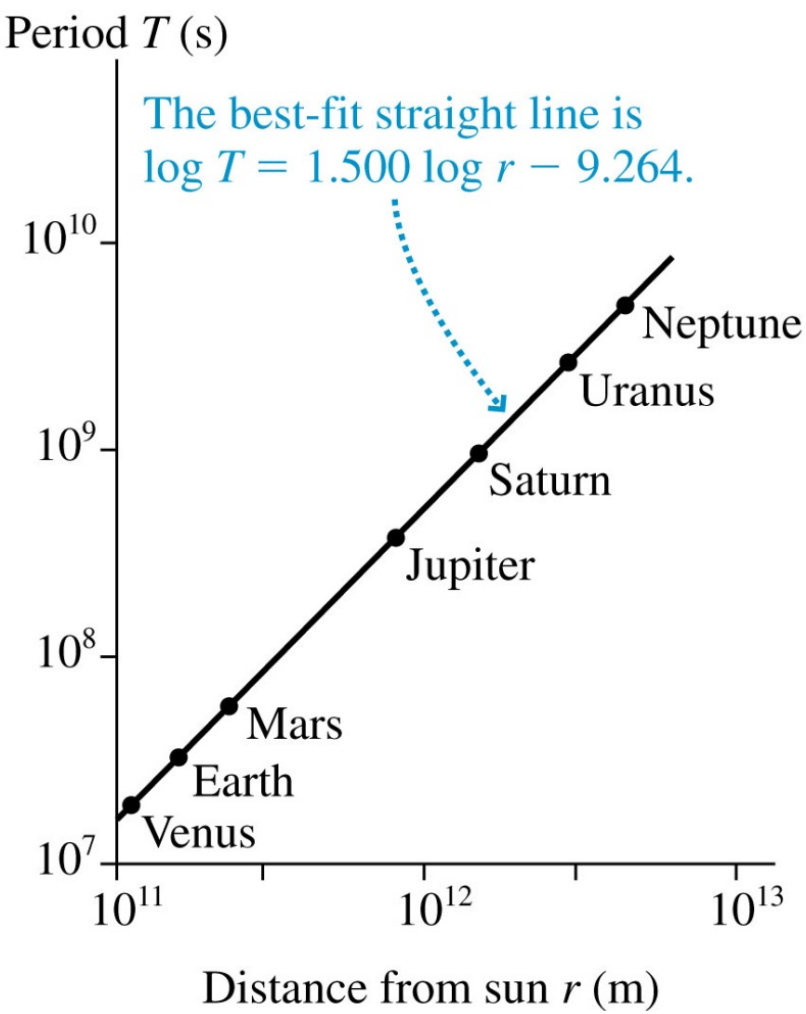


*Conservation of angular momentum

The following is a log-log plot of the orbital period (T) compared to the distance to the sun(r). What is the relationship between T and r?

1. $\log T = C\, r$
2. $T^2 = C\, r^3$
3. $T = C\, r$
4. $T^3 = C\, r^2$

For some constant C



Kepler's Laws experimental observations

3. Square of orbital period is proportional to cube of semimajor axis.

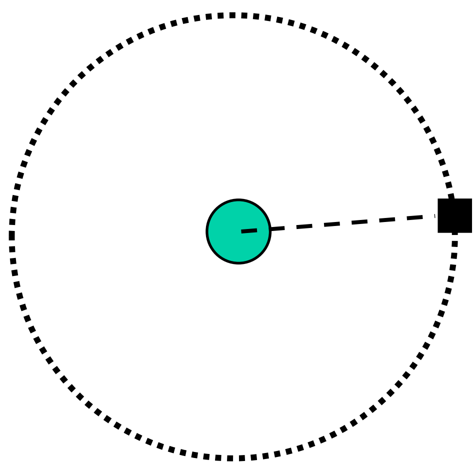
- We can actually deduce this (for ***circular orbit***) from gravitational law
- assume gravity responsible for acceleration in orbit →

Orbits of Satellites

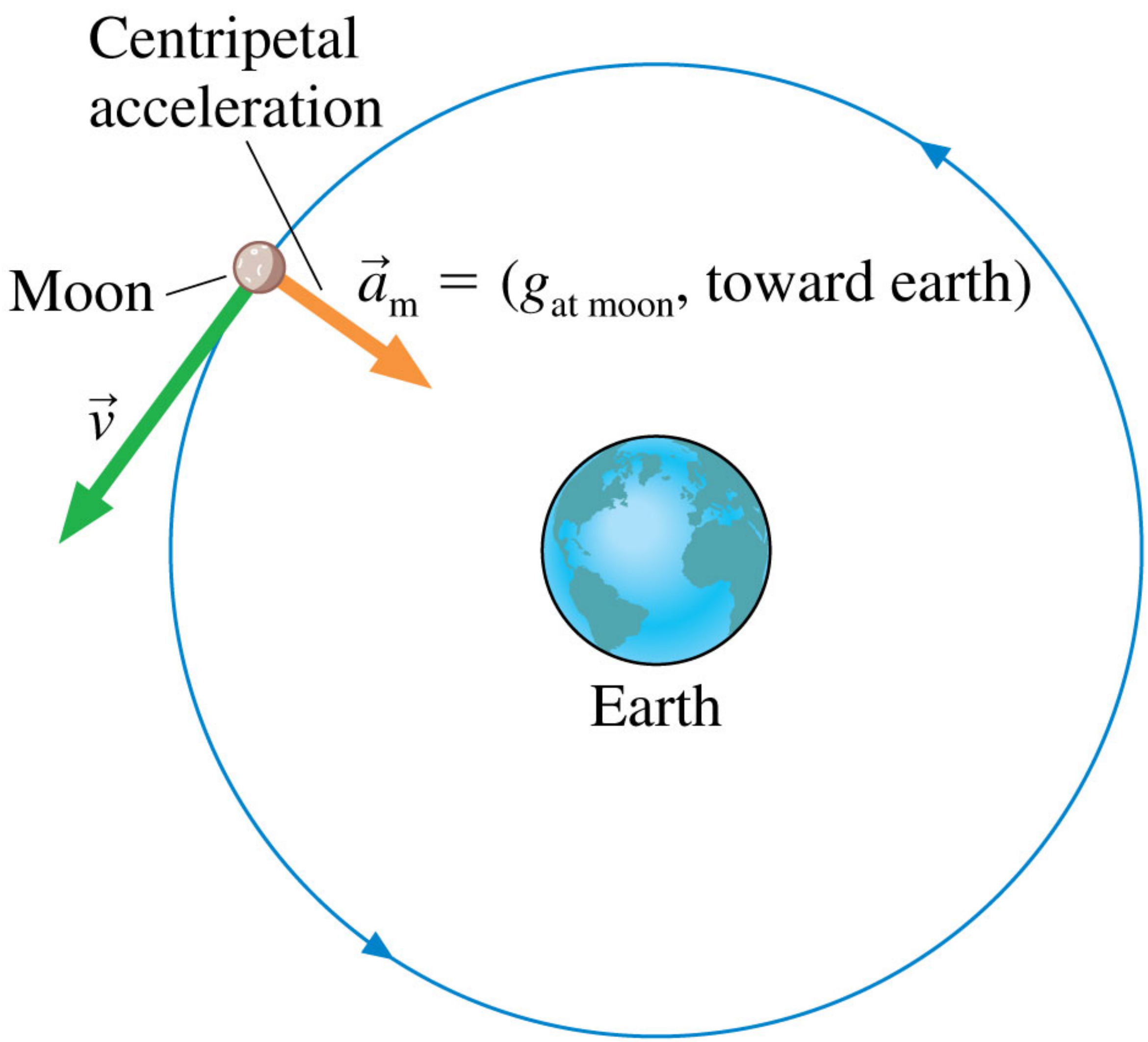
- Following similar reasoning to Kepler's 3rd law →

$$GM_E M_{sat}/r^2 = M_{sat} v^2/r$$

$$v = (GM_E/r)^{1/2}$$



The moon is in free fall around the earth.



Gravitational Field

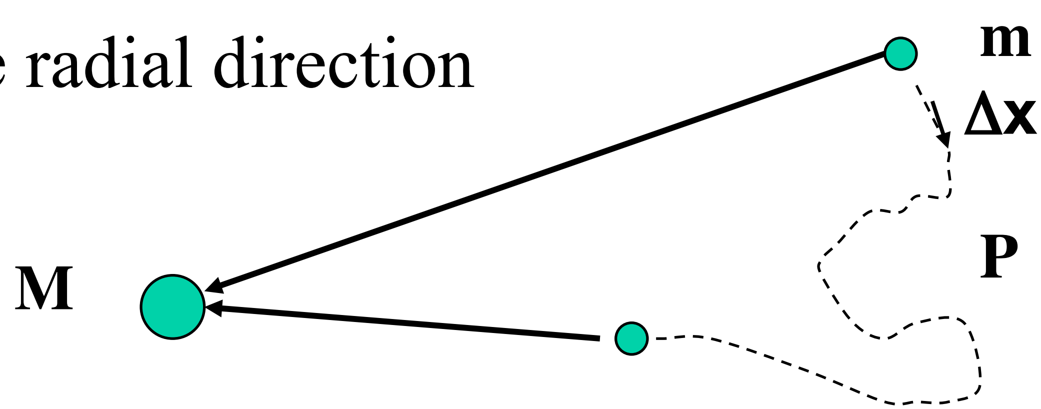
- Newton never believed in ***action at a distance***
- Physicists circumvented this problem by using new approach – imagine that every mass creates a ***gravitational field*** Γ at every point in space around it
- Field tells the magnitude (and direction) of the gravitational force on some test mass placed at that position →$F = m_{test}\Gamma$

Gravitational Potential Energy

Work done moving small mass along path P

$W = \Sigma F \cdot \Delta x$

But F acts along the radial direction



Therefore, only component of F to do work is along r

$W = - \ \Sigma F(r)\ \Delta r$

Independent of P!

Gravitational Potential Energy

Define the gravitational potential energy U(r) of some mass m in the field of another M as the work done moving the mass m in from infinity to r

➔ $U = \Sigma F(r) \Delta r = -GMm/r$

A football is dropped from a height of 2 m. Does the football's gravitational potential energy increase or decrease ?

1. decreases
2. increases
3. stays the same
4. depends on the mass of football

Gravitational Potential Energy near Earth's surface

$U = -GM_E m/(R_E+h) = -(GM_E m/R_E)\ 1/(1+h/\ R_E)$

For small $h/R_E \rightarrow (GM_E m/R_E^2)h = mgh$!!
as we expect

Called the "flat earth approximation"

Energy conservation

- Consider mass moving in gravitational field of much larger mass M
- Since $W = -\Delta U = \Delta K$ we have:

$$\Delta E = 0$$

$$\text{where } E = K+U = 1/2mv^2 - GmM/r$$

- Notice $E < 0$ if object ***bound***

Escape speed

- Object can just escape to infinite r if E=0

→ $(1/2)mv_{esc}^2 = GM_Em/R_E$
→ $v_{esc}^2 = 2GM_E/R_E$

- Magnitude ? 1.1×10^4 m/s on Earth
- What about on the moon ? Sun ?

Consequences for planets

- Planets with large escape velocities can retain light gas molecules, e.g. Earth has an atmosphere of oxygen, nitrogen
- Moon does not
- Conversely Jupiter, Sun manage to retain hydrogen

Sample problem: A less-than-successful inventor wants to launch small satellites into orbit by launching them straight up from the surface of the earth at a very high speed.

A) with what speed should he launch the satellite if it is to have a speed of 500 m/s at a height of 400 km? Ignore air resistance.

B) By what percentage would your answer be in error if you used a flat earth approximation (i.e. $U = mgh$)?

Black Holes

- Suppose light travels at speed c
- Turn argument around – is there a value of M/R for some star which will not allow light photons to escape ?
- Need $M/R = c^2/2G$ ➔ density = 10^{27} kg/m^3 for object with R = 1m approx
- Need very high densities – possible for collapsed stars