PHY 211 – Exam 1A

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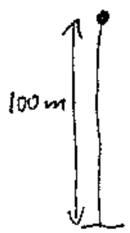
SUID: ______________

It is very important that you print your name and SUID at the top of the exam page. Please do it before you read any questions!

Document your work. Use the back of each sheet if you run out of space.

1.[25 pts total] A mortar style fireworks shell must by detonated at a height of 100 m.

a. [9 pts] If the shell is to detonate at the top of its trajectory, what must its launch speed be? Assume that it is shot straight up.



$\Delta y = 100\,m$, $a = -g = -9.8\,\frac{m}{s^2}$

$v_f = 0$

$v_f^2 = v_i^2 + 2a\Delta y$

$0 = v_i^2 - 2g\Delta y$

$\Rightarrow v_i^2 = 2g\Delta y \Rightarrow v_i = \sqrt{2g\Delta y} = \sqrt{2 \times 9.8 \times 100}$

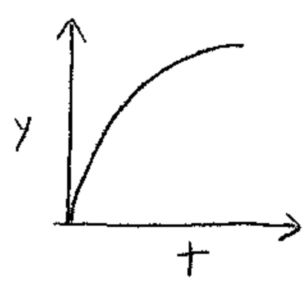
$= \boxed{44.3\,\frac{m}{s}}$

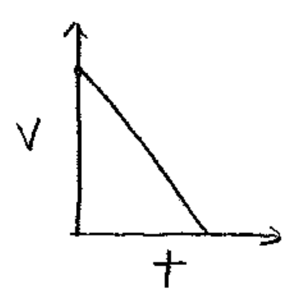
b. [8 pts] For how many seconds must the detonation fuse burn if it is lit at the instant of launch?

$v_f = v_i + at \Rightarrow 0 = v_i - gt \Rightarrow v_i = gt$

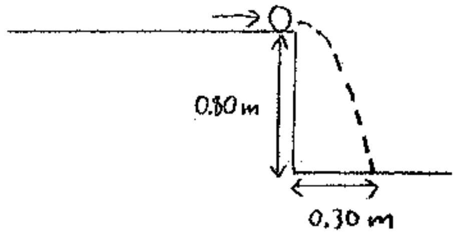
$\Rightarrow t = \frac{v_i}{g} = \frac{44.3}{9.8} = \boxed{4.52\,s}$

c. [8 pts] Draw the position versus time graph and the velocity versus time graph for the shell.





2. [25 pts total] A marble rolls off of a table and lands on the (level) floor. The edge of the table is 0.80 m above the floor. The horizontal distance between where the marble lands and the edge of the table is 0.30 m.



$V_{ox} = ?\ ,\ V_{oy} = 0$

a. [7 pts] How long does it take the marble to hit the floor?

$a_x = 0\ ,\ a_y = -g$

$y_o = 0.80\ m$

$x_f = 0.30\ m$

$x = \cancel{x_o} + V_{ox} t + \cancel{\frac{1}{2} a_x t^2}$

$y = y_o + V_{oy} t + \frac{1}{2} a_y t^2$

$0 = y_o + \cancel{V_{oy} t} - \frac{1}{2} g t^2$

$-y_o = -\frac{1}{2} g t^2 \Rightarrow y_o = \frac{1}{2} g t^2 \Rightarrow t = \sqrt{\frac{2 y_o}{g}}$

$t = \sqrt{\frac{2 \times 0.80}{9.8}} = \boxed{0.404\ s}$

b. [6 pts] What is the marble's speed when it leaves the table?

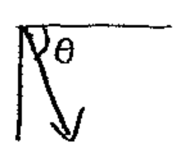
$x = V_{ox} t \Rightarrow V_{ox} = \frac{x}{t} = \frac{0.30}{0.40} = \boxed{0.75\ \frac{m}{s}}$

c. [12 pts] What is the final velocity of the marble? Give both the magnitude of the velocity and the angle that the velocity vector makes with the horizontal.

$V_{fx} = V_{ox} = 0.75\ \frac{m}{s}$

$V_{fy} = \cancel{V_{oy}} - g t = -9.8 \times 0.404 = 3.96\ \frac{m}{s}$

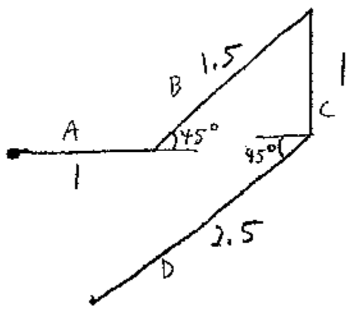
$V_f = \sqrt{V_{fx}^2 + V_{fy}^2} = \boxed{4.03\ \frac{m}{s}}$



$\tan \theta = \frac{V_{fy}}{V_{fx}} \Rightarrow \boxed{\theta = 79.3^{\circ} \text{ below x-axis}}$

3. [25 pts total] You leave your house and walk east for 1.0 h, northeast for 1.5 h, south for 1.0 h, and southwest for 2.5 h, always moving at the same speed.

a. [16 pts] Realizing it is going to get dark soon, you head directly home. How long does it take to walk directly home if your speed stays the same as it was on every leg of the walk?



$$\vec{A} = \hat{i}\ , \quad \vec{B} = \overset{1.06}{1.5\cos 45^\circ}\,\hat{i} + \overset{1.06}{1.5\sin 45^\circ}\,\hat{j}$$

$$\vec{C} = -\hat{j}\ , \quad \vec{D} = -\overset{1.77}{2.5\cos 45^\circ}\,\hat{i} - \overset{1.77}{2.5\cos 45^\circ}\,\hat{j}$$

$$\vec{A} + \vec{B} + \vec{C} + \vec{D} = 0.29\,\hat{i} - 1.71\,\hat{j}$$

$$|0.29\,\hat{i} - 1.71\,\hat{j}| = \sqrt{0.29^2 + 1.71^2}$$

$$= \boxed{1.73 \text{ hr}}$$

b. [5 pts] If the entire round-trip distance traveled is 10 km, what is your average speed in m/s?

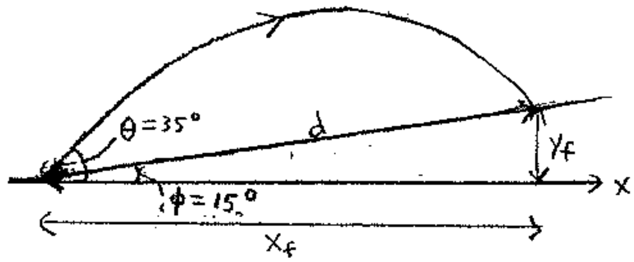
$$|\vec{V}_{avg}| = \frac{d}{\Delta t} \Rightarrow \frac{10\ \cancel{km}}{7.73\ \cancel{hr}}\ \frac{10^3\ m}{1\ \cancel{km}}\ \frac{1\ \cancel{hr}}{3600\ s} = \boxed{0.359\ \frac{m}{s}}$$

$$\Delta t_{tot} = 1 + 1.5 + 1 + 2.5 + 1.73 = 7.73\ hr$$

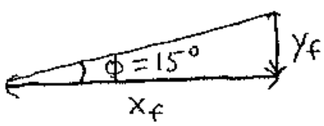
c. [4 pts] What is the average velocity of the round-trip journey?

$$\vec{V}_{avg} = \frac{x_f - x_i}{\Delta t} = \boxed{0}$$

4. [25 pts total] You launch a cannon ball at 35 m/s at θ=35^0 with respect to the horizontal at the bottom of a hill that has a slope of φ=15^0 (see figure below). Assume the ball is fired at ground level and neglect air resistance.



a. [3 pts] Express the final position in the y-direction y_f in terms of the final position in the x-direction x_f and the angle φ.



$$\frac{y_f}{x_f} = \tan\phi \Rightarrow \boxed{y_f = x_f \tan\phi}$$

b. [4 pts] Write down equations showing how the x and y coordinates of the ball change with time t.

i. x(t) = $\cancel{x_0} + V_0 \cos\theta\, t$

ii. y(t) = $\cancel{y_0} + V_0 \sin\theta\, t - \frac{1}{2} g t^2$

c. [6 pts] Using your results from parts a and b, find y_f as a function of the magnitude of the initial velocity v_0, the time t, and θ and φ. Express your result in terms of an algebraic formula with variables, rather than numbers.

$$x_f = V_0 \cos\theta\, t, \quad y_f = x_f \tan\phi \Rightarrow \boxed{y_f = V_0 t \cos\theta \tan\phi}$$

d. [8 pts] Use the previous results to solve for the time the ball is in the air. How long is this in seconds?

$$y_f = V_0 \cancel{t} \cos\theta \tan\phi = V_0 \sin\theta \cancel{t} - \frac{1}{2} g t^2$$

$$\Rightarrow \quad V_0 \cos\theta \tan\phi - V_0 \sin\theta = -\frac{1}{2} g t$$

$$t = -\frac{2}{g}\left(V_0 \cos\theta \tan\phi - V_0 \sin\theta\right) = \frac{2V_0}{g}\left(\sin\theta - \cos\theta \tan\phi\right)$$

e. [4 pts] Now compute x_f, y_f, and the distance d up the hill that the ball lands.

$$x_f = V_0 \cos\theta\, t$$

$$= 35 \cos 35° \times 2.53 = \boxed{72.5\ m} \quad \text{or } 119.8\ m$$

$$y_f = x_f \tan\phi = 72.5 \tan 15° = \boxed{19.4\ m} \quad \text{or } 32.1\ m$$

$$d = \sqrt{x_f^2 + y_f^2} = \boxed{75.1\ m} \quad \text{or } 124\ m$$

$$t = \frac{2 \times 35}{9.8}\left(\sin 35° - \cos 35° \tan 15°\right)$$

$$t = \boxed{2.53\ s} \quad \text{or } 3.25\ s$$

Formula Sheet for Physics 211

$$\vec{v} = \frac{d\vec{x}}{dt}; \quad \vec{a} = \frac{d\vec{v}}{dt}; \qquad \vec{v}_{\text{avg}} = \frac{\Delta\vec{x}}{\Delta t}; \quad \vec{a}_{\text{avg}} = \frac{\Delta\vec{v}}{\Delta t}$$

$$\Delta x = x(t_2) - x(t_1) = \text{signed area under the } v(t) \text{ curve from } t_1 \text{ to } t_2 = \int_{t_1}^{t_2} v(t)dt$$

$$\Delta v = v(t_2) - v(t_1) = \text{signed area under the } a(t) \text{ curve from } t_1 \text{ to } t_2 = \int_{t_1}^{t_2} a(t)dt$$

$$v_x = v_{0x} + a_x t; \qquad x = x_0 + v_{0x}t + \frac{1}{2}a_x t^2;$$
$$v_x^2 = v_{0x}^2 + 2a_x\Delta x$$

$$\vec{v} = v_x\hat{i} + v_y\hat{j}; \qquad v = |\vec{v}| = \sqrt{v_x^2 + v_y^2}$$

$$g = 9.8\frac{m}{s^2}$$